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# Involuntary Unemployment in a Competitive Labour Market *

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**Abstract:** High and persistent unemployment, especially in the sector of low-skilled work, is usually attributed to the presence of minimum wages or unemployment benefits creating a reservation wage at which labour demand of firms is insufficient. While not refusing this view this paper argues that also without these obstacles a situation is likely to occur which is similar to unemployment and in a rigorous sense even worse: people not finding a job in the sector of "honest work" at a wage sufficient to meet minimum consumption are forced to recourse to less desirable activities. The topic is analyzed in an OLG-context with two working periods for each generation.

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# 1 Introduction

Whenever it comes to a discussion of the underlying reasons of high and persistent unemployment, especially in the sector of low-skilled labour, the emphasis usually lies on factors like wage rigidities (enforced by government or unions or both) or unemployment benefits making it unattractive to accept a job offer below a certain wage level[1]. Typical policy implications are then a reduction of payments to the unemployed and measures to increase downward flexibility of the wage rate. Although the present paper does not in principle deny the possible meaningfulness of such measures, it points to the problem, that involuntary unemployment might even exist under the conditions of a fully flexible labour market.

Earlier approaches[2] in this regard developed in conjunction with the introduction of market power into macroeconomic models tried to show that imperfect competition on the markets for goods can lead to a situation in which labour demand is insufficient to absorb labour supply at all positive wage levels. One of the main problems with these approaches, however, is the standard assumption of a fully inelastic labour supply curve, which is, of course, unrealistic and also hard to justify from a theoretical point of view, at least for very low levels of the real wage.

In contrast to this the present paper focusses on two aspects having to do with the supply side on the labour market. The central claims in this regard are the following:

- There is a certain level of minimum consumption that cannot be dispensed with. Furthermore, there are reasons to assume that this level increases with the overall development of the economy. (Think, e.g., of train-journeys that are nowadays no longer available below a certain quality standard like second-class waggons; coaches with wooden chairs are not provided any more in many countries.)
- Even in modern, industrialized economies the majority of people does not possess enough capital to be able to finance this minimum consumption without any income from working.

The question thus arises, what happens, if also income from working (in addition to potential capital income) does not suffice to cover the minimum consumption requirements. Although such a situation is surely unlikely to occur for all types of labour, it can very well arise for skill-levels below a certain threshold. The crucial point is thus, what workers in this segment of the labour market will actually do if a situation like the one just discussed occurs. In principal, there are only four possible solutions:

[1]It is clearly impossible to give a complete list even of the most relevant literature dealing with this item. Thus, only Blanchard/Katz (1997) (especially pp. 51–54 and 66–68) and Nickell (1997) (here among others especially pp. 67–73) shall be mentioned here as two arbitrarily chosen examples in this regard.

[2]See D'Aspremont/Dos Santos Ferreira/Gérard-Varet (1989), (1990) and (1991) as the most prominent examples for this direction of research.

- To accept not to be able to finance minimum consumption.
- To try to get support from government, relatives or friends.
- To try to get self-employed.
- To try to acquire income from "alternative sources."

If now the first alternative is ruled out (since minimum consumption represents a binding constraint) and the second by assumption[3] the question remains whether becoming an entrepreneur is a reasonable and promising way out. Although this possibility should not be neglected, there are at least two issues worth mentioning. First, working as a self-employed usually requires a certain amount of physical capital in most cases. This in turn means that people not possessing enough resources (and thus collaterals) will find it difficult to acquire the necessary credits. Second, even if some people are able to solve this problem somehow, not all of them will end up as successful entrepreneurs.

For those who will not and all others who – anticipating a possible failure – did not take the risk right from the beginning only the fourth alternative remains. It is now time to become more precise about what these "alternative sources" actually mean. The basic idea behind is the simple fact that not all activities directed at the achievement of income contribute to a society's well-being. Clearly, a large variety of examples exists in this regard, ranging from legal to semi-legal exploitations of information asymmetries up to clear-cut criminal behaviour. One comparably harmless example can be seen in many advertising activities beyond the level considered useful or funny by the consumer (with marketing by telephone as one especially annoying case). Another example is the whole class of salesmen having to sell "refrigerators to Eskimos", i.e. having to convince consumers to buy things which are at least not optimal for them. Further examples include working in firms producing goods with considerable shortcomings[4] but selling them as high-quality products.

Although in real life it is often difficult to make a clear distinction between "honest" and "dishonest" work[5], it will be assumed in the following that the economy under consideration is made up by the following two sectors:

- Sector I, comprising "honest" activities enlarging the the consumption possibilities of the society and
- Sector II, comprising all activities solely directed at a deflection of income produced elsewhere in the economy into one's own pocket.

[3]Recall that support from government is intentionally neglected here for conceptual reasons, since the outcome under fully free market conditions is the issue to be investigated here.

[4]For a recent investigation on the economic damage caused by problems of this kind in Eastern Europe transition economies see Todorova (2004).

[5]A more elaborated version of the model to be developed might contain, e.g., a variety of professions each of which being characterized by a special convex combination between honest and dishonest work.

Together with the following two additional assumptions the explicit consideration of such a "second sector" opens the door for an alternative explanation for involuntary unemployment:

- It is always possible to gain enough income in the second sector to survive. This includes a sufficient "wage rate" in this sector as well as unconstrained employment opportunities[6].
- People have a strong moral aversion against working in the second sector and will do so only if otherwise they cannot finance the subsistence level of consumption.

If now a situation occurs in which the wage rate in sector I is at the subsistence level but labour demand at that rate is insufficient to absorb the entire labour supply, part of the working population will be forced to recourse to sector II. From the society's point of view this is in no way better than a situation where these people were unemployed and receiving benefits. Furthermore, workers being engaged in sector II would strictly prefer a job in sector I. Bringing both aspects together, there is obviously an excess supply of labour in the emerging equilibrium if the notion of "labour" is confined to "honest" activities. Note furthermore, that the wage rate in sector I cannot fall, since in this case nobody could afford to work in this sector any more; the resulting excess demand for labour would then immediately drive up the wage rate to its previous level.

The question, however, remains whether the problem just described still prevails if one takes into account that the labour market does not open only once during an individual's life time. Obviously, in a multi-period context it is not cogent to immediately turn to the second sector in the case of being unemployed in sector I in one period or facing a very low wage rate there. If one had the opportunity to accumulate enough capital in the past (or if one expects to able to do so in the future) it is also possible to abstain from working at all for a number of periods. On the other hand it is clear that this number must not be too large. Thus, in how far the possibility exists to avoid working in sector II at all depends on the time path of employment opportunities in the first sector and the associated sequence of wages which can be earned there. These magnitudes, in turn, are influenced by the labour supply decisions of the individuals just sketched and therefore to be treated as endogenous variables. In order to capture these aspects while keeping things otherwise as simple as possible the model outlined so far will be embedded into an OLG-context with two working periods for each individual.

Despite its importance for a society's welfare the literature on the topic introduced here seems to be rare. Although its single ingredients like honest or dishonest behaviour or the implications of minimum consumption constraints are meanwhile widely discussed within other contexts (like, e.g., in conjunction with growth models with regard to the latter) the concrete problem highlighted in this paper has apparently not attracted so much attention. What still comes closest to the approach developed here are models dealing

[6]Future extensions of the model might weaken this assumption by assuming instead something like a "production function" for income deflection in the second sector.

with different aspects concerning the intertemporal allocation of labour supply under the presence of unemployment like, e.g. Snow/Warren (1991) or Yaniv (1991), which are at least remotely related to the topic under discussion.

In the next section the basic argument outlined so far will be given a formal content, first within a simple one-period model. The extension to an OLG-model will then take place in section 3 in which the microeconomic decision problem will be dealt with. The resulting dynamics on the labour market will then be investigated in section 4. It will be shown, that despite a higher degree of complexity of the analysis the core argument can be made also in this context. Section 5 will then summarize the main results together with some proposals for future research.

# 2 The basic model

In this first section the model's core shall be presented in the simplest way possible. Therefore it will be assumed first that individuals live one period only and that different generations enter the scene one after the other without any connection between them. The production technology involves three factors: low-skilled labour, high-skilled labour and physical capital. The division of the total labour force (the size of which is assumed to be constant over time) between the two groups is given exogenously. Capital is assumed to be perfectly internationally mobile, whereas the same is neither the case for unskilled nor for skilled labour, which are bound to stay in their home country. The main reason for this three factor approach is to confine the kind of involuntary unemployment to be investigated here to the sector of low-skilled labour since arguments involving minimum consumption issues would be far less convincing when applied to the labour force as a whole. Furthermore, in the case of only one – homogeneous – type of labour the assumption of a small open economy ("open" with regard to the capital stock) and a correspondingly given international rate of interest $\bar{r}$ would imply a horizontal labour demand curve[7] at the wage rate determined by $\bar{r}$ via the factor price frontier. In contrast to this, the presence of an exogenously given and – by assumption – always fully employed number of high-skilled workers ensures a downward sloping demand curve for their low-skilled counterparts.

Preferences are given as follows. Utility of a typical household is assumed to depend in a linear way[8] on consumption $c$ and disutility from working $l$:

$$U \quad = \quad \gamma c - l, \quad \gamma > 0. \tag{1}$$

[7] Note that the same problem would arise in the long run equilibrium of a closed economy in which the rate of interest had to coincide with the – parametrically given – individual's rate of time preference (enlarged by the rate of depreciation, population growth and technological progress, depending on the concrete model).

[8] Since the approach developed here will be embedded into an OLG-structure in the next section it is recommendable to keep things as easy as possible because already the introduction of wage-dependent labour supply into an otherwise standard OLG-model alone leads to considerable complexities (see Reichlin (1986) or Nourry (2001) for extensions in this regard).

In order to ease the subsequent expositions it is assumed that each individual has one unit of labour at his disposal which has to be fully allocated either to the first or the second sector or to leisure. Furthermore, utility maximization has to take place under the constraint that a certain minimum consumption $\bar{c}$ has to be met under all circumstances, independently of the individual's preferences. Let now $w^{(I)}$ be the real wage rate for low-skilled labour in the first sector and $\bar{w}^{(II)}$ the corresponding rate in the second sector which is supposed to be constant and larger than $\bar{c}$. For sake of simplicity it is also assumed that the fraudulent behaviour in sector II is entirely at the expense of the group of high-skilled workers (the real wage "net of fraud" of which can be ensured to be well above the maximum of $w^{(I)}$ and $\bar{w}^{(II)}$ by an appropriate choice of the production function).

Denoting now as $x$ and $y \in \{0; 1\}$ the individual's choice variables by which the chosen alternative ("working in sector I" and "working in sector II") is switched on (and the other one switched off) and letting $b$ and $d >> b$ represent disutility from working in sector I and sector II, respectively, the utility maximization problem for a low-skilled worker can now be formulated as follows:

$$\max_{x,y} \quad U \quad = \quad \gamma\left(x \cdot w^{(I)} + y \cdot \bar{w}^{(II)}\right) - x \cdot b - y \cdot d \tag{2}$$

$$\text{s.t.}: \quad x \cdot w^{(I)} + y \cdot \bar{w}^{(II)} \geq \bar{c} \tag{3}$$

$$x, y \in \{0; 1\} \quad \text{and} \quad x + y \leq 1 \tag{4}$$

The last constraints ensure that for $(x, y)$ only the combinations $(1, 0), (0, 1)$ and $(0, 0)$ are admissible. Thus, if one decides, e.g., to work in sector I ($x = 1$), one cannot work in sector II as well ($\Rightarrow y = 0$) and vice versa. Of course it is also possible not to work at all ($x = y = 0$), but this alternative would violate condition (3).[9] Whether this condition can be fulfilled by exercising a job in sector I depends on the current wage rate there and can therefore not be decided in general. On the other hand it shall be assumed that the real income attainable in sector II is always larger than the minimum consumption requirement:

$$\bar{w}^{(II)} \quad > \quad \bar{c} \tag{5}$$

The crucial assumption for the subsequent considerations is now that individuals are strongly reluctant to work in sector II and will do so only if otherwise the minimum consumption requirement cannot be met. In formal terms this can be expressed as follows:

$$\gamma w^{(I)} - b \quad > \quad \gamma \bar{w}^{(II)} - d \tag{6}$$

for all values of $w^{(I)}$ being larger or equal to $\bar{c}$.

