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Agreements**

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# Union Wages, Hours of Work and the Effectiveness of Partial Coordination Agreements

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## Abstract

Small monopoly trade unions decide upon the wage rate per hour and the hours of work subject to firm's demand for union members. Since the resulting Nash equilibrium is characterized by excess unemployment, we study the employment and welfare effects when trade unions try to coordinate their policies. Firstly, we consider a joint agreement about marginal wage moderation, where trade unions remain free to choose the hours of work non-cooperatively. Secondly, we analyze in which way a joint change in the hours of work affects employment and welfare if trade unions are free to choose the wage rate.

*JEL Classification:* C72, J51

*Keywords:* unemployment, wage setting, hours of work, partial cooperation

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# 1 Introduction

Among industrialized nations, especially European countries suffer from high and persistent unemployment rates caused by real wages that are above the market clearing level. As one basic reason, it has been emphasized that the existence of trade unions is, in general, not compatible with full employment. Since (involuntary) unemployment is traded off with the wage rate accruing to its members, this is the ‘price’ trade unions are willing to accept when maximizing the well-being of their members. The economic literature has therefore pointed out that redesigning the tax system might provide a potential remedy to such a distortion on the labor market (see, for instance, Richter and Schneider 2001 and Koskela and Schöb 2002a,b). In fact, tax policy can either be used to manipulate the labor demand elasticity or to directly subsidize the labor market to lower the wage rate and boost employment, respectively.

Within the well-known monopoly-union framework, the pure ability to exert market power on the labor market is the basic reason for excessive wage claims and involuntary unemployment. In addition, however, wages and unemployment might be even higher if a country comprises many sector-specific monopoly trade unions, each of which imposing an externality on the rest of the economy when deciding upon the wage rate unilaterally. The result of such a decentralized equilibrium might be referred to as *excess unemployment*. Prominent examples of such externalities are the interactions between trade unions and the government sector, which can be interpreted as a fiscal externality. To the extent unemployed union members receive unemployment benefits from a government-run insurance program, each individual trade union is not fully aware of the true costs of wage induced layoffs since the additional expenses for unemployment benefits are spread over all employees within the country. Thus, increasing each union’s financial responsibility of running the unemployment benefit system of its members has a wage moderating effect (see Holmlund and Lundborg 1988 as well as Sinko 2004). Another example is the potential hump-shaped pattern of the real wage rate depending on the degree of centralization in the wage setting (Calmfors and Driffill 1988). For intermediate levels of centralization, an increase in the union’s wage rate might raise the price level of the firms’ output, representing a loss in real wage for all union members in other sectors. In contrast, for the extreme cases of fully decentralized and centralized wage setting, such a price increase is either not possible due to the existence of close substitutes or falls back on all union members as an increase in the general price level, respectively.<sup>1</sup> This hump-shaped relation, however, is alleviated the more

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<sup>1</sup>See Calmfors (1993) for other types of externalities dealt with in the literature.

countries are integrated in a world market producing highly substitutable goods (Danthine and Hunt 1994).

Simple stylized facts support the view that the unemployment rate is considerably lower in countries with centralized bargaining (e.g., Austria, Norway and Sweden) than in economies with a very decentralized bargaining structure (United Kingdom, United States, France); see Mares (2006). In addition, there is also empirical evidence indicating that among the two extreme cases it is full centralization that performs better in terms of employment compared with decentralized bargaining structure (see, e.g., Belot and van Ours 2001). More detailed evidence by Belot and van Ours (2004) or Nickell et al. (2005) suggests that the interactions with other labor market institutions seem to matter. They find that in the presence of decentralized trade unions unemployment is higher when there is a high degree of employment protection or union density.

In the present paper, we abstract from the potential externalities mentioned afore. Rather, we restrict our wage setting analysis to a quite fundamental form of a prisoners' dilemma situation among small decentralized (monopoly) trade unions. The basic externality at work in this paper is as follows. When a trade union claims a higher wage, with the corresponding loss in employment being the cost of this additional wage income, it imposes an external effect on all other trade unions simply because the unemployed members of the latter now face a lower probability of getting re-employed. We choose this *unemployment externality* to be the driving force of excess unemployment in our setting. In addition, to draw a more realistic picture of trade union behavior, we also allow each trade union to decide upon both the wage rate per hour and the hours of work per employee in the sector. Since both union instruments affect the firm's labor demand, both are able to impose an externality on all other sectors. Obviously, the resulting equilibrium entails room for improvements in terms of welfare and employment. This is the starting point of the present paper. Our basic question will then be the following. Even in the absence of any government intervention, can trade unions effectively benefit from coordination agreements that aim at internalizing this externality? Clearly, if all trade unions are perfectly able to agree on both available instruments, the answer is in the affirmative. But what happens when trade unions are unable to commit themselves to a joint agreement that captures *both* the wage rate and the hours of work? Can the internalization of external effects work if only *partial* coordination is possible in the sense that only one of the unions instruments is cooperatively chosen, whereas the respective other instrument can nevertheless be freely chosen by all trade unions involved?

The focus of the present paper is therefore the following. Assume that a country

cannot simply move from a very decentralized structure to a centralized one by installing a trade union that is common to all firms in the country. What is then the potential scope for cooperation among all decentralized trade unions? Is there a chance to mimic centralized trade unions by jointly agreeing on some projects but still retaining the decentralized structure as such?

As one example, we might refer to the German *Alliance for Jobs* (Bündnis für Arbeit), i.e. central negotiations between trade unions, employers' representatives and the government to boost employment, where the metal sector trade union (IG Metall) was the first to announce that it would leave the negotiations if the wage rate appears on the agenda. In fact, the main (and only) purpose of the trade union leaders was to negotiate on the working time by reducing overtime or weekly hours of work and promoting early retirement programs.

On the other hand, many European countries have undertaken some effort to establish social pacts between trade unions and the governments (see Mares 2006). These pacts often comprise wage moderation in return for changes in tax policy or social security regulation. In most cases, however, the hours of work are not explicitly on the agenda.<sup>2</sup>

Rather than analyzing a multi-party contract between trade unions and other potential bargaining parties such as the government or employers, we study the effectiveness of partial agreements among decentralized trade unions only. In particular, our approach differs from the previous literature primarily because it deals with decentralized trade unions. In contrast, Calmfors (1985), Booth and Schiantarelli (1987) as well as Booth and Ravallion (1993) simplify their analysis to some extent by assuming that all workers are members of a centralized trade union. However, as has been set out above, countries with a centralized union structure have remarkably lower unemployment rates since they do not suffer excess unemployment. This is an important difference since centralized unions have no intrinsic motivation to further use their instruments to change the employment level. Instead, these authors have to rely on exogenous reductions in working time and derive an ambiguous employment effect when taking the subsequent wage response into account.

To address this issue in the presence of decentralized unions, the paper is organized in the following way. In section 2, we set up a simple model of decentralized monopoly trade unions deciding upon the wage rate per hour and the hours of work. Since the Nash equilibrium implies unemployment that is higher than under centralized wage setting, section 3 discusses different forms of cooperation among trade

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<sup>2</sup>As one exception, the Dutch Wassenaar agreement explicitly stated that wage moderation was exchanged for a reduction in working time.

unions. In particular, we distinguish between full cooperation and the more realistic scenario of partial cooperation. Section 4 summarizes and concludes.

## 2 The model

We consider a single small open economy that consists of a fixed large number of identical firms or sectors producing a homogenous output good. The good is sold to the world market at a constant price.

Turning to the firm level first, we assume that each firm produces the homogenous output good  $X_i$  using ‘labor’ as the only variable input according the production function  $X_i = F(L_i)$ , where  $F' > 0$ ,  $F'' < 0$  and the index  $i$  refers to an individual sector. The production function is common to all firms within the country. For notational convenience, other factors are assumed to be fixed in supply and are therefore suppressed in our formulation. We define ‘labor’  $L_i$  as effective labor input that comprises both the number of employed workers  $l_i$  in the sector and the hours of work per employed worker  $h_i$ . Following, e.g., Booth and Schiantarelli (1987) as well as Booth and Ravallion (1993), we allow working time and employment to be less than perfectly substitutable. Effective labor input is therefore specified as follows:

$$L_i = (h_i)^\alpha l_i, \quad (1)$$

where  $0 \leq \alpha \leq 1$  in order to capture potential decreasing returns to scale of a longer working day, e.g., due to fatigue effects. This specification implies

$$\frac{\partial L_i}{\partial l_i} \frac{l_i}{L_i} = 1,$$

i.e. for given hours of work per employee, a one percentage increase in employment  $l_i$  always translates into a one percentage increase in effective labor input. In contrast, for the hours of work we have

$$\frac{\partial L_i}{\partial h_i} \frac{h_i}{L_i} = \alpha \leq 1,$$

i.e. for employment kept constant, a one percentage increase in working hours does not increase effective labor input by more than one percentage. In particular, note that  $\alpha = 1$  is the special case of employment and working hours being ‘perfect substitutes’. For this case, effective labor input is simply given by the total working hours of all employees. On the other hand,  $\alpha < 1$  indicates that the hours of work are less than perfectly substitutable to employment  $l_i$ .

Since we suppose the wage rate  $w_i$  per hour as well as the hours of work  $h_i$  to be choice variables of a sector-specific monopoly trade union, each firm takes these

variables as given. Normalizing the constant output price to one, each firm then has the right to manage, i.e. it chooses employment  $l_i$  so as to maximize its profit:

$$\max_{l_i} F(L_i) - w_i h_i l_i, \quad (2)$$

subject to (1). This yields

$$(h_i)^\alpha F'(L_i) = w_i h_i$$

and implicitly defines labor demand of firm  $i$  as  $l_i = l_i(w_i, h_i)$ .

Following a change in the wage rate or the hours of work, respectively, each firm will adjust employment according to:

$$\frac{\partial l_i}{\partial w_i} = \frac{h_i}{(h_i)^{2\alpha} F''} < 0, \quad (3)$$

$$\frac{\partial l_i}{\partial h_i} = \frac{w_i(1-\alpha)}{(h_i)^{2\alpha} F''} - \alpha \frac{l_i}{h_i} < 0. \quad (4)$$

In terms of elasticities, we are able to express the labor demand elasticity with respect to the hours of work as a weighted average of the labor demand with respect to the wage rate and  $-1$ , with  $\alpha$  being the weight:

$$\frac{\partial l_i}{\partial h_i} \frac{h_i}{l_i} = (1-\alpha) \frac{\partial l_i}{\partial w_i} \frac{w_i}{l_i} - \alpha. \quad (5)$$

The interpretation of (5) is straightforward. For the extreme case  $\alpha = 0$ , the hours of work would collapse to a pure cost factor, equivalent to the wage rate. Both labor demand elasticities would therefore coincide. On the other hand, for the special case of  $\alpha = 1$ , the hours of work are perfectly substitutable to employment. It is only the total working hours, i.e.  $L_i = l_i h_i$ , that is relevant to the firm as the input factor of production. For a given factor price of the effective labor input,  $L_i$ , there is a one-to-one relation between  $l_i$  and  $h_i$  in terms of percentages. Since the working hours enter the production function, labor demand is more elastic with respect to the wage rate than the hours of work, the exception being  $\alpha = 0$ .

For later use, note that, in general, the labor demand elasticities are not constant in the level of effective labor input, but will change in response to changes in the union's policy instruments. Defining  $\varepsilon_{l,w}$  and  $\varepsilon_{l,h}$  as the elasticities of labor demand with regard to the wage rate and the hours of work,<sup>3</sup> respectively, we have

$$\frac{\partial \varepsilon_{l,h}}{\partial j} = (1-\alpha) \frac{\partial \varepsilon_{l,w}}{\partial j}, \quad j = w, h, \quad (6)$$

$$\frac{\partial \varepsilon_{l,w}}{\partial w} = \frac{\varepsilon_{l,w}}{w} \left[ 1 - \varepsilon_{l,w} \left( 1 + \frac{F'''}{F''} L \right) \right] \begin{matrix} \leq \\ \geq \end{matrix} 0, \quad (7)$$

$$\frac{\partial \varepsilon_{l,w}}{\partial h} = (1-\alpha) \frac{\varepsilon_{l,w}}{h} \left[ 1 - \varepsilon_{l,w} \left( 1 + \frac{F'''}{F''} L \right) \right] \begin{matrix} \leq \\ \geq \end{matrix} 0. \quad (8)$$

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<sup>3</sup>The index  $i$  has been dropped for notational convenience.