Given now a production technology as described above and a labour supply in the low-skilled sector of sufficient size, a situation like the one depicted in Figure 1 can occur:

[9] It will become admissible for a single period, however, in some cases of the model's extended version to be developed in section 3.

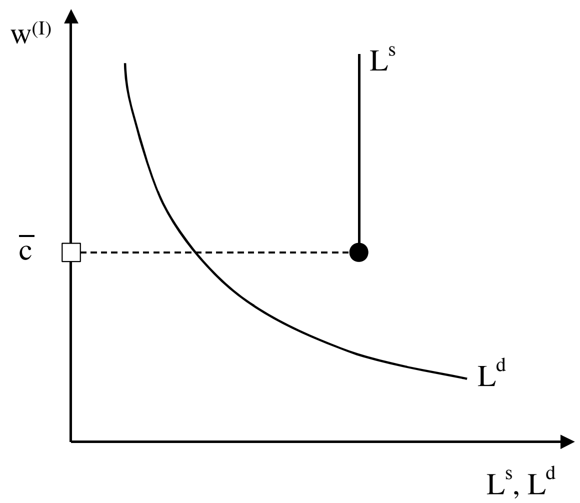

Figure 1

Obviously, labour demand is insufficient at wage rate $w^{(1)} = \bar{c}$, so that people not finding an employment opportunity in sector I are forced to work in sector II although they would strictly prefer a job in the first sector despite the lower wage there. On the other hand, the wage rate in sector I cannot fall below $\bar{c}$, since otherwise nobody could afford to work in this sector any more. The sudden decline of labour supply to zero in this case would then immediately lead back to the previous wage rate.

As already mentioned in the introduction the equilibrium is characterized by involuntary unemployment for two reasons. First, although people being engaged in sector II are not unemployed in the sense that they would not do anything, the type of work exercised in that sector is surely not in the interest of society. Giving these people tax-financed unemployment benefits in the size of what they actually get by their activities would have the same effect on the income of the groups involved and economize on disutility from working[10] in sector II. Second, at the equilibrium wage rate $\bar{c}$ all individuals working in sector II would strictly prefer a job in sector I.

Clearly the model described in this section is quite simple, thereby neglecting many aspects that might be of importance and influence the outcome. From the many extensions worthwhile and necessary to be investigated only one shall be discussed here. It deals with the question, how the results will be affected if the labour market opens more than one time for one generation. A natural starting point in this regard is to embed the approach outlined here into an OLG-model with two working periods. The next section will show that despite a considerably higher degree of complexity the fundamental result will survive.

[10]Note that the possibility of additional damages caused by sector II as far as purely criminal activities are concerned are not taken into account in this model. Thus, the current approach still underestimates the welfare losses associated with the second sector.

# 3 The OLG-variant of the model

It is now assumed that individuals work for two periods and, for sake of simplicity, that the entire consumption takes place at the very end of this time horizon. Alternatively, one might think of a third period of life in which people only consume but are no longer involved in the economic process. Recall in conjunction with this the assumption of a small open economy with perfect international mobility of capital, so that saving of the young and dissaving of the old generation do not affect the capital stock used in production.

Denoting with $\theta$ a representative individual's rate of time preference and with subindex $i \in \{1; 2\}$ the corresponding period of life, utility as of period 1 is now given by:

$$U = \frac{\gamma}{1+\theta} \cdot c - l_1 - \frac{1}{1+\theta} l_2 \tag{7}$$

Although the optimization problem for a low-skilled worker is similar to that in section 1, one has now to keep in mind that in the presence of unemployment in the second period of life there is uncertainty on whether a job in sector I will then be available or not. The decision problem to be solved first – to seek an employment opportunity in period 1 in the first sector or not[11] – has therefore to be formulated as follows (with $x_i$ and $y_i, i \in \{1; 2\}$ denoting the "switching variables" as defined in section 2 for the first and second working period, respectively):

$$\begin{aligned} \max_{x_1, y_1, x_2, y_2} E(U) = p_2 \cdot \Bigg( & \frac{\gamma}{1+\theta} \cdot \Big[(1+\bar{r})(x_1 w_1^{(I)} + y_1 \bar{w}^{(II)}) + x_2 w_2^{(I)} + y_2 \bar{w}^{(II)}\Big] \\ & - x_1 b - y_1 d - \frac{1}{1+\theta} x_2 \tilde{b} - \frac{1}{1+\theta} y_2 \tilde{d} \Bigg) \\ + (1-p_2) \cdot \Bigg( & \frac{\gamma}{1+\theta} \cdot \Big[(1+\bar{r})(x_1 w_1^{(I)} + y_1 \bar{w}^{(II)}) + y_2 \bar{w}^{(II)}\Big] \\ & - x_1 b - y_1 d - \frac{1}{1+\theta} y_2 \tilde{d} \Bigg) \end{aligned} \tag{8}$$

$$\text{s.t.:} \quad (1+\bar{r})(x_1 w_1^{(I)} + y_1 \bar{w}^{(II)}) + x_2 w_2^{(I)} + y_2 \bar{w}^{(II)} \geq \bar{c} \tag{9}$$

$$(1-p_2) \cdot \Big[(1+\bar{r})(x_1 w_1^{(I)} + y_1 \bar{w}^{(II)}) + \bar{w}^{(II)}\Big] \geq (1-p_2) \cdot \bar{c} \tag{10}$$

$$x_i, y_i \in \{0; 1\} \quad \text{and} \quad x_i + y_i \leq 1, \quad i = 1, 2 \tag{11}$$

The variable $p_2$ denotes the probability of getting a job offer in sector I in the second period of life. Under the additional assumptions that

- the wages and employment probabilities in the second period of life are perfectly anticipated by each individual and
- the employment probability is the same for each individual and thus equal to the employment rate[12]

[11] Alternatively one could ask, whether a job offer in sector I in the first period of life should be accepted or not.

[12] Note the variety of possible extensions of this approach if this simplifying assumption is relaxed.

the variable $p_2$ could be replaced by $\frac{L(w_2^{(I)})}{L_2^s(w_2^{(I)})}$ with the numerator denoting demand in sector I for unskilled labour in the second period of life, given a wage rate of $w_2^{(I)}$, and the denominator representing the corresponding labour supply.

The objective function is the expected utility as of the first period of life, provided that in the case of $x_1 = 1$ (and correspondingly $y_1 = 0$) the maximizing individual really gets a job offer in the first period. The variables $b$ and $d$ denote disutility from working in sector I and II in the first working period as before; $\tilde{b}$ and $\tilde{d}$ are the corresponding counterparts for the second period and can be larger, equal or smaller than $b$ or $d$.

The first constraint (9) simply states that in the case where the individual's plans can actually be realized the minimum consumption requirement must be met. According to the second constraint (10) the same has also to be true in the case that the individual is involuntarily unemployed in the second period and thus forced to work in sector II. The factor $(1 - p_2)$ is pre-multiplied on both sides in order to switch off this constraint if full employment prevails in the second period, i.e. if $p_2 = 1$ so that a job offer at $w_2^{(I)}$ is guaranteed. This is important in cases where $w_2^{(I)} > \bar{w}^{(II)}$ and $x_2 = 1$ (and thus $y_2 = 0$), since here condition (10) is sharper than (9). For $p_2 = 1$, however, this constraint is not valid and must therefore be prevented from becoming binding in this case. Note furthermore, that in the case where the individual decides not to work in the second period at all (i.e. $x_2 = y_2 = 0$) – which might be the optimal solution if $w_1^{(I)}$ is sufficiently high – condition (9) is automatically sharper than (10), so that no further action has to be taken to "switch off" this constraint for this situation.

Since for the first period there are only three different decisions possible here ($(x_1, y_1) = (1, 0)$ or $(0, 1)$ or $(0, 0)$), the easiest way to solve the problem is simply to evaluate the objective function for all three alternatives (thereby taking into account the different possible constellations for the second period) and to compare the corresponding values. This has the additional advantage, that one gets automatically also the "second best" solution if $x_1^* = 1$ (and $y_1^* = 0$) is optimal but cannot be realized due to a missing job offer in the first period.

Prior to these calculations a further condition shall be postulated which reduces the number of constellations to be considered in the following:

$$(1 + \bar{r})\bar{w}^{(II)} \quad < \quad \bar{c} \tag{12}$$

Thus, it is assumed that working in the first period of life in sector II while not working at all in the second period does not suffice to finance minimum consumption. Note, that this also implies $\bar{w}^{(II)} < \bar{c}$, i.e. the reverse constellation (not working in the first period and working in sector II in the second) is not admissible, too. Together with condition (11) ($x_i + y_i \leq 1, i = 1, 2$) which states that being employed in one sector excludes a

Thus, one might think of versions where young people have a higher probability of getting a job offer than old ones (or vice versa) or where people having already been employed in their first period of life have a higher chance of keeping their job than an unemployed to become employed the first time.

simultaneous employment in the other this reduces the number of admissible solutions from 16 to 6: $(x_1, y_1, x_2, y_2) = (1,0,1,0), (1,0,0,1), (0,1,1,0), (0,1,0,1), (1,0,0,0)$ and $(0,0,1,0)$. Note that the last constellation requires a sufficiently high wage rate in sector I in the second period of life as well as the absence of unemployment at that time (since otherwise there would be a risk of not getting $w_2^{(I)}$). It will be assumed here, however, that at wage rates allowing members of one generation to abstain from working in their first period of life labour demand in sector I falls short of $N$ so that full employment in the second period of life cannot be established. Thus, also alternative $(0,0,1,0)$ can be ruled out. This means in particular that working in the first period of life cannot be dispensed with so that in the case of no job offer in sector I the individual is forced to immediately work in sector II.

Although it has been assumed that working in sector II in one period only while not working at all in the other does not suffice to finance $\bar{c}$, the latter shall be possible if one is engaged in this sector in both periods, i.e.:

$$(1+\bar{r})\bar{w}^{(II)} + \bar{w}^{(II)} \quad = \quad (2+\bar{r})\bar{w}^{(II)} \quad > \quad \bar{c} \tag{13}$$

For the following evaluations it is now advisable to consider two possible constellations concerning $w_2^{(I)}$, the real wage rate in sector I in the second period of life, separately:

<u>Case I</u>: $\quad w_2^{(I)} \geq \bar{w}^{(II)}$

In conjunction with case I employment opportunities in the second period of life are essential. If unemployment prevails at that time, the maximizing individual has to take into account the possibility of getting no job in sector I in the second working period. In this case the maximal income attainable in that period is $\bar{w}^{(II)}$ to be earned in sector II. This in turn sets a lower limit for the acceptance of a job in sector I in the first period which is denoted by $w_a^r$ in the following:

$$(1+\bar{r})w_a^r + \bar{w}^{(II)} \quad = \quad \bar{c} \quad \Longleftrightarrow \quad w_a^r \quad = \quad \frac{\bar{c} - \bar{w}^{(II)}}{1+\bar{r}} \tag{14}$$

If $w_1^{(I)}$ falls short of this limit, the individual has to take up a job in sector II in the first period, even if he is offered a job in sector I. Thus, $w_a^r$ is a reservation wage for a young worker stemming from the minimum consumption requirement. With regard to condition (13) it follows immediately that $w_a^r < \bar{w}^{(II)}$. This, however, means that from a purely financial point of view there is an incentive to work in sector II instead of sector I already in the first period as far as $w_1^{(I)}$ is smaller than $\bar{w}^{(II)}$. If, however, disutility from working in sector II is sufficiently large, this incentive vanishes. In particular, if

$$d - b \quad > \quad \frac{\gamma}{1+\theta}(1+\bar{r})(\bar{w}^{(II)} - w_a^r) \tag{15}$$

is fulfilled, a job offer in sector I will be accepted whenever $w_1^{(I)}$ exceeds $w_a^r$. Normally, a job in the second period of life (preferably in the first sector, but in the case of unemployment there in the second one) has to be accepted then as well in order to cover $\bar{c}$.

For values of $w_1^{(I)}$ exceeding a certain threshold, however, working in the second period could be dispensed with. More precisely, whenever $w_1^{(I)}$ is larger than or equal to a value $\hat{w}$ with

$$(1+\bar{r})\hat{w} \quad \geq \quad \bar{c} \tag{16}$$

the question arises whether to accept a job offer in the second period and in which sector. Here a voluntary employment in sector II shall be ruled out by the following assumption:

$$\gamma\bar{w}^{(II)} - \tilde{d} \quad < \quad 0, \tag{17}$$

i.e. the disutility from working in that sector is larger than the additional consumption it makes possible. Thus, the combination $(1,0,0,1)$ will never be chosen whenever $w_1^{(I)}$ exceeds $\hat{w}$. The question remains, however, whether a job offer in sector I should be accepted in the second working period (provided that such an offer exists). This will obviously the case if and only if

$$\gamma w_2^{(I)} - \tilde{b} \quad > \quad 0, \tag{18}$$

holds true, by which simultaneously the *unconstrained* reservation wage for the second period is defined:

$$\gamma w_2^R - \tilde{b} = 0 \quad \Longleftrightarrow \quad w_2^R = \frac{\tilde{b}}{\gamma}. \tag{19}$$

Thus, whenever $w_1^{(I)}$ equals or exceeds $\hat{w}$ and $w_2^{(II)} \geq w_2^R$ holds true, $(1,0,1,0)$ will constitute the optimal solution, provided that in both periods a job in the first sector is available. If $w_2^{(I)}$, however, falls short of $w_2^R$ but inequality $w_1^{(I)} \geq \hat{w}$ still applies, $(1,0,0,0)$ is the best choice.

The results obtained so far will undergo some changes if there is full employment in the second period[13] with the consequence that the reservation wage induced by the consumption constraint falls short of $w_a^r$ if $w_2^{(I)} > \bar{w}^{(II)}$. The new reservation wage, $\tilde{w}_a^r$, is now given by

$$(1+\bar{r})\tilde{w}_a^r + w_2^{(I)} = \bar{c} \quad \Longleftrightarrow \quad \tilde{w}_a^r = \frac{\bar{c} - w_2^{(I)}}{1+\bar{r}} \tag{20}$$

If, additionally, condition (15) is replaced by the stronger one

$$d - b \quad > \quad \frac{\gamma}{1+\theta}(1+\bar{r})\bar{w}^{(II)}, \tag{21}$$

the individual will indeed seek an employment opportunity in sector I in the first period as far as $w_1^{(I)}$ does not fall short of the new reservation wage $\tilde{w}_a^r$. A special case occurs

[13] Note that this scenario has been ruled out only for wage levels $w_2^{(I)}$ high enough to allow for leisure in the first period of life. For values of $w_2^{(I)}$ below this threshold full employment in the second period of life is still a possibility.

if the latter is smaller than zero (which is equivalent to $w_2^{(I)} > \bar{c}$). With regard to the consumption constraint it is now possible to work only in the second period of life. The unconstrained reservation wage for the acceptance of a job offer in the first period, denoted by $w_1^R$, is then given by:

$$\gamma(1+\bar{r})w_1^R - b = 0 \quad \Longleftrightarrow \quad w_1^R = \frac{b}{\gamma(1+\bar{r})} \tag{22}$$

As already mentioned above, however, a situation giving rise to a choice between working and leisure in the first period will not emerge (due to the assumed location of the labour demand curve). With regard to the reverse constellation, that $w_1^{(I)}$ allows for leisure in the second period of life, the same considerations apply as in the previous case (with unemployment in the second working period).

Summing up, one can state for case I ($w_2^{(I)} \geq \bar{w}^{(II)}$):

- If unemployment prevails in the second period and condition (15) (or even (21)) and (17) is fulfilled, then $(x_1, y_1, x_2, y_2) = (1, 0, 1, 0)$ (or $(1, 0, 0, 0, )$ if $w_1^{(1)}$ is sufficiently large and $w_2^{(I)}$ not large enough) is the best choice, provided that the wage rate $w_1^{(I)}$ equals or exceeds $w_a^r$. If there is no job offer in sector I or if $w_1^{(I)}$ falls short of $w_a^r$, however, working in sector II already in the first period is unavoidable.

- If there is full employment in the second period and condition (21) prevails, a job offer in sector I will be accepted whenever $w_1^{(1)}$ equals or exceeds $\tilde{w}_a^{(r)}$, provided that the second period's wage $w_2^{(I)}$ is not so large to allow for leisure in the first period (which has been excluded, see above). If there is no employment opportunity in sector I or if $w_1^{(I)}$ is smaller than $\tilde{w}_a^r$, working in sector 2 in the first period is necessary again.

The question, however, remains whether the reservation wage $\tilde{w}_a^r$, derived under the full-employment assumption for the second period of life, is really relevant. Provided that the *actual* wage rate $w_1^{(I)}$ in the first period is somewhere between $\tilde{w}_a^r$ and $w_a^r$[14], each member of the young generation must work also in his second period of life (according to (12) also those currently working in sector II). Due to $w_2^{(I)} \geq \bar{w}^{(II)}$ (as supposed in the case under consideration), the entire *formerly* young generation will then seek for a job in sector I. The *currently* young generation of *that* period in turn will do just the same since constellations allowing for leisure in the first period of life have been ruled out and since the assumed wage constellation makes working in sector I preferable to a job in sector II even from a purely financial point of view. (This once again stems from $w_2^{(I)} \geq \bar{w}^{(II)}$ and the fact that $w_2^{(I)}$ is $w_1^{(I)}$ in the eyes of the young generation just mentioned.)

Thus, total labour supply for sector I will then be $2N$ (with $N$ denoting the – time invariant – size of each generation), whereas labour demand (to be discussed in more detail later on) at $w_2^{(I)}$ will be assumed to be smaller. If each group is equally rationed on

[14]Note that $\tilde{w}_a^r \leq w_a^r$ must always hold true under the conditions of Case I.

the labour market, full employment for a young generation in its second period of life is impossible. This in turn means that $w_a^r$ is the unique reservation wage (enforced by the minimum consumption constraint) for a young generation with regard to a job offer in the first sector in the first period of life, provided that $w_2^{(I)} \geq \bar{w}^{(II)}$ holds true.

Case II: $w_2^{(I)} < \bar{w}^{(II)}$

It is now assumed that $w_2^{(I)}$ falls short of $\bar{w}^{(II)}$. The reservation wage induced by the consumption constraint is again given by equation (20), but now its value is larger than $w_a^r$ (the reservation wage in the case of $\bar{w}^{(II)}$ in the second period). For the further considerations it is now recommendable to make a distinction on whether $w_1^{(I)}$ is larger or smaller than $\tilde{w}_a^{(r)}$:

(i) Let first $w_1^{(I)}$ be larger than or equal to $\tilde{w}_a^{(r)}$. Since the maximum wage which can be obtained in the second period is $\bar{w}^{(II)}$ (by working in sector II), working in period 1 cannot be dispensed with if covering of $\bar{c}$ is required. The optimizing individual has now again to choose between the same five alternatives as before (provided that a job offer exists for the first period):

1. $(x_1, y_1, x_2, y_2) = (1, 0, 1, 0)$ (i.e. working in sector I in the first period of life and planning to do the same in the second period)

2. $(x_1, y_1, x_2, y_2) = (1, 0, 0, 1)$ (i.e. working in sector I in the first period of life and planning to work in sector II in the second period)

3. $(x_1, y_1, x_2, y_2) = (0, 1, 1, 0)$ (i.e. working in sector II in the first period of life and planning to work in sector I in the second period)

4. $(x_1, y_1, x_2, y_2) = (0, 1, 0, 1)$ (i.e. working in sector II in both periods of life)

5. $(x_1, y_1, x_2, y_2) = (1, 0, 0, 0)$ (i.e. working in sector I in the first period of life only and planning not to work at all in the second period; clearly, this alternative presupposes a sufficiently high wage rate in sector I in the first period).

Evaluating the objective function (8) for each alternative and comparing them pairwise then leads to the following results. First, alternative (1) is better than alternative (2) whenever

$$\tilde{d} - \tilde{b} \quad > \quad \gamma(\bar{w}^{(II)} - w_2^{(I)}) \tag{23}$$

is fulfilled. In the following, the sharper condition

$$\tilde{d} - \tilde{b} \quad > \quad \gamma \bar{w}^{(II)} \tag{24}$$

will always be assumed to prevail. From this follows immediately, that alternative (3) will be preferred to alternative (4). Note, however, that total income is smaller in alternative

(3) and does not necessarily suffice to finance $\bar{c}$. Furthermore, this alternative is dominated by alternative (1) if

$$d - b \quad > \quad \frac{\gamma}{1+\theta}(1+\bar{r})(\bar{w}^{(II)} - w_1^{(I)}) \tag{25}$$

holds true. This condition, however, is already guaranteed by inequality (21) which in turn – together with (24) – makes alternative (1) superior to alternative (4). Thus, whenever the two conditions just mentioned prevail, the maximizing individual will always try to get a job in sector I in the first and second period of life. Only if $w_1^{(I)}$ exceeds the threshold value $\hat{w}$ (according to (16)) it is possible to choose alternative (5), i.e. not to work at all in the second period. If the unconstrained reservation wage $w_2^R$ for working in the second period (as defined by (19)) exceeds $\bar{w}^{(II)}$ (as will be assumed in the following), this alternative will then also actually be chosen (due to $w_2^{(I)} < \bar{w}^{(II)}$).

(ii) Let now $w_1^{(I)}$ be smaller than $\tilde{w}_a^{(r)}$. The decisive problem is here that one must work in sector II in at least one period – even if one faces a job offer in sector I in both periods of life – since otherwise the minimum consumption $\bar{c}$ could not be financed. If $w_1^{(I)} < w_a^r$, i.e. if the wage rate in sector I in the first period of life is even smaller than the reservation wage *on the basis of* $\bar{w}^{(II)}$ *in the second period of life* it is even unavoidable to work in sector II in both periods. The important question is now, how to decide if $w_a^r < w_1^{(I)} < \tilde{w}_a^{(r)}$. In this case, from the five possibilities discussed in conjunction with case I only (2), (3) and (4) remain. It is now easy to show that alternative (2) dominates (3) whenever

$$d - b - \frac{1}{1+\theta} \cdot p_2 \cdot (\tilde{d} - \tilde{b}) \quad > \quad \frac{\gamma}{1+\theta}[(1+\bar{r})(\bar{w}^{(II)} - w_1^{(I)}) - p_2(\bar{w}^{(II)} - w_2^{(I)})], \tag{26}$$

holds true. In the following the sharper condition (obtained by setting $p_2 = 1$ on the LHS and $p_2 = 0$ on the RHS and dropping $w_1^{(I)}$ there)

$$d - b \quad - \quad \frac{1}{1+\theta}(\tilde{d} - \tilde{b}) \quad > \quad \gamma\frac{1+\bar{r}}{1+\theta}\bar{w}^{(II)}, \tag{27}$$

will always be assumed to prevail. Note furthermore, that in addition to being inferior to (2), alternative (3) is not admissible for all values of the second period's wage $w_2^{(I)}$, since for the latter being small enough the minimum consumption $\bar{c}$ can no longer be financed. In addition to this it is worth mentioning that in the special (but not unlikely) case of $b = \tilde{b}$ and $d = \tilde{d}$ the above inequality (27) is very easy to fulfill even in the case of a very small value of $\theta$; it fully suffices in this case that the difference $d - b$ is sufficiently large, i.e. that – with regard to its moral implications – working in sector II is much more unattractive than in sector I.