Indeed, standard one-factor production functions produce  $F''' > 0$  as a property and support the above ambiguity. According to (7) and (8), the elasticity of the above elasticities with regard to  $h$  and  $w$  are also connected by the parameter  $\alpha$ , i.e.

$$\frac{\partial \varepsilon_{l,w}}{\partial h} \frac{h}{\varepsilon_{l,w}} = (1 - \alpha) \frac{\partial \varepsilon_{l,w}}{\partial w} \frac{w}{\varepsilon_{l,w}}. \quad (9)$$

The parameter  $\alpha$  therefore has two impacts on the labor demand elasticities. On the one hand, it determines the extent to which the labor demand elasticity  $\varepsilon_{l,h}$  can be influenced by either  $w$  or  $h$  compared with  $\varepsilon_{l,w}$  [see equation (6)]. For the extreme case  $\alpha = 1$ , it is constant at  $\varepsilon_{l,h} = -1$ . On the other hand, according to (9), the impact of the hours of work on the labor demand elasticity  $\varepsilon_{l,w}$  differs from the wage impact on this elasticity by the factor  $(1 - \alpha)$ . Intuitively, the change in the labor demand elasticity  $\varepsilon_{l,w}$  crucially depends on the effective labor input  $L$ .<sup>4</sup> In turn, the hours of work have a positive direct effect on  $L$  given the number of employed workers, which is not the case for the wage rate. The negative indirect effect on  $L$  due to the reduction in employment can offset the former effect only for  $\alpha = 1$ . Otherwise a negative impact of effective labor input remains.

As mentioned above, both the hours of work in sector  $i$  as well as the wage rate in this sector are determined by a corresponding sector-specific monopoly trade union. Each union's membership is assumed to be fixed throughout. As is usual in the literature on trade union behavior, each small monopoly trade union acts as a Stackelberg leader towards the firm. Thus, when choosing  $w_i$  and  $h_i$  it takes into account that the firm retains the 'right to manage' according to labor demand  $l_i = l_i(w_i, h_i)$ . However, each sector-specific trade union is assumed to be sufficiently small and is therefore unable to influence the countrywide employment level and thus, in turn, the probability that unions members are employed in the rest of the economy.

Since trade unions represent the preferences of their member, we have to specify the utility of union members. Each member's utility function is assumed to be additive and linear in income. If employed in firm  $i$ , the household works  $h_i$  hours and receives a wage rate of  $w_i$  per hour, both variables being determined by the trade union the household is organized in. Since employment is associated with forgone leisure, we capture the disutility of supplying labor by the term  $e(h_i)$ ,  $e' > 0$ ,  $e'' \geq 0$  with  $e(0) = 0$ .<sup>5</sup> Thus, an unemployed household receives a zero utility level.

The objective of the union  $i$  is to maximize the members' welfare which is given

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<sup>4</sup>The labor demand elasticity with respect to the wage rate is given by  $\varepsilon_{l,w} = F'(L)/[LF''(L)]$ .

<sup>5</sup>See, e.g., Earle and Pencavel (1990) for a similar procedure.



by

$$\max_{w_i, h_i} V_i = l_i(w_i, h_i) [w_i h_i - e(h_i)] + [m_i - l_i(w_i, h_i)] (1 - u) [wh - e(h)], \quad (10)$$

where  $m_i$  denotes the fixed number of union members of which  $l_i$  are employed in sector  $i$  and  $m_i - l_i$  in the rest of the country. The subindex  $i$  refers to the individual union-firm relationship and variables without index denote countrywide averages which cannot be affected by a small trade union. In particular,  $u$  denotes the countrywide unemployment rate such that the probability of re-employment ( $1 - u$ ) is therefore given by  $l/m$ . Since the number of trade unions and firms is fixed and we restrict our attention to symmetric outcomes, all variables without index  $i$  are a measure of countrywide values. In the objective function (10) we have assumed that union members are perfectly mobile between firms within the country under consideration (Hoel 1991). A union member who is not employed in firm  $i$  receives the average wage  $w$  and works for  $h$  hours since these numbers prevail in the rest of the country. On the other hand, if a union member is not employed outside firm  $i$ , which happens with probability  $u$ , her payoff is zero since  $w = 0$  and  $e(0) = 0$ .<sup>6</sup>

Taking into account the firm's response to changes in trade union 'policy variables', each union's first-order conditions require, respectively,

$$\frac{\partial V_i}{\partial w_i} = 0 \Rightarrow l_i h_i + \frac{\partial l_i}{\partial w_i} \left\{ w_i h_i - e(h_i) - \frac{l}{m} [wh - e(h)] \right\} = 0 \quad (11)$$

and

$$\frac{\partial V_i}{\partial h_i} = 0 \Rightarrow l_i [w_i - e'(h_i)] + \frac{\partial l_i}{\partial h_i} \left\{ w_i h_i - e(h_i) - \frac{l}{m} [wh - e(h)] \right\} = 0, \quad (12)$$

i.e. for both instruments, the marginal benefit (at constant employment) must be equal to its marginal cost (due to the reduction in employment). For later use, note that the second-order conditions must satisfy

$$\frac{\partial^2 V_i}{\partial w_i^2} = 2h_i \frac{\partial l_i}{\partial w_i} + \frac{\partial^2 l_i}{\partial w_i^2} \left\{ w_i h_i - e(h_i) - \frac{l}{m} [wh - e(h)] \right\} < 0$$

and

$$\frac{\partial^2 V_i}{\partial h_i^2} = \frac{\partial^2 l_i}{\partial h_i^2} \left\{ w_i h_i - e(h_i) - \frac{l}{m} [wh - e(h)] \right\} + 2 [w_i - e'(h_i)] \frac{\partial l_i}{\partial h_i} - l_i e''(h_i) < 0.$$

In a symmetric equilibrium, each union has solved the same problem. We are therefore able to write  $w_i = w$  and  $h_i = h$ . For the analysis in section 3, it proves

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<sup>6</sup>Recall that we fully ignore the government sector and therefore abstract from unemployment benefits accruing to unemployed union members.

convenient to express the first-order conditions in the symmetric Nash equilibrium in terms of elasticities. We obtain

$$\frac{wh}{[wh - e(h)] \left(1 - \frac{l}{m}\right)} + \varepsilon_{l,w} = 0 \quad (13)$$

and

$$\frac{h[w - e'(h)]}{[wh - e(h)] \left(1 - \frac{l}{m}\right)} + \varepsilon_{l,h} = 0, \quad (14)$$

i.e. for both ‘policy’ instruments of the trade union, a one percentage increase of this instrument must balance the percentage gain in utility with the percentage reduction in employment (see, e.g., Booth 2002). Union members are only willing to supply labor if they receive a positive rent from doing so. Hence,  $wh - e(h) > 0$  and the resulting Nash equilibrium is characterized by unemployment,  $l/m < 1$  [see equation (13)]. Since trade unions also determine the hours of work, each union member who is employed will be *underemployed* in the Nash equilibrium,  $w > e'(h)$  [see equation (14)]. Substituting (13) into (14) yields the following relation between the employment effects of the union’s policy instruments

$$\frac{\varepsilon_{l,h}}{\varepsilon_{l,w}} = \frac{w - e'(h)}{w}, \quad (15)$$

i.e. the relative percentage employment effects of the union’s instruments must be equal to the relative percentage benefits of the two policy instruments. As a common feature in the presence of monopoly power, the Nash equilibrium implies that labor demand is elastic in the wage rate,  $\varepsilon_{l,w} < -1$ . This follows from straightforward manipulation of expression (13). Recalling equation (5), a similar property applies to the labor demand elasticity regarding the hours work, i.e.  $\varepsilon_{l,h} \leq -1$ , where the case of  $\varepsilon_{l,h} = -1$  holds for  $\alpha = 1$ . The expression in (5) then also allows us to infer that, in terms of percentages, the hours of work cannot have a stronger impact on labor demand than the wage rate, i.e.  $\varepsilon_{l,h} \geq \varepsilon_{l,w}$ .

Since we restrict our attention to symmetric equilibria we are able to rewrite the second-order conditions as

$$\frac{\partial^2 V}{\partial w^2} = \frac{\partial l}{\partial w} h \left(2 + \frac{F'''}{F''} L\right) < 0 \quad (16)$$

and

$$\frac{\partial^2 V}{\partial h^2} = \frac{\partial^2 l}{\partial h^2} [wh - e(h)] \left(1 - \frac{l}{m}\right) + 2[w - e'(h)] \frac{\partial l}{\partial h} - le''(h) < 0.$$

Note that central wage setting, e.g., by a countrywide monopoly trade union, will also entail unemployment and a wage rate that exceeds the marginal disutility of labor. However, the unemployment rate will be lower compared with the decentralized

scenario (see Appendix 1 for details). Consequently, there is room for improvement in terms of employment if all decentralized unions coordinate their policies. The next section examines to which extent such cooperation can be effective, depending on the policy instruments that are included in such an agreement.

### 3 Cooperation among decentralized unions

When discussing coordination, we restrict our analysis to marginal steps, starting from the symmetric uncoordinated Nash equilibrium. As a point of reference, we first discuss full cooperation in subsection 3.1, where all unions agree to jointly change one policy instrument while keeping the remaining variable constant. Subsections 3.2 and 3.3 then relax the latter assumption, i.e. we analyze the effectiveness of a joint change in the wage (hours of work) when all trade are still free to choose their hours of work (wage rate) non-cooperatively. Note that we do not attempt to explain why the one or the other form of cooperation is established. Our basic motivation is that it seems to be unrealistic that the participants of such a cooperation are able or willing to agree on several issues.

#### 3.1 Full cooperation in the wage rate and the hours of work

We refer to the special case of full cooperation in  $w$  and  $h$  if all trade unions agree to jointly change one of their policy instruments while keeping the remaining instrument constant. Let us first consider a joint agreement that prescribes to marginally *reduce* the wage rate at constant hours of work. For notational clarity, we omit the index  $i$  if joint changes are considered, since all sectors are identical and face the same reactions. Such an agreement boosts employment in each sector:

$$\left. \frac{\partial l}{\partial w} \right|_{dh=0} = \frac{h^{1-2\alpha}}{F''} < 0,$$

and thus, in turn, output and profits according to, respectively,

$$\left. \frac{\partial X}{\partial w} \right|_{dh=0} = F' h^\alpha \left. \frac{\partial l}{\partial w} \right|_{dh=0} < 0$$

and

$$\left. \frac{\partial [X - whl]}{\partial w} \right|_{dh=0} = -hl < 0. \quad (17)$$

For each sector, these numbers are quantitatively identical to the effects of changes in the wage rate carried out by the respective trade union, unilaterally.

Most importantly, such coordination in the wage rate now affects each union's outside utility by lowering the countrywide probability of being unemployed. The welfare impact can therefore be written as

$$\frac{\partial V}{\partial w} \Big|_{dh=0} = \frac{m-l}{m} \left\{ hl + [wh - e(h)] \frac{\partial l}{\partial w} \Big|_{dh=0} \right\} = -\frac{l}{m} hl < 0. \quad (18)$$

Hence, if all trade unions are able to commit on both a joint wage moderation and a given number of working hours, all unions are better off. As a consequence of the envelope theorem, all wage impacts which are present in the case of a unilateral change in  $w_i$  have no welfare impact, since the symmetric uncoordinated Nash equilibrium serves as the starting point. Thus, (in case of a joint wage cut) the only relevant effects are the reduction of wage income for the households not employed in firm  $i$  and the higher re-employment probability if not employed in sector  $i$ . Using the first-order condition (13), allows us to unambiguously sign this expression.

Correspondingly, a second form of cooperation comprises a joint change in the hours of work with the wage rate kept constant at its previous level. Again, a positive employment effect emerges if all unions reduce  $h$  :

$$\frac{\partial l}{\partial h} \Big|_{dw=0} = \frac{w(1-\alpha)}{h^{2\alpha} F''} - \alpha \frac{l}{h} < 0.$$

In turn, the output effect as well as the impact on firm profits depend on  $\alpha$  since:

$$\frac{\partial X}{\partial h} \Big|_{dw=0} = \frac{w(1-\alpha)F'}{h^\alpha F''} \leq 0$$

and

$$\frac{\partial [X - whl]}{\partial h} \Big|_{dw=0} = -wl(1-\alpha) \leq 0. \quad (19)$$

Thus, we can only expect positive effects on output and profits, if employment and working hours are not perfectly substitutable to the firm. Intuitively, recall that for  $\alpha = 1$  effective labor input is constant in the hours of work since the direct effect is exactly balanced by the indirect effect via reduced employment.