Finally, alternative (2) also dominates (4) in the case of

$$d - b \quad > \quad \gamma\frac{1+\bar{r}}{1+\theta}(\bar{w}^{(II)} - w_1^{(I)}). \tag{28}$$

With (27) given, however, this condition is automatically fulfilled.

Summing up, also case II (i.e. $w_2^{(I)} < \bar{w}^{(II)}$) yields $w_a^r$ as the reservation wage for the young generation, provided that conditions (24) and (27) are met. These conditions, in turn, only require that the individuals' aversion against the fraudulent type of work in sector II is sufficiently strong.

What can now be said with regard to the behaviour of the old generation in a given period, say $t$? The answer clearly depends on whether the wage earned the previous period, $w_{t-1}$[15], together with the interest payments on it, already suffices to cover $\bar{c}$ or not. If this is the case and if working in the second period of life is considered desirable at all, labour will only be supplied to the first sector, because from (23) one immediately gets:

$$\gamma w_t^{(I)} - \tilde{b} \quad > \quad \gamma \bar{w}^{(II)} - \tilde{d} \tag{29}$$

Thus, additional utility from working in sector I is larger than that from working in sector II. Furthermore, the latter is even negative according to (24). However, also a job offer in sector I will only be accepted if $w_t^{(I)}$ is larger than (or equal to) $w_2^R$, the unconstrained reservation wage for the second period as defined in (19).

If, on the other hand, wage earnings (plus interest payments) from the first period are smaller than $\bar{c}$, working in the second period is indispensable. Whether it has to take place in the first or the second sector is determined – apart from the availability of a job in sector I – by the concrete size of $w_{t-1}$ and $w_t^{(I)}$. Depending on whether an old individual has worked in the first or the second sector in his first period of life two reservation wages emerge for an acceptance of a job offer in sector I in $t$ (which constitutes the second working period here).

In the case that $w_{t-1}$ (i.e. the wage actually earned by a concrete member of the old generation in the previous period) was equal to $w_{t-1}^{(I)}$ (the equilibrium wage rate in sector I at that time) this reservation wage, denoted as $w_b^r$ in the following, is given by:

$$(1+\bar{r})w_{t-1}^{(I)} + w_b^r \quad = \quad \bar{c} \quad \Longleftrightarrow \quad w_b^r \quad = \quad \bar{c} - (1+\bar{r})w_{t-1}^{(I)} \tag{30}$$

If the individual under consideration, however, had to work in sector II during his first period of life, $w_{t-1}$ was equal to $\bar{w}^{(II)}$. The corresponding reservation wage in the second period, denoted as $w_c^r$, is then defined via:

$$(1+\bar{r})\bar{w}^{(II)} + w_c^r \quad = \quad \bar{c} \quad \Longleftrightarrow \quad w_c^r \quad = \quad \bar{c} - (1+\bar{r})\bar{w}^{(II)}. \tag{31}$$

Note, that the positivity of the RHS of (31) stems from assumption (12), according to which working in sector II in the first period alone does not suffice to cover $\bar{c}$. Furthermore, a comparison between (31) and (14) reveals that $w_c^r < w_a^r$ due to (13). In contrast to this, a similar statement is not possible in conjunction with $w_b^r$, since its size depends

[15]Note that this wage may differ between different members of the old generation, since some of them have perhaps worked in sector I in their first period of life while others had to work in sector II. Thus, $w_{t-1}$ must not be confused with the equilibrium wage rate in sector I in period $t-1$, $w_{t-1}^{(I)}$.

on $w_{t-1}^{(I)}$, the equilibrium wage in sector I of the previous period. All three reservation wages, however, are assumed to be smaller than $w_2^R$ (which can easily be achieved by an appropriate choice of $\tilde{b}$ and $\gamma$ in (19)).

# 4 The dynamics of labour market equilibria

Corresponding to the three reservation wages just derived there are three different groups of workers in a given period of time:

- Group $a$: The members of the young generation.
- Group $b$: The part of the old generation who has worked in sector I in the previous period and thus earned $w_{t-1}^{(I)}$.
- Group $c$: The part of the old generation who had to work in sector II in the previous period, thereby receiving $\bar{w}^{(II)}$.

Under certain conditions to be outlined in the following a steady state position will emerge at the subsistence level of the real wage rate in sector I, denoted by $w_{sub}$:

$$(1+\bar{r})w_{sub} + w_{sub} \quad = \quad \bar{c} \quad \Longleftrightarrow \quad w_{sub} \quad = \quad \frac{\bar{c}}{2+\bar{r}} \tag{32}$$

Thus, $w_{sub}$ is just sufficient to cover $\bar{c}$ if earned in both periods of life. With the aid of (13) one immediately gets

$$w_a^r < w_{sub} < \bar{w}^{(II)} \tag{33}$$

Let now the following situation prevail in a given period $t$:

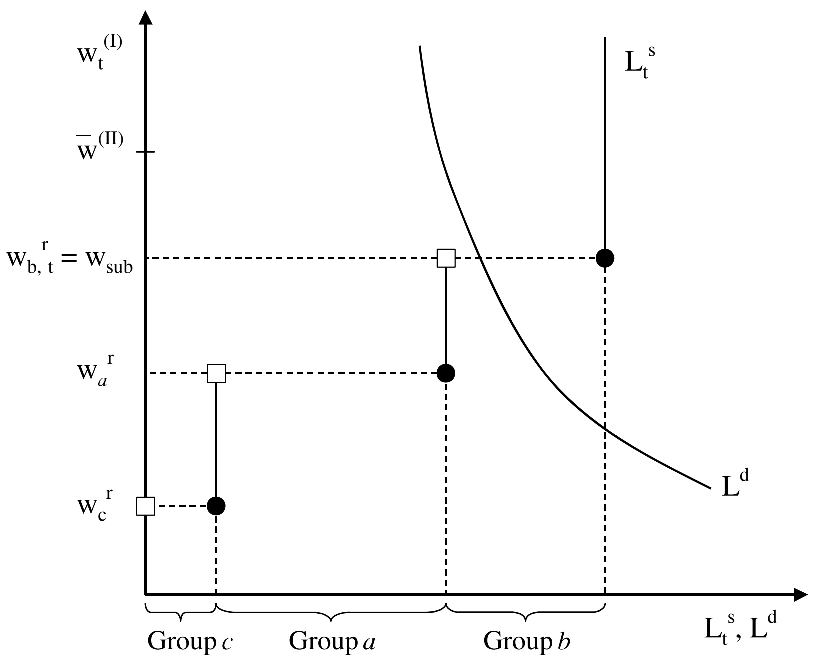


Figure 2

In this case, the equilibrium wage $w_t^{(I)}$ obviously coincides with $w_{sub}$, the subsistence wage just introduced. Note, that the picture presupposes that $w_{sub}$ is also the reservation wage of that part of the old generation who has been employed in sector 1 in $t-1$, $w_{b,t}^r$. According to (32), however, this is always the case, when $w_{sub}$ was the equilibrium wage of the previous period. Thus, the situation depicted in Figure 2 does not only presuppose $w_{sub}$ as the equilibrium wage of $t-1$, but it also means that $w_{sub}$ will be the reservation wage of "group b" in the subsequent period, $t+1$.

With $L(.)$ denoting the – time-invariant! – labour demand curve (for low-skilled labour in the first sector) and $N$ the size of one generation (as already mentioned) the employment rate in Figure 2 is given by $\frac{L(w_{sub})}{2N}$ and due to the assumption of equal rationing for all three groups of workers the size of "group b" in $t+1$ will be $\frac{L(w_{sub})}{2N} \cdot N = \frac{1}{2}L(w_{sub})$. This in turn means, that the sum of "group a" and "group c" in $t+1$ must equal $N + [N - \frac{1}{2}L(w_{sub})]$. If now the labour demand curve has the "correct" location, the equilibrium in $t+1$ will also be at $w_{sub}$ and the same picture will be repeated over and over again, thus constituting a stationary state. The only requirement in this regard is that labour demand at $w_{sub}$ is larger than the sum of "group a" and "group c", but smaller than total labour supply $2N$:

$$\begin{aligned} 2N - \frac{1}{2}L(w_{sub}) < L(w_{sub}) < 2N \quad &\Longleftrightarrow \quad \frac{4}{3}N < L(w_{sub}) < 2N \\ &\Longleftrightarrow \quad \frac{2}{3} < \frac{L(w_{sub})}{2N} < 1 \end{aligned} \tag{34}$$

Thus, an employment rate at $w_{sub}$ between $\frac{2}{3}$ and 1 is necessary and sufficient for a stationary state at that wage rate.

The next questions to be answered are now, whether this steady state is unique and stable. As the subsequent considerations will show, neither the first nor the second property is fulfilled. In order to show this, it is essential to see that each possible constellation falls within one of the following four, mutually exclusive cases. The first distinction to be made in this regard concerns the question, whether $w_b^r$ is larger or smaller than $w_a^r$:[16]

[16]Recall that – in contrast to the constant values of $w_a^r$ and $w_c^r$ – $w_b^r$ does not solely depend on the model's parameters but also on the size of $w_{t-1}^{(I)}$.