Finally, the joint change in the hours of work (at a constant  $w$ ) yields the following welfare effect:

$$\frac{\partial V}{\partial h} \Big|_{dw=0} = \frac{m-l}{m} \left\{ l[w - e'(h)] + [wh - e(h)] \frac{\partial l}{\partial h} \Big|_{dw=0} \right\} = -\frac{l}{m} l[w - e'(h)] < 0. \quad (20)$$

Trade unions will therefore be better off if they agree to reduce working time and can commit themselves to keep the wage rate constant. Again, the only relevant effects stem from households who are not employed in firm  $i$ . Even though the joint cut in the hours of work reinforces the underemployment of employed households, this loss

in total income is exactly compensated by the gain in employment in this sector. In addition, however, the joint reduction in  $h$  increases employment in all other sectors and thus, in turn, the probability of employment outside sector  $i$ . Hence, using the first-order conditions of the symmetric Nash equilibrium, which serves as a starting point of coordination, the expression in (20) is unambiguously negative.

### 3.2 Partial cooperation in the wage rate

In contrast to the preceding subsection, we now reject the idea that unions are able to form an agreement on both the wage rate and the hours of work. Instead, we now suppose that trade unions are only able to agree on even smaller projects. In particular, we consider a joint change in only *one* of the two instruments ( $w$  or  $h$ ), whereas the coordination arrangement does not cover the remaining variable. The latter can then freely be chosen by all trade unions.

In this subsection, we suppose that all trade unions have agreed to jointly reduce their wage rate to benefit from the subsequent reduction in the unemployment rate. Since the trade unions were assumed to be small, such a reduction in the unemployment rate (i.e. the probability that unions members earn no wage income at all) has not been possible for each union individually. However, since the hours of work are not a part of the agreement that stipulates the joint reduction in the wage rate, each union might now perceive its individual choice of  $h_i$  to be incorrect and aims at a corresponding adjustment.

To be more detailed, recall that the hours of work have been determined according to the first-order condition  $\partial V_i / \partial h_i = 0$ . Again, it is convenient to rewrite this condition in terms of elasticities, using the fact that the starting point (as well as the final equilibrium) is symmetric [see equation (14)]. In the uncoordinated (symmetric) optimum, the net benefit from changing the hours of work (by a marginal unit) can therefore be written as

$$NB(h) = h [w - e'(h)] + \varepsilon_{l,h} [wh - e(h)] \left(1 - \frac{l}{m}\right) = 0. \quad (21)$$

Since the wage coordination will, in general, alter this condition, each union has the incentive to use the hours of work to restore this condition again. In doing so, each single union will, again, treat the (un)employment rate as constant. However, as all unions face the same incentive to adjust their hours of work, the countrywide (un)employment rate will be subject to changes during this adjustment. Thus, to find out the extent to which all unions have finally adjusted their hours of work in

response to the initial wage coordination, we need to determine

$$\frac{dh}{dw} = -\frac{\partial NB(h)}{\partial w} \cdot \left( \frac{\partial NB(h)}{\partial h} \right)^{-1}, \quad (22)$$

where, again, the changes in  $w$  and  $h$  are carried out by all countries and the index  $i$  is suppressed for clarity. Equation (22) then gives us the uncoordinated, but joint adjustment of  $h$  that is necessary to restore  $\partial V_i / \partial h_i = 0$ , if all unions face a coordinated change in the wage rate and the final equilibrium is symmetric again.

Note that joint changes in  $h$  or  $w$ , respectively, trigger the same employment reaction for each trade union as has been the case for unilateral changes; see the effects on  $l_i$  in (3) and (4). In addition, however, the employment level of the whole country, i.e.  $l$ , is changed. Due to our assumption of symmetric unions, these responses are equivalent to the ones given by (3) and (4).

In detail, we have

$$\begin{aligned} \frac{\partial NB(h)}{\partial h} &= (w - e') (1 + \varepsilon_{l,h}) - \varepsilon_{l,h} \frac{l}{m} \left[ w - e' + \varepsilon_{l,h} \frac{wh - e(h)}{h} \right] \\ &+ \frac{\partial \varepsilon_{l,h}}{\partial h} [wh - e(h)] \left( 1 - \frac{l}{m} \right) - he'', \end{aligned} \quad (23)$$

which must be negative in sign for the sake of stability of the initial Nash equilibrium. To see this, bear in mind that the expression  $NB(h)$  stated in (21) gives each union's marginal net benefit from altering the hours of work (which must be zero to establish an optimum for the individual trade union). Now suppose that the hours of work for all unions are slightly lower than in the Nash equilibrium so that this net benefit is positive and each union has an incentive to increase its working hours. Since all unions face the same incentive to increase the hours of work, the corresponding *joint* increase must reduce the net benefit from increasing this variable to reach a stable Nash equilibrium, i.e. to eliminate the incentive to change the hours of work any further. Thus, whether unions choose higher or lower working hours following a wage coordination solely depends on the sign of the first term in (22):

$$\text{sign} \left\{ \frac{dh}{dw} \right\} = \text{sign} \left\{ \frac{\partial NB(h)}{\partial w} \right\}.$$

This term becomes

$$\frac{\partial NB(h)}{\partial w} = h + \varepsilon_{l,h} h \left( 1 - \frac{l}{m} \right) - \varepsilon_{l,w} \varepsilon_{l,h} \frac{l}{m} \frac{wh - e(h)}{w} + \frac{\partial \varepsilon_{l,h}}{\partial w} [wh - e(h)] \left( 1 - \frac{l}{m} \right), \quad (24)$$

where its sign is ambiguous *a priori*, depending on how a joint reduction in the wage rate affects the marginal benefit and marginal cost of changing the hours of work. For all employed union members, the joint wage cut lowers the additional

rent from raising the working hours which *ceteris paribus* renders an increase in working hours less interesting for the trade union; see the first term on the right hand side of (24). On the other hand, a joint reduction in the wage rate also lowers the total rent from being employed. As a consequence of the lower opportunity cost for members being laid off (due to an increase in  $h$ ), unions will call for more working hours, *ceteris paribus*; see the second term in (24). The third term then captures that the coordinated wage cut is able to reduce the countrywide unemployment rate. The cost of increasing the working hours are therefore reduced due to the higher re-employment probability for unemployed union members. Finally, the last term on the right hand side of (24) represents the way in which the trade union's marginal cost of increasing the hours of work are affected by a joint change in the wage rate. In particular, the impact on the labor demand elasticity is important, which is ambiguous in sign. Depending on whether the labor demand elasticity with respect to  $h$  is augmented (alleviated), in absolute terms, when a collective wage cut is carried out, increasing the hours of work becomes more (less) costly to the trade unions. In general, we are not able to conclude whether or not the trade unions' response is to have eventually raised their working hours in the new Nash equilibrium. A clear-cut statement can only be made if the labor demand elasticity with respect to the hours of work remains constant or becomes less elastic following the initial wage cut, i.e.  $\partial\varepsilon_{l,h}/\partial w \leq 0$ . For this case, we unambiguously find that trade unions have responded by increasing the hours of work; see Appendix 2 for details.

### Employment effect

The overall effect on employment is given by the sum of the initial employment effect due to the joint reduction in the wage rate (at constant hours of work) and the subsequent joint change in the hours of work (at a constant wage rate), where the latter must be weighted with the extent to which all unions have finally adjusted  $h$  in the new Nash equilibrium:

$$\frac{dl}{dw} = \left. \frac{\partial l}{\partial w} \right|_{dh=0} + \frac{dh}{dw} \left. \frac{\partial l}{\partial h} \right|_{dw=0}. \quad (25)$$

For notational parsimony, we suppress the characterization  $dh = 0$  and  $dw = 0$  in what follows if the respective variable is kept constant. We solely use the partial derivative notation in what follows. Plugging (23) and (24) into (22) and substituting the result [together with (13) and (15)] into expression (25), we derive at (see Appendix 3):

$$\frac{dl}{dw} = \left( \frac{\partial NB(h)}{\partial h} \right)^{-1} \cdot \left( \frac{\partial \varepsilon_{l,h}}{\partial w} w \frac{\varepsilon_{l,h}}{\varepsilon_{l,w}} - \frac{\partial \varepsilon_{l,h}}{\partial h} h - \varepsilon_{l,w} \frac{h}{w} e'' \right) l. \quad (26)$$

Obviously, the overall employment effect is ambiguous *a priori*. To provide an intuitive explanation for this result, a suitable starting point is to think of the zero employment effect as the benchmark scenario. Loosely speaking, if the hours of work were an instrument that perfectly mimics the wage rate, the trade union would be able to exactly return to where they started, i.e. the initial Nash equilibrium. The employment effect would be zero in this case. To the extent these instruments have different impacts on the first-order condition  $NB(h) = 0$ , a non zero employment effect emerges. Equation (26) can therefore be interpreted as describing the differential effect of  $w$  and  $h$ , respectively, that are able to constitute an employment effect. Since  $\partial NB(h)/\partial h < 0$ , the signs of the terms in brackets on the right hand side of (26) are important. A positive (negative) expression indicates that a joint wage cut boosts (reduces) total employment *ceteris paribus*.

Let us first turn to the last term in brackets which depends on the change of the marginal disutility of the hours of work. As  $\varepsilon_{l,w} < 0$  and  $e'' \geq 0$  this effect is *ceteris paribus* associated with a non-negative overall employment effect. It will vanish for the extreme case of a constant marginal disutility of labor ( $e'' = 0$ ). What is the intuition behind this mechanism? The starting point of coordination is the symmetric Nash equilibrium which is characterized by each trade union's optimal choice of the wage rate and the hours of work (and thus, in turn, employment). The joint wage cut then disturbs this equilibrium such that each union perceives the allocation as no longer the individually most preferred one. Technically, the collective change in the wage rate changes the first-order condition which gives the optimality rule for the hours of work. Since all trade unions are still allowed to change the hours of work, they use this instrument to restore this condition again. Intuitively, trade unions try to use the hours of work to imperfectly mimic the initial equilibrium that has been most preferred from an individual point of view. In the course of the adjustment, the marginal disutility of the working hours is changed. This serves as one channel to partially restore  $NB(h) = 0$ . However, the initial joint change in the wage rate could not have an impact on the marginal disutility of working time. Thus, in an attempt to go back to the initial Nash equilibrium by increasing the hours of work in response to the joint wage cut, the marginal disutility of labor is increased. Since this mechanism already restores the first-order condition  $NB(h) = 0$  to some extent, the total change in  $h$  is lower than what would have been necessary to perfectly undo the initial stimulus, i.e. the reduction in  $w$ .

Turning to the intuition of the first two terms in (the brackets of) equation (26), we should again be detailed in the interpretation of the effects which are at work. Let us first consider the impact on the elasticity of labor demand  $\varepsilon_{l,h}$ . Starting with the initial collective wage cut, the labor demand elasticity might become less elastic



( $\partial\varepsilon_{l,h}/\partial w < 0$ ) or more elastic ( $\partial\varepsilon_{l,h}/\partial w > 0$ ). In turn, this has repercussions on the way trade unions use their hours of work as an adjustment device. To be more specific, in the former case, where  $\partial\varepsilon_{l,h}/\partial w < 0$ , when competing back to the initial Nash equilibrium by raising  $h$ , trade unions use this adjustment too excessively. The reason is that the initial reduction in the wage rate ‘distorts’ the choice of  $h$  in a way that makes unions more aggressive in using the hours of work. This is a case in which, *ceteris paribus*, the overall employment effect is to have even lower employment after the joint wage cut. The opposite holds for  $\partial\varepsilon_{l,h}/\partial w > 0$ , where labor demand becomes more elastic so that unions do not use the hours of work to fully go back to the employment level that prevailed in the initial Nash equilibrium. An analogous interpretation applies when all unions use the working hours as an instrument to respond to wage agreement. This time it is the adjustment, i.e. the joint increase in  $h$ , that renders labor demand more or less elastic. For  $\partial\varepsilon_{l,h}/\partial h < 0$ , the joint adjustment (that is to say, the increase in  $h$ ) implies that labor demand becomes more elastic with respect to the hours of work. As a consequence, the adjustment is not carried out to the ‘full extent’, i.e. to restore the initial Nash equilibrium, as it becomes more costly. For  $\partial\varepsilon_{l,h}/\partial h > 0$ , in contrast, the joint increase in  $h$  renders the labor demand elasticity less elastic which, in turn, induces the unions to raise the working hours even further.