Case $A$: $0 < w^r_{b,t} \leq w^r_a$

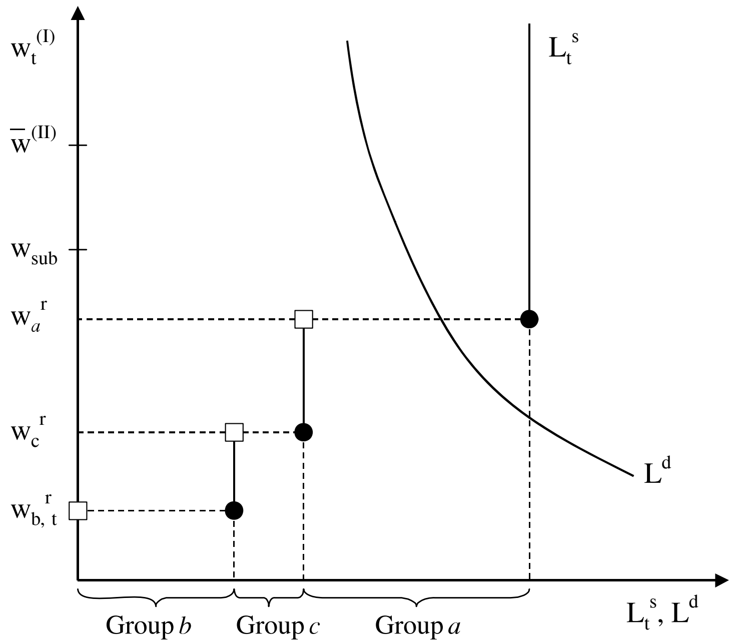


Figure 3

Note that $L(w^r_a) > N$ already follows from (34) and $w^r_a < w_{sub}$. On the other hand, the additional assumption

$$L(w^r_a) < 2N \tag{35}$$

has been made here. Note furthermore, that the location of the equilibrium[17] and the corresponding wage rate will not change if $w^r_{b,t}$ is larger than $w^r_c$ or even equal to $w^r_a$. As can be directly be obtained from Figure 3, the equilibrium wage rate $w^{(I)}_t$, the employment rate of the young generation in sector 1, $V_t$, as well as the reservation wage of "group b" in the next period, $w^r_{b,t+1}$, are given as follows:

$$w^{(I)}_t = w^r_a, \quad V_t = \frac{L(w^r_a)}{2N}, \quad w^r_{b,t+1} = \bar{c} - (1+\bar{r})w^r_a = \bar{w}^{(II)} \tag{36}$$

Case $B$: $w^r_{b,t} > w^r_a$

In contrast to the previous situation, this constellation allows for three different subcases, which have to be treated separately. From a formal point of view, the distinction is best

[17]Of course the term "equilibrium" is somewhat problematic in this context since it describes a situation with excess supply on the labour market. (Note in this regard, that people not finding a job in sector I are not indifferent between having a job there or not). Thus, the expression "equilibrium" has to be understood here as the wage and employment rate that finally emerge under the labour supply and demand schedule given.

made with the aid of that wage rate $w_{ac,t}$ at which labour demand $L$ equals the combined size of “group a” and “group c” in the current period:

$$L(w_{ac,t}) = \underbrace{N}_{\text{group a}} + \underbrace{(1 - V_{t-1})N}_{\text{group c}} \tag{37}$$

Note, that this definition makes $w_{ac,t}$ a function of $V_{t-1}$, the employment rate of the previous period: $w_{ac,t} = w_{ac,t}(V_{t-1})$.

<u>Subcase $B1$</u>: $w_{ac,t} < w^r_a < w^r_{b,t}$

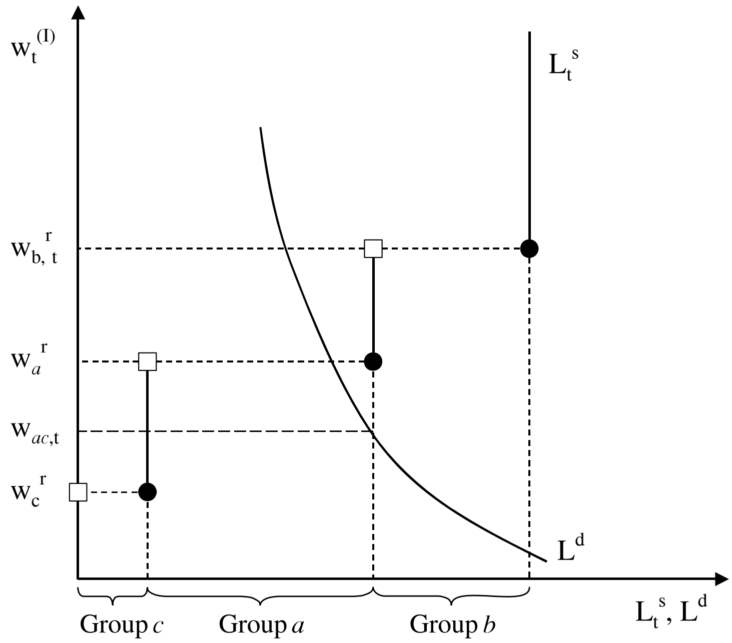


Figure 4

In this case, the unemployment rate in the previous period $t-1$ was so large, that the combined labour supply of the resulting “group c” and the “new” “group a” exceeds labour demand at $w^r_a$, which again turns out to be the equilibrium wage in the current period $t$. It is important to note that this wage rate is too low to allow “group b” to cover $\bar{c}$, so that the members of the latter are now forced to work in sector II at $\bar{w}^{(II)} > w^r_a$. This fact has to be taken into account when determining the employment rate of the young generation, $V_t$:

$$w_t^{(I)} = w^r_a, \quad V_t = \frac{L(w^r_a)}{N + (1 - V_{t-1})N}, \quad w^r_{b,t+1} = \bar{c} - (1 + \bar{r})w^r_a = \bar{w}^{(II)} \tag{38}$$

Subcase $B2$: $w_a^r \leq w_{ac,t} < w_{b,t}^r$

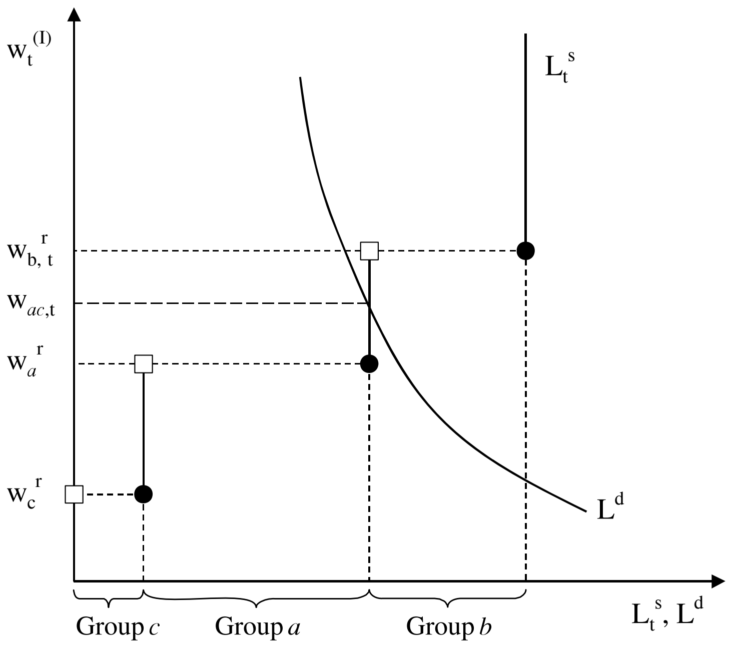


Figure 5

"Group c" is now small enough (and $w_{b,t}^r$ large enough) to allow for full employment of this group and "group a". Once again, "group b" cannot afford to work in sector I at the resulting equilibrium wage rate which now coincides with $w_{ac,t}$:

$$w_t^{(I)} = w_{ac,t} \geq w_a^r, \quad V_t = 1, \quad w_{b,t+1}^r = \bar{c} - (1+\bar{r})w_{ac,t} \leq \bar{w}^{(II)} \tag{39}$$

Subcase $B3$: $w_a^r < w_{b,t}^r \leq w_{ac,t}$

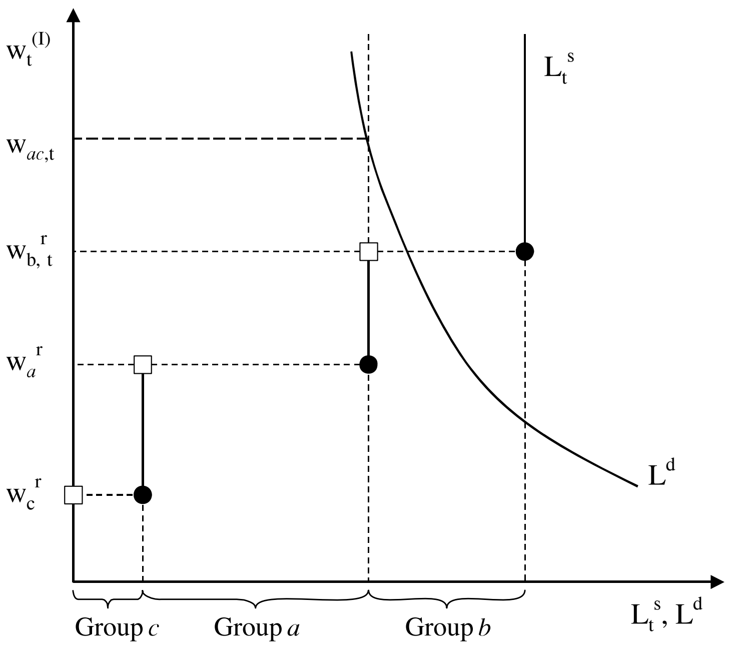


Figure 6

The sizes of “group c” and $w^r_{b,t}$ are now so small, that labour demand at $w^r_{b,t}$ equals or exceeds the combined labour supply of “group a” and “group c”. The resulting equilibrium values are now:

$$w_t^{(I)} = w^r_{b,t}, \quad V_t = \frac{L(w^r_{b,t})}{2N}, \quad w^r_{b,t+1} = \bar{c} - (1+\bar{r})w^r_{b,t} \leq \bar{w}^{(II)} \tag{40}$$

Note, that the above three subcases also cover a situation in which the wage rate of the previous period, $w_{t-1}^{(I)}$, was large enough to allow “group b” to abstain from working in the current period $t$. In this case $w^r_{b,t}$, which is now zero or even negative, has only to be replaced by $w_2^R$ as defined in (19). The qualitative properties do not even change, if “group b” comprises all members of the old generation. In this case, however, situation $B1$ can no longer occur since labour supply at $w^r_a$ is now equal to $N$ (the size of “group a”) and thus smaller than $L(w^r_a)$. Recall again, that the reverse case, in which the members of a young generation do not work in their first period of life in expectation of a sufficiently high wage rate in connection with full employment in the next period has been ruled out by the assumption that labour demand does not allow for full employment at such wages.

As the above considerations already indicate, the temporary equilibrium values $w_t^{(I)}$ and $V_t$ depend on the corresponding values of the previous period: $w_{t-1}^{(I)}$ determines the size of $w^r_{b,t}$ and $V_{t-1}$ the size of the current groups b and c, respectively. Case $A$ and the three subcases subsumed under case $B$ therefore constitute a nonlinear first-order system of difference equations in the two dynamic variables $w_t^{(I)}$ and $V_t$. As its formal description is not very helpful for the subsequent analysis, it is omitted here.

A further important observation can immediately be derived from the investigations carried out so far. First, one immediately gets $w_t^{(I)} \geq w^r_a \quad \forall t$. This means that the same must have been true in $t-1$ which in turn puts an upper limit on $w^r_{b,t}: \quad w^r_{b,t} \leq \bar{w}^{(II)}$. Second, provided that $w_{t-1}^{(I)}$ was not so high to allow “group b” in $t$ to abstain from working, $w_t^{(I)}$ cannot exceed the maximum of $w^r_a$ and $w^r_{b,t}$ (note in conjunction with this, that $w_t^{(I)} = w_{ac,t}$ is only possible if $w_{ac,t} < w^r_{b,t}$). Together with $w^r_{b,t} \leq \bar{w}^{(II)}$ this has the consequence

$$w^r_a \quad \leq \quad w_t^{(I)} \quad \leq \quad \bar{w}^{(II)}. \tag{41}$$

This interval for $w_t^{(I)}$ immediately raises the question about the size of labour demand at its boundaries. Whereas from (33), (34) and (35) one already gets $\frac{4}{3}N < L(w^r_a) < 2N$, the size of $L(\bar{w}^{(II)})$ is still unknown, especially whether it is larger or smaller than $N$. An answer can be obtained by the following consideration. According to (32) the value for $w_{sub}$ can be determined by the choice of the two parameters $\bar{c}$ and $\bar{r}$. If now $\bar{w}^{(II)}$ is chosen such that it coincides with $w_{sub}$, it must also be equal to $w^r_a$. From (34) one then gets immediately

$$\frac{4}{3}N \quad < \quad L(\bar{w}^{(II)}) \quad = \quad L(w_{sub}) \quad = \quad L(w^r_a) \quad < \quad 2N \tag{42}$$

and thus especially $L(\bar{w}^{(II)}) > N$. If now $\bar{w}^{(II)}$ is raised a little bit, $w^r_a$ will decline, but there will be no substantial change in the above inequality as far as the increase of $\bar{w}^{(II)}$

is not too large. Instead of (42) one gets

$$\frac{4}{3}N \quad < \quad L(\bar{w}^{(II)}) \quad < \quad L(w_{sub}) \quad < \quad L(w_a^r) \quad < \quad 2N, \tag{43}$$

which will be assumed to hold throughout the entire further analysis. Note, that this assumption also makes sense with regard to the economic story to be told, since very high values of $\bar{w}^{(II)}$ in comparison to $w_{sub}$ would make it unlikely that people decide to work in sector II only if forced by the consumption constraint to do so.