The factor  $\varepsilon_{l,h}/\varepsilon_{l,w} \leq 1$  then corrects for the following. First, as explained above the elasticity  $\varepsilon_{l,h}$  is altered when employment changes due to both the initial coordination stimulus as well as the trade unions’ response. Second, note that the impact of the wage rate on labor demand is larger than the impact of  $h$  (of equal size) on labor demand. Thus, the factor  $\varepsilon_{l,h}/\varepsilon_{l,w} \leq 1$  scales down the  $\partial\varepsilon_{l,h}/\partial w$ -effect compared to the  $\partial\varepsilon_{l,h}/\partial h$ -effect.

### Some special cases

In order to judge the direction of the employment effect, it might be interesting to examine some special cases. First, for  $\alpha = 1$  and  $e'' = 0$  there is no employment effect, i.e. the agreement which marginally changes the wage rate but fails to cover the hours of work is not able to affect employment. The former denotes the special case of employment and working hours being perfect substitutes in determining the effective labor input to production. It ensures that the labor demand elasticity  $\varepsilon_{l,h}$  remains constant at  $\varepsilon_{l,h} = -1$  and is therefore not affected by the collective wage cut nor the joint adjustment of the working hours. The latter may be the even more restrictive scenario of a constant marginal disutility of labor ( $e'' = 0$ ), an assumption that is frequently used in the literature on union wage setting (see, for instance, Boeters and Schneider 1999 or Koskela and Schöb 2002a). As has been

pointed out before,  $e'' = 0$  eliminates one mechanism that prevents the trade unions to exactly return to the employment level of the initial Nash-equilibrium. Clearly, for  $\alpha = 1$  and  $e'' > 0$  the joint wage cut can effectively boost total employment even if trade unions engage in ‘competition’ by using the working hours as an instrument to maximize their well-being.

Turning to the more relevant scenario of labor and capital being less than perfectly substitutable ( $\alpha < 1$ ), we now have a case where the repercussions on the labor demand elasticity  $\varepsilon_{l,h}$  become important. The impact on  $\varepsilon_{l,h}$  seems to be stronger the smaller is  $\alpha$ ; see equation (6). However, we have to take into account that this labor demand elasticity is not only affected by the joint wage cut, but also by the subsequent joint reaction in the hours of work. It is therefore important to examine in which direction this value is eventually influenced after the working time adjustment is carried out. First, inspecting equations (6) to (8) reveals that  $h \cdot \partial\varepsilon_{l,h}/\partial h = (1 - \alpha)w \cdot \partial\varepsilon_{l,h}/\partial w$ . As has been set out before, the reason for the differential impact goes back to the different effects on effective labor input because the hours of work have a positive direct effect on  $L$ , which is not the case for the wage rate. Since the  $h \cdot \partial\varepsilon_{l,h}/\partial h$ -effect is smaller than the  $w \cdot \partial\varepsilon_{l,h}/\partial w$ -effect by the factor  $(1 - \alpha)$  and the  $w \cdot \partial\varepsilon_{l,h}/\partial w$ -effect itself is scaled down by the ratio  $\varepsilon_{l,h}/\varepsilon_{l,w} \leq 1$ , we are left with comparing  $\varepsilon_{l,h}/\varepsilon_{l,w}$  and  $(1 - \alpha)$ . Inspecting equation (5) then shows that  $\varepsilon_{l,h}/\varepsilon_{l,w} - (1 - \alpha) = -\alpha/\varepsilon_{l,w}$ , i.e. the  $w \cdot \partial\varepsilon_{l,h}/\partial w$ -effect is stronger than the  $h \cdot \partial\varepsilon_{l,h}/\partial h$ -effect by a factor that is proportional to the parameter  $\alpha$ . In fact, for  $\alpha = 0$  (together with  $e'' = 0$ ) again, a zero employment effect emerges. This time, the reason is twofold. First, both labor demand elasticities coincide so that the scaling factor in front of the  $\partial\varepsilon_{l,h}/\partial w$ -effect vanishes. Second, referring back to equations (6) to (8), a percentage change in the wage rate or the hours of work have an equal impact on the labor demand elasticity  $\varepsilon_{l,h}$ .

Summing up, for  $e'' = 0$  and  $0 < \alpha < 1$ , the sign of the overall employment effect depends on whether the initial wage cut renders the labor demand elasticity  $\varepsilon_{l,h}$  more or less elastic:

$$\text{sign} \left\{ \left. \frac{dl}{dw} \right|_{e''=0} \right\} = -\text{sign} \left\{ \frac{\partial\varepsilon_{l,h}}{\partial w} \right\}.$$

Thus, if the initial joint wage cut renders the labor demand elasticity with respect to the hours of work more elastic (less elastic), i.e.  $\partial\varepsilon_{l,h}/\partial w > 0$  ( $\partial\varepsilon_{l,h}/\partial w < 0$ ), the overall level of employment in each sector will be higher (lower) when trade unions have optimally responded to the reduction in  $w$  by using the hours of work.

## Welfare effect

Since the collective reduction in the wage rate is associated with a joint adjustment

in working hours and employment, a natural question is whether or not all union members eventually benefit from such a partial cooperation after the adjustment has taken place. The overall effect on union members' welfare is written as:

$$\frac{dV}{dw} = \frac{\partial V}{\partial w} + \frac{dh}{dw} \frac{\partial V}{\partial h}, \quad (27)$$

i.e. it comprises the initial welfare gain of a joint reduction in the wage rate (at a constant  $h$ ), i.e.  $\partial V/\partial w$ , and the subsequent welfare effect of the joint adjustment of the hours of work (at a constant  $w$ ), i.e.  $\partial V/\partial h$ , weighted with the magnitude of the adjustment. Both welfare terms on the right hand side of equation (27) have already been determined in subsection 3.1. Plugging (18), (20) and (22) into (27), the overall welfare effect is then given by (see Appendix 4):

$$\frac{dV}{dw} = \left( \frac{\partial NB(h)}{\partial h} \right)^{-1} \cdot \left( \frac{\partial \varepsilon_{l,h}}{\partial h} h - \frac{\varepsilon_{l,h}}{\varepsilon_{l,w}} \frac{\partial \varepsilon_{l,h}}{\partial w} w + \frac{h}{w} \varepsilon_{l,w} e'' \right) \frac{whl}{\varepsilon_{l,w}} \frac{l}{m}. \quad (28)$$

Not surprisingly, the direction of the welfare effect is identical to the employment effect of partial wage coordination, i.e. boosting employment is welfare enhancing. Since the starting point of coordination is the uncoordinated Nash equilibrium and the wage rate, the hours of work and thus employment are optimally chosen by each individual trade union, welfare effects can only arise if collective marginal actions can influence the countrywide employment level as it determines the unions' outside option. Consequently, the interpretation of (26) also applies to the welfare effect of (28).