Before turning to the dynamics of the various cases discussed above it is useful first to summarize the findings obtained so far by a diagram which shows the current situation at time $t$ in dependence on the values of $w_{t-1}^{(I)}$ and $V_{t-1}$:

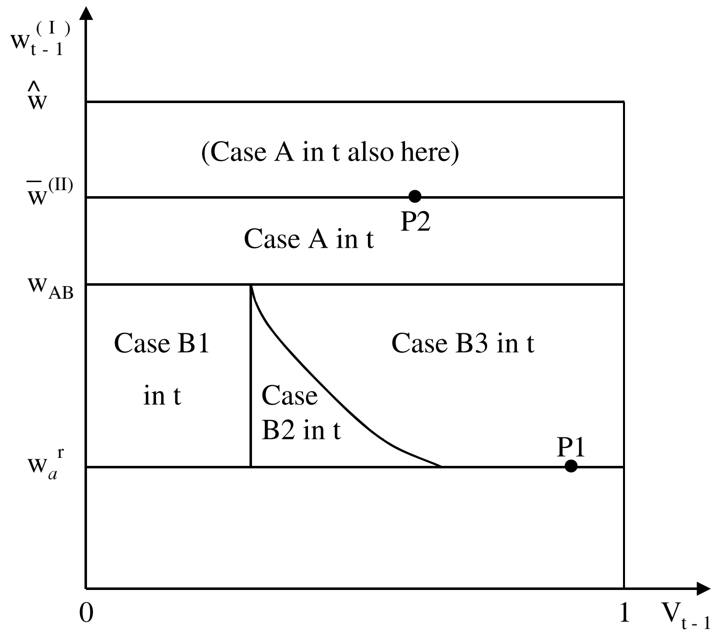


Figure 7

The borderlines between the various cases are obtained as follows. First, from the value of $w_{t-1}^{(I)}$ one directly gets the corresponding value for $w_{b,t}^r$ via (30) and thus the information whether case $A$ or case $B$ prevails. The transition between both constellations occurs at $w_{AB}$, which is defined as follows:

$$w_{b,t}^r(w_{t-1}^{(I)} = w_{AB}) = w_a^r \quad \Longleftrightarrow \quad (1+\bar{r})w_{AB} + w_a^r = \bar{c} \quad \Longleftrightarrow \quad w_{AB} = \frac{\bar{c} - w_a^r}{1+\bar{r}} \tag{44}$$

Thus, between $w_{t-1}^{(I)} = w_{AB}$ and $w_{t-1}^{(I)} = \bar{w}^{(II)}$ the reservation wage $w_{b,t}^r$ of "group b" in period $t$ falls short of $w_a^r$ so that in this region case $A$ prevails, independently of $V_{t-1}$. Note that the same is also true for values of $w_{t-1}^{(I)}$ larger than $\bar{w}^{(II)}$, provided that $w_{t-1}^{(I)}$ is not so large to allow the previously employed members of the old generation to abstain from working in $t$ (this constellation will be considered later). Recall furthermore, that

$w_{t-1}^{(I)} < w_a^r$ is not possible because in this case no member of the previously young generation could have afforded to work in sector I so that labour supply was at best equal to $N$, the size of the old generation in $t-1$ (and even this is possible only under special circumstances). As labour demand in this region is, however, larger than $\frac{4}{3}N$, the resulting excess demand must have driven up the wage rate in $t-1$ to a level equal to $w_a^r$ or above.

For values of $w_{t-1}$ between $w_a^r$ and $w_{AB}$ the value of $w_{b,t}^r$ exceeds $w_a^r$ so that one of the constellations subsumed under case $B$ must prevail. The borderlines between the three subcases $B1$, $B2$ and $B3$ are then determined as follows. From figure 4 and figure 5 as well as from (37) and the definitions of the different cases one gets for the dividing line between case $B1$ and $B2$:

$$L(w_a^r) = \underbrace{N}_{\text{group a}} + \underbrace{(1 - V_{t-1}^{B1/B2})N}_{\text{group c}} \iff V_{t-1}^{B1/B2} = 2 - \frac{L(w_a^r)}{N} \tag{45}$$

Thus, there is a unique employment rate $V_{t-1}^{B1/B2}$ at which the transition between the two subcases takes places and which is independent of $w_{t-1}^{(I)}$.

In a similar way the borderline between $B2$ and $B3$ is determined. Figure 5 and 6 and the definitions of the cases lead to:

$$\begin{aligned} L(w_{b,t}^{r,B2/B3}) &= \underbrace{N}_{\text{group a}} + \underbrace{(1 - V_{t-1}^{B2/B3})N}_{\text{group c}} \\ \iff \quad L(\bar{c} - (1+\bar{r})w_{t-1}^{(1),B2/B3}) &+ N \cdot V_{t-1}^{B2/B3} - 2N = 0 \end{aligned} \tag{46}$$

Applying the implicit function theorem yields a downward sloping curve along which this equation is fulfilled:

$$\frac{\partial LHS(46)}{\partial w_{t-1}^{(1),B2/B3}} = \underbrace{L'(.)}_{(-)} \cdot (-(1+\bar{r})) > 0; \quad \frac{\partial LHS(46)}{\partial V_{t-1}^{B2/B3}} = N > 0;$$

$$\Longrightarrow \quad \frac{dw_{t-1}^{(1),B2/B3}}{dV_{t-1}^{B2/B3}} = -\frac{(+)}{(+)} < 0$$

That this curve must intersect the "$w_a^r$-line" at a value of $V_{t-1}$ strictly smaller than 1 can be immediately seen by considering the extreme case of $w_{t-1}^{(I)} = w_a^r, V_{t-1} = 1$. Since the latter means that no "group c" exists in $t$ and the former that $w_{b,t}^r = \bar{w}^{(II)}$, subregime $B3$ must cogently prevail at this point (take again into account the assumptions on the labour demand curve made so far). Then, however, the same must still be the case for some values of $V_{t-1}$ smaller than one.

Finally, it can be shown that the borderline between $B1$ and $B2$ and that between $B2$ and $B3$ intersect at $w_{t-1}^{(1)} = w_{AB}$:

$$\begin{aligned} V_{t-1}^{B1/B2} = V_{t-1}^{B2/B3}\big|_{w_{t-1}^{(I)}=w_{AB}} \quad &\iff \quad 2 - \frac{L(w_a^r)}{N} = 2 - \frac{1}{N}L(\bar{c} - (1+\bar{r})w_{AB}) \\ &\iff \quad L(w_a^r) = L(w_a^r) \quad \square \end{aligned}$$

It is now time to consider the dynamics of the different "regimes." First, the successor of case $A$ shall be determined. From (36) it is already known that in this situation the equilibrium wage $w_t^{(I)}$ equals $w_a^r$ with the consequence of $w_{b,t+1}^r = \bar{w}^{(II)} > w_a^r$. Thus, case $A$ is succeeded by case $B$ in $t+1$. What remains to be clarified is the corresponding subcase which turns out to be $B3$. In order to see this, it suffices to show that labour demand $L$ at the wage rate $\bar{w}^{(II)}$ exceeds the size of group "a" and "c" in $t+1$. On the basis of case $A$ prevailing in $t$ group "c" in $t+1$ makes up $N - \frac{1}{2}L(w_a^r)$. Thus, it has to be demonstrated that

$$L(\bar{w}^{(II)}) \quad > \quad \underbrace{N}_{\text{group a}} + \underbrace{N - \frac{1}{2}L(w_a^r)}_{\text{group c}} \quad = \quad 2N - \frac{1}{2}L(w_a^r)$$

This, however, immediately follows from inequality (43) according to which

$$L(\bar{w}^{(II)}) > \frac{4}{3}N = 2N - \frac{2}{3}N = 2N - \frac{1}{2} \cdot \underbrace{\frac{4}{3}N}_{\overset{(43)}{<} L(w_a^r)} > 2N - \frac{1}{2}L(w_a^r) \quad \square$$

Thus, if case $A$ prevails in $t$, the economy is mapped to case $B3$ in $t+1$. More precisely, it is always mapped to a point like $P1$ in Figure 7, since $w_t^{(I)}$ was equal to $w_a^r$ in situation $A$ (note in conjunction with this, that the horizontal line at $w_{t-1}^{(I)} = w_a^r$ still belongs to the corresponding subcase of $B$).

The next question naturally concerns the successor of the situation just described, i.e. the regime reached in $t+2$. According to (40) the equilibrium wage in $t+1$, $w_{t+1}^{(I)}$, was equal to $w_{b,t+1}^r$ which in turn coincided with $\bar{w}^{(II)}$ due to $w_t^{(I)} = w_a^r$. Thus, the reservation wage of group "b" in $t+2$ must be:

$$w_{b,t+2}^r \quad = \quad \bar{c} - (1+\bar{r})\bar{w}^{(II)} \quad = \quad w_c^r$$

according to (31). Since $w_c^r < w_a^r$, the above equation directly yields $w_{b,t+2}^r < w_a^r$ so that in $t+2$ again situation $A$ prevails. Due to $w_{t+1}^{(I)} = \bar{w}^{(II)}$ this corresponds to a point like $P2$ in figure 7 (with $w_{t-1}^{(1)}$ on the vertical axis now representing $w_{t+1}^{(I)}$ from the "point of view of period $t+2$"). From this position, however, the economy will move to position $P1$ in the next period $(t+3)$ as already shown above.

These findings already lead to a first important result. Whenever the economy starts in a situation characterized by case $A$ in $t$, it will be "caught" in a period-two-cycle between point $P1$ and $P2$ from $t+1$ on. Note that in $P1$ the *current* employment rate $V_t$ is always equal to $L(\bar{w}^{(II)})/2N$ and thus only dependent on the parameters of the model. The same is true for point $P2$ where the *current* employment rate equals $L(w_a^r)/2N$. Since furthermore $L(w_a^r) > L(\bar{w}^{(II)})$ and due to the fact that on the axes in figure 7 the wage and employment rate of the *previous* period are depicted the corresponding previous employment rate $V_{t-1}$ of point $P1$ is larger than that of point $P2$.

Note, however, that although constellation $A$ always leads to point $P1$ in the subsequent period (independently of the *exact* initial position "within case $A$"), a similar statement does not hold for case $B3$ in general. Up to now it was only shown that one concrete point in regime $B3$, $P1$, was mapped to point $P2$ and thus to regime $A$. It has now to be clarified, what happens, if the economy starts somewhere else in regime $B3$.

The first issue to be mentioned in this regard is that the steady state at $w_{sub}$ described in figure 2 lies within regime $B3$. The question is now what happens, if the initial situation slightly differs from this constellation but still falls within case $B3$. If $w_t^{(I)}$ is the corresponding equilibrium wage in period $t$, the reservation wage of group "b" at time $t+1$ is then given by (30):

$$w_{b,t+1}^r = \bar{c} - (1+\bar{r})w_t^{(I)} \tag{47}$$

Provided that regime $B3$ is not left in $t+1$, this wage coincides with the equilibrium wage in this period, so that

$$w_{t+1}^{(I)} = \bar{c} - (1+\bar{r})w_t^{(I)} \tag{48}$$

While the steady state of this first-order difference equation is clearly $w_{sub}$, the fact that $w_t^{(I)}$ on the RHS is multiplied by $-(1+\bar{r}) < -1$ immediately reveals its alternating instability. This in turn leads to the question, whether regime $B3$ will not be left sooner or later. This will surely be the case if in one period $w_{b,t}^r$ falls short of $w_a^r$, because this is the constellation of regime $A$. If this situation occurs, e.g., in period $t+1$ (i.e. $w_{b,t+1}^r \leq w_a^r$), whereas in period $t$ the economy was still in regime $B3$, the following inequality must hold:

$$\bar{c} - (1+\bar{r})w_{b,t}^r \quad = \quad w_{b,t+1}^{(I)} \quad \leq \quad w_a^r \quad \Longleftrightarrow \quad w_{b,t}^r \quad \geq \quad \frac{\bar{c} - w_a^r}{1+\bar{r}}$$

Due to $w_{b,t}^r = \bar{c} - (1+\bar{r})w_{t-1}^{(I)}$ (according to (30)) the above inequality can be rewritten as

$$w_{b,t}^r \quad = \quad \bar{c} - (1+\bar{r})w_{t-1}^{(I)} \quad \geq \quad \frac{\bar{c} - w_a^r}{1+\bar{r}} \quad \Longleftrightarrow \quad w_{t-1}^{(I)} \quad \geq \quad \frac{\bar{r}\bar{c} + w_a^r}{(1+\bar{r})^2} \tag{49}$$

It is easily shown that this wage rate lies between $w_{sub}$ and $w_a^r$: both parts of the inequality chain $w_a^r < RHS(49) < w_{sub}$ finally lead to $(2+\bar{r})w_a^r < \bar{c}$ which is clearly fulfilled since earning $w_a^r$ in both periods of life does not suffice to cover $\bar{c}$.

This means that in figure 7 regime $B3$ can be subdivided in the following way:

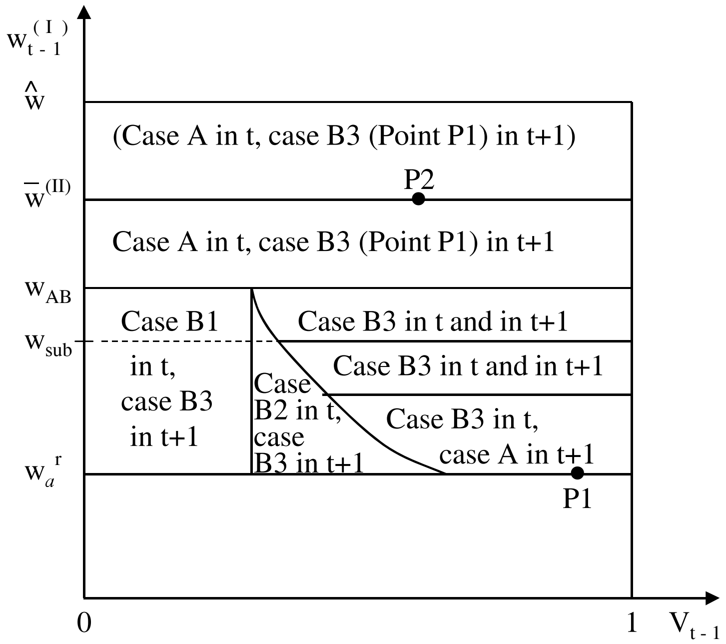


Figure 8

As just shown, the subregion of $B3$ between $w_a^r$ and the RHS of (49) has the property that the economy will be in regime $A$ in the subsequent period $t+1$. For the rest of area $B3$, however, figure 8 claims that this regime will not be left in $t+1$. This shall be shown now. First, it has to be noted that in this part of $B3$ $w_{b,t+1}^r$ is larger than $w_a^r$ so that situation $A$ cannot occur in $t+1$. Second, however, it has to be clarified whether the labour demand at $w_{b,t+1}^r$ exceeds the combined size of group "a" and "c" in that period. Since $B3$ prevails in $t$, the employment rate in this period, $V_t$, equals $L(w_t^{(I)})/2N$. Consequently, the size of group "a" and "c" in $t+1$, $N+(1-V_t)N$, is equal to $2N - \frac{1}{2}L(w_t^{(I)})$. The reservation wage of group "b" at that time, $w_{b,t+1}^r$, is given by $\bar{c} - (1+\bar{r})w_t^{(I)}$, so that the condition to be checked becomes

$$L(\bar{c} - (1+\bar{r})w_t^{(I)}) > 2N - \frac{1}{2}L(w_t^{(I)}) \quad \Longleftrightarrow \quad L(\bar{c} - (1+\bar{r})w_t^{(I)}) + \frac{1}{2}L(w_t^{(I)}) > 2N \quad (50)$$

Note that the entries in $L(.)$ on the LHS are identical to each other only in the special case of $w_t^{(I)} = w_{sub}$; otherwise one of them is larger than the other. In conjunction with the negative dependence of $L$ on $w_t$ it is thus possible to obtain a lower limit for the LHS of (50) by inserting for both entries of $L(.)$ the largest value of $w_t^{(I)}$ that is possible for regime $B3$. This value is $\bar{w}^{(II)}$ due to $w_t^{(I)} = w_{b,t}^r = \bar{c} - (1+\bar{r})w_{t-1}^{(I)}$ in case $B3$ and $w_{t-1}^{(I)} \geq w_a^r$. Thus, the LHS of (50) is always extremely underestimated by the expression $L(\bar{w}^{(II)}) + \frac{1}{2}L(\bar{w}^{(II)}) = \frac{3}{2}L(\bar{w}^{(II)})$. (Note that even in the case that one of the two wages involved on the LHS of (50)) is indeed equal to $\bar{w}^{(II)}$ the other one is necessarily smaller). If now $\frac{3}{2}L(\bar{w}^{(II)}) > 2N$ is fulfilled, the same must be the case for the "true" LHS of (50). According to (43), however, it is already ensured that $L(\bar{w}^{(II)}) > \frac{4}{3}N$, which directly proves the relationship just mentioned. Thus, whenever the economy starts in regime $B3$ such that $w_{b,t+1}^r$ does not fall short of $w_a^r$, it will find itself again in regime $B3$ in $t+1$.

This result also answers the question about the long run outcome of an initial position in this regime. Provided that the economy does not start exactly at the steady state ($w^* = w_{sub}; V^* = L(w_{sub})/2N$), it will first show an alternating behaviour within regime $B3$ with an ever increasing distance to $(w^*; V^*)$. Sooner or later, however, the reservation wage of group "b" will fall short of $w_a^r$ which leads the economy to regime $A$. As already shown, however, this regime is mapped to point $P1$ (once again belonging to $B3$) which in turn is followed by $P2$ (belonging to $A$). Having reached this period-two-cycle, the economy will remain there forever. Thus, every starting point within $B3$ (except $(w^*; V^*)$) will finally lead to an alternation between the two points just mentioned.

What remains to be shown is now, that the same is true for regime $B2$ and $B1$ as already claimed by figure 8. They are both mapped to regime $B3$ in the next period of time as can be demonstrated as follows. If situation $B2$ prevails in $t$, the young generation is always fully employed, i.e. $V_t = 1$. As a consequence, the size of group "c" in $t+1$ equals zero and the combined size of group "a" and "c" is equal to $N$. Furthermore, since the equilibrium wage $w_t^{(I)}$ is at least $w_a^r$, the reservation wage of group "b" in $t+1$ is smaller or equal to $\bar{w}^{(II)}$. If now labour demand at that wage exceeds $N$ (group "a" + group "c"), the same is true (more than ever) for wages below. The relation $L(\bar{w}^{(II)}) > N$, however, again follows directly from (43). Therefore, regime $B2$ must always be followed by regime $B3$.

Finally, the same property shall be proved for constellation $B1$. This regime is characterized by an equilibrium wage $w_t^{(I)} = w_a^r$ and an employment rate of the young generation equal to $V_t = \frac{L(w_a^r)}{N+(1-V_{t-1})N}$. Thus, the size of group "a" and "c" in $t+1$ is $N + (1-V_t)N = 2N - \frac{L(w_a^r)}{2-V_{t-1}}$ and the reservation wage of group "b" in $t+1$ equals $w_{b,t+1}^r = \bar{c} - (1+\bar{r})w_a^r = \bar{w}^{(II)}$. In order to arrive in regime $B3$ in $t+1$, the following inequality has thus to hold:

$$L(w_{b,t+1}^r = \bar{w}^{(II)}) \quad > \quad 2N - \frac{L(w_a^r)}{2 - V_{t-1}}$$

Due to the RHS being smaller than $2N - \frac{1}{2}L(w_a^r)$, the above inequality is always fulfilled whenever the sharper condition

$$L(w_{b,t+1}^r = \bar{w}^{(II)}) \quad > \quad 2N - \frac{1}{2}L(w_a^r)$$

holds true. This, however, again follows from (43): $\frac{4}{3}N < L(\bar{w}^{(II)}) \iff 2N < \frac{3}{2}L(\bar{w}^{(II)}) = L(\bar{w}^{(II)}) + \frac{1}{2}L(\bar{w}^{(II)}) < L(\bar{w}^{(II)}) + \frac{1}{2}L(w_a^r) \iff 2N - \frac{1}{2}L(w_a^r) < L(\bar{w}^{(II)})$ □

Taking together the above findings already leads to a strong result: independently of the regime in which the economy starts, it will always end up in the alternating cycle defined by the two points $P1$ and $P2$. This conclusion is valid up to that value of $w_{t-1}^{(I)}$ that would allow an individual employed in sector I to abstain from working in the next period, i.e. $\hat{w}$ as defined in (16).

It is worthwhile to have a closer look at the cycle between $P1$ and $P2$. Both constellations are characterized by a labour supply exceeding labour demand in sector I. Those people not getting a job there are forced to work in sector II although they would strictly prefer an occupation in sector I. As already outlined at the beginning, activities in sector II do not contribute to the society's well-being but are only directed at the exploitation of information asymmetries, so that from a social planner's point of view this is in no way better than a situation with the corresponding part of the working force being unemployed and receiving support from government. Thus, the central result obtained in the elementary model of section I is also reproduced here, although the long run is no longer characterized by the stationary state at $w_{sub}$. Furthermore it should be noted that the cycle contains a certain imbalance between two successive generations. Whereas a member of a young generation starting in point $P2$ earns $(1+\bar{r})w_a^r + \bar{w}^{(II)} = \bar{c}$ in his life if employed in sector I in both periods, a corresponding member of the next generation (starting at $P1$) will end up with $(1+\bar{r})\bar{w}^{(II)} + w_a^r > \bar{c}$. Note furthermore, that people working in sector II in both periods of life have on the one hand an even higher income $((2+\bar{r})\bar{w}^{(II)})$ but nevertheless a lower welfare due to the significantly higher inconvenience from working in sector II.

The next question is now, what happens, if the economy starts with a wage rate above $\hat{w}$, i.e. the wage rate allowing for leisure in the second period of life. Since under "normal" conditions the wage rate cannot exceed $\bar{w}^{(II)}$, however, it first has to be clarified how such a situation can occur at all. The answer can well be connected with an issue of utmost political interest: the existence of a binding minimum wage $w_0$ (fixed by a treaty between trade unions and firms or by government), combined with the question about the consequences that arise, if this minimum wage is abolished at a certain point of time.