### Repercussion on profit income

Even though the cooperation is among decentralized trade unions only, repercussion on each sector's profit income may be important in terms of a more comprehensive welfare analysis. Since employment is optimally chosen by each firm in the Nash equilibrium, any employment effects, which partial wage coordination might have, cannot affect profits. Following a joint wage cut with a subsequent adjustment in the hours of work, profit income is affected according to:

$$\frac{d[X - whl]}{dw} = -hl \left[ \frac{dh}{dw} \frac{w}{h} (1 - \alpha) + 1 \right]. \quad (29)$$

Interestingly, for the extreme case of  $\alpha = 1$ , the impact on profit income is independent on whether the hours of work are kept constant during the wage coordination [see equation (17) in subsection 3.1] or can be freely chosen by trade unions afterwards. Note that the reason for this, somewhat surprising, result cannot be that trade unions voluntarily choose not to adjust their working hours. On the contrary,

for employment and working hours being perfect substitutes ( $\alpha = 1$ ), the labor demand elasticity  $\varepsilon_{l,h}$  remains constant and trade unions will react to a wage cut by increasing the hours of work [see equation (24) and the discussion thereafter]. To be more specific, since  $\varepsilon_{l,h} = -1$ , the joint adjustment of the hours of work does not alter effective labor input  $L = lh$ , which, in turn, implies that profits remain unchanged in the course of the joint adjustment of the working hours. Thus, recipients of profit income should not be concerned whether the wage coordination agreement among trade unions covers the hours of work or not, given that employment and working hours are perfect substitutes.

### 3.3 Partial cooperation in the hours of work

As has been set out before, we observe that the hours of work are on the agenda of joint agreements (between trade unions and firms), while unions refuse to talk about the wage rate jointly. The German *Alliance for Jobs* is a prominent example. It may therefore be interesting to consider the consequences of a partial coordination in the hours of work.

From a theoretical perspective, Calmfors (1985) and Booth and Schiantarelli (1987) have analyzed a reduction in working time and its repercussion on employment when the wage rate is subject to changes afterwards. However, their starting point is a country-wide trade union that covers all workers so that unemployment is less severe than in countries with decentralized trade unions. The focus of these previous studies is therefore not on excess unemployment due to externalities among unions.

To make the scenario suitable to many other countries, let us therefore assume that, on a national level, all decentralized unions agree to marginally change (reduce) their working hours, whereas the choice of the wage rate is not subject to the joint agreement and can therefore be adjusted afterwards in an optimal manner from each union's perspective.

This time, each union is free to choose the wage rate such that in the new equilibrium the first-order condition  $\partial V_i / \partial w_i = 0$  must be restored. Again, the net benefit from changing the wage rate by a marginal unit can be rewritten in terms of elasticities as [see equation (13)]:

$$NB(w) = wh + \varepsilon_{l,w} [wh - e(h)] \left(1 - \frac{l}{m}\right) = 0,$$

where we have used the property of a symmetric equilibrium. For the wage adjust-

ment, we then need an expression for

$$\frac{dw}{dh} = -\frac{\partial NB(w)}{\partial h} \cdot \left( \frac{\partial NB(w)}{\partial w} \right)^{-1}, \quad (30)$$

with

$$\frac{\partial NB(w)}{\partial w} = h(1 + \varepsilon_{l,w}) + \frac{\partial \varepsilon_{l,w}}{\partial w} [wh - e(h)] \left( 1 - \frac{l}{m} \right) - \varepsilon_{l,w} \frac{l}{m} \left( h + \frac{wh - e(h)}{w} \varepsilon_{l,w} \right).$$

Note that for a similar stability reason as in the previous subsection we must have  $\partial NB(w)/\partial w < 0$ . Consequently, if a joint change in the hours of work raises (reduces) the net marginal benefit from a wage increase, all trade unions will react by increasing (lowering) the wage rate. The direction of the wage adjustment is then solely given by the sign of

$$\begin{aligned} \frac{\partial NB(w)}{\partial h} &= w + \varepsilon_{l,w}(w - e') \left( 1 - \frac{l}{m} \right) - \frac{wh - e(h)}{h} \frac{l}{m} \varepsilon_{l,h} \varepsilon_{l,w} \\ &\quad + \frac{\partial \varepsilon_{l,w}}{\partial h} [wh - e(h)] \left( 1 - \frac{l}{m} \right). \end{aligned} \quad (31)$$

Proceeding in an analogous way to the previous subsection, it is instructive to have a closer look at the impact of a joint *reduction* in the hours of work on the marginal benefit and marginal cost of increasing the wage rate. The first term in the upper line of equation (31) captures that a joint marginal reduction in the working hours also reduces the marginal benefit from raising the wage rate since all employed union members receive less total wage compensation. Since the reduction in the hours of work also lowers the total rent from being employed, wage moderation *ceteris paribus* becomes less interesting for the unions; see the second term in (31). Reducing the hours of work in all sectors will boost the countrywide employment level and therefore increase the re-employment probability of unemployed union members. Unions will *ceteris paribus* respond with more aggressive wage claims, see the third term in equation (31). Finally, we have to take into account that a joint cut in  $h$  has an impact on the labor demand elasticity with respect to the wage rate. This effect is captured by the last term in (31) and can go in either direction. If the collective reduction in the hours of work renders the labor demand elasticity  $\varepsilon_{l,w}$  more (less) elastic, the wage rate becomes more (less) costly (at the margin) as a trade union instrument. For  $\partial \varepsilon_{l,w}/\partial h \leq 0$ , we can unambiguously infer that unions react to the cut in  $h$  by raising the wage rate (see Appendix 5 for details). This condition would be sufficient to conclude that the employment effect is smaller than the one under full cooperation.

In fact, Hunt (1999) presents empirical evidence that German unions claim higher wages following a reduction in standard