Since now the region above $\hat{w}$ is considered the *unconstrained* reservation wage of group "b," $w_2^R$ according to (19), comes into play. Thus, two issues are of importance now: first the concrete position of $w_2^R$ and second the relative position of the minimum wage $w_0$ vis-à-vis $w_2^R$. In addition to this, also the size of labour demand at $\hat{w}$ is of importance. It will be assumed that the latter exceeds $N$, the size of one generation, which can formally be expressed by

$$\hat{w} \quad < \quad w_N \quad \text{with} \quad L(w_N) \quad = \quad N, \tag{51}$$

i.e. with $w_N$ denoting the wage rate at which labour demand equals $N$. In conjunction with this it also makes sense to formalize a further assumption which had already been made earlier in order to exclude the possibility of abstaining from working in the first period of life. Letting $\tilde{w} = \bar{c}$ denote the corresponding wage rate from which on the alternative just mentioned becomes possible, the assumpion

$$L(\tilde{w}) \quad < \quad N \tag{52}$$

makes sure that in this region labour demand falls short of supply, since after leisure in the first period the whole young generation would have to seek an employment opportunity in sector I thereafter. In the case of unemployment, however, the availability of such a job for a concrete individual under consideration is not ensured so that not working in the first period would jeopardize the minimum consumption. Since this has to be avoided under all circumstances, (52) ensures that each young generation has to work in its first period of life.

Summing up, the following chain of inequalities applies in the following:

$$\bar{w}^{(2)} \quad < \quad \hat{w} \quad < \quad w_N \quad < \quad \tilde{w} \tag{53}$$

Note that while $w_N$ depends on $N$ and the labour demand function $L(.)$ the variables $\hat{w}$ and $\tilde{w}$ are dependent on $\bar{c}$ and $\bar{r}$. Thus, the last two parameters in conjunction with $\bar{w}^{(2)}$ can be chosen in such a way that (53) is fulfilled. Since $w_2^R$ depends additionally on $\tilde{b}$ and $\gamma$ (according to (19)) which do not play any role in (53), the relative position of $w_2^R$ is not yet determined by these inequalities.

There is now a plenty of cases that arises from the various possible relative positions of $w_2^R$ and $w_0$, the legal minimum wage. It shall be mentioned here only, that in most cases the economy finally ends up in the "(P1,P2)-cycle" once the minimum wage constraint is released. Therefore, only one case shall be given special attention here which makes in exception in this regard and arises if $w_2^R > w_N$ and $\hat{w} < w_0 < w_N$.
As far as $w_0$ prevails the employment rate $V_t$ will increase according to the following dynamic equation

$$V_t \quad = \quad \frac{L(w_0)}{2N - V_{t-1}N} \quad = \quad \frac{L(w_0)}{N} \cdot \frac{1}{2 - V_{t-1}} \tag{54}$$

which directly derives from the fact that due to $w_0 > \hat{w}$ "group b" is not forced to work in $t$ and with regard to $w_0 < w_N < w_2^R$ it will also actually abstain from working. Labour supply is thus reduced to the size of group "a" and group "c" which is equal to $N + (1 - V_{t-1})N$.

It is easily checked that a steady state of this first-order difference equation must obey to the equation

$$\bar{V}(w_0) \quad = \quad 1 \pm \sqrt{1 - \frac{L(w_0)}{N}}. \tag{55}$$

Obviously, the sign in front of the square root must be negative due to $\bar{V} \leq 1$. Even in this case, however, a solution for $\bar{V}$ does only exist for $L(w_0) \leq N$ which in turn means $w_0 \geq w_N$. Since this is in contrast to the situation considered here, a first conclusion is that a fixed minimum wage between $\hat{w}$ and $w_N$ will not lead to stable, self-sustaining situation with regard to the associated employment rate, at least as far as $w_0$ is binding. The question thus arises how the concrete dynamics of $V_t$ look like. From equation (54) (together with $L(w_0) > N$) one gets:

$$V_t = \underbrace{\frac{L(w_0)}{N}}_{>1} \cdot \frac{1}{2 - V_{t-1}} > \frac{1}{2 - V_{t-1}} \quad \Longrightarrow \quad V_t - V_{t-1} > \frac{(V_{t-1} - 1)^2}{2 - V_{t-1}} > 0. \tag{56}$$

Thus, whenever the economy starts at a value of $V_0$ below 1, the employment rate will increase in the course of time until in a certain period labour demand at $w_0$ is equal or even larger than the combined size of group "a" and "c". Thus, there is now full employment at a wage rate equal to or even above $w_0$. As a consequence, a group "c" will no longer be present in the subsequent period which is then characterized by the following constellation:

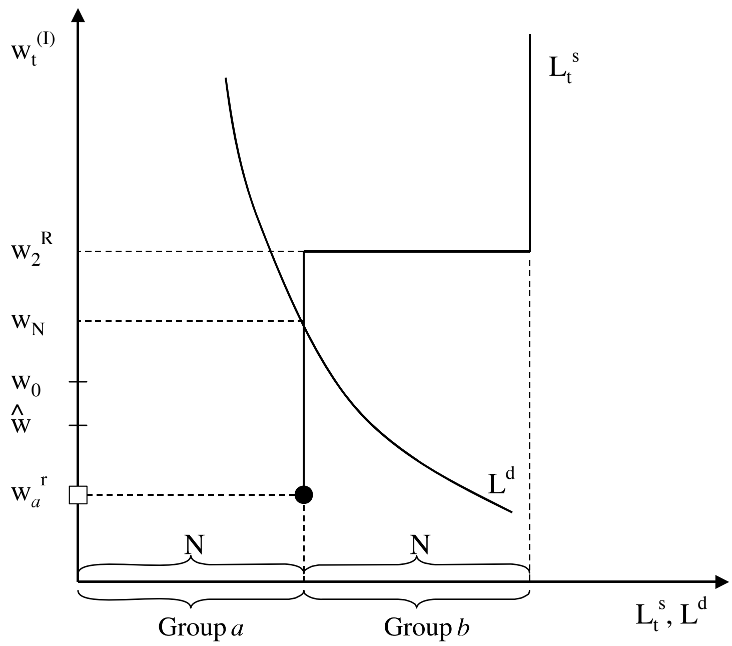


Figure 9

This in turn means, that the same situation will prevail again one period later and will thus be repeated over and over again. In other words: there is now a second steady state ($w^* = w_N; V^* = 1$) with *voluntary* unemployment only, since workers do not want to work at $w_N$ in their second period of life. There are two issues worth mentioning in this regard. First, despite the fact that the legal minimum wage is absolutely necessary to make this outcome possible, it is no longer binding in the emerging steady state. Second, nobody is forced to work in sector II any more so that there are now no activities in this sector. It should be noted, however, that in spite of these attractive properties the production volume of the economy is smaller than in the "(P1,P2)-cycle", since the input of low-skilled labour is lower whereas the number of high-skilled workers has not changed. Thus, the advantage for the low-skilled workers is at the cost of the latter.

What are now the policy implications of the model? The main result can be seen in the fact that an abolition of impediments against wage flexibility does not necessarily lead to a satisfactory situation on the labour market, even apart from considerations in conjunction with a "fair" distribution of income. Thus, even if it were possible to enforce a free labour market by appropriate political actions (as removing minimum wage legislation or curtailing unions' power) government would be well advised to take additional measures to prevent (as far as possible) the problems described above. What seems to be especially promising in this regard are wage subsidies which – if chosen adequately – can here even lead to a Pareto-improvement in comparison to the pure market solution. As could be shown, full employment in the first sector could be established instead of the "(P1,P2)-cycle" emerging otherwise by a suitable size of these subsidies with everybody being better of. It does not even matter, if the wage subsidy is offered for jobs in the second sector as well, since having the choice between the two sectors nobody would voluntarily choose to work in the second in the presence of full employment in sector I and equal pay in both cases.

# 5 Conclusions and outlook

In this paper it was shown that even under the condition of a fully free labour market, i.e. in the absence of unions or government, a situation might occur which is characterized by an excess supply of labour in the sector of "honest work", at least as far as the segment of low-skilled labour is concerned. As the central reasons for this result two factors were identified: first, the necessity to finance minimum consumption which introduces a lower boundary for the real wage and the possibility that at this wage rate labour demand falls short of labour supply. People not finding a job under these conditions have then to recourse to activities that do not contribute to the society's well-being and have primarily to do with the exploitation of information asymmetries in a broad sense. Despite a strong moral aversion against this type of "work" there is no alternative if minimum consumption requirements shall be met.

The topic was first introduced in a one-period framework and then extended to an OLG-model containing two working periods for each generation. The reason for this extension was the fact, that in a multi-period context additional possibilities arise for each individual to react to unemployment in a given period which cannot be captured by a one-period approach. It turned out that despite a considerably higher degree of complexity the basic argument is still valid also in this framework albeit now in form of a period-two-cycle. This cycle is attractive for most initial conditions, even when starting at a legal minimum wage (enforced by unions or government) which is released after a certain period. In this case, however, also a full-employment steady state can emerge, but only if the minimum wage just mentioned was so high that each generation could afford to abstain from working in the second period of life. With regard to the policy implications of the model it was argued that a suitable intervention by government like wage subsidies for the segment of low-skilled labour can lead to a Pareto-improvement in comparison to the outcome under free market conditions.

There are two further issues worth mentioning. The first concerns the question, whether the whole approach is still convincing if technical progress is assumed to take place or whether the latter will inevitably lead to a secular shift of the demand curve for unskilled labour to the right with the consequence that sooner or later full-employment equilibria will emerge with a real wage well above $\bar{c}$. Although such a possibility cannot be ruled out, it is, on the other hand, not cogent as technical progress might also take place in a skilled-biased manner (see, e.g., Aghion/Howitt (1998), p. 298 ff. or Juhn/Murphy/Pierce (1993) among the meanwhile vast literature on this item). If capital input is determined as before by the international real rate of interest and if high-skilled labour is immobile and fully employed, the demand curve for unskilled labour may very well even shift to the left if the rate of labour-augmenting technical progress takes place in the way just mentioned. Thus, there is at least no guarantee, that the problems highlighted by the present model will lose their importance in conjunction with technological innovations in the production process. Moreover, for the same reasons as just outlined the problem might come back after a period where it had already vanished.

Furthermore, as already mentioned in the introduction, also minimum consumption requirements might increase in the course of time, since due to technical progress taking place in the economy under consideration many goods or services which cannot be dispensed with are no longer available below a certain quality standard. In addition to this certain products like manuals having been a luxury good still ten or fifteen years ago are nowadays often even necessary to be eligible for a job; even a private car is sometimes required in this regard. Thus, even if an individual indeed tries to boil down his consumption to a level as low as possible he will not be able to prevent this level from rising in the course of time.

Taking both aspects together there is obviously no cogent reason why the problem dealt with in this paper should disappear if technical progress is taken into account in the course of the economic development of a society. On the other hand it goes without saying that the model discussed here can only be regarded as a first step towards a really satisfactory treatment of the topic, since many important aspects have still to be added. Among the most important issues to be included in this regard are surely the extension to a framework with more than only two periods in which the allocation of labour takes place, the introduction of an unemployment insurance (emerging in the private sector for conceptual reasons), an endogenization of the income that can be earned in the "second sector" (making it dependent, e.g., on the number of people being engaged there) and a – partial – international mobility of high-skilled labour (the factor in the "background"). Although things will become even more involved in these cases the importance of the problem analyzed in this paper is surely a sufficient justification to undertake the effort.

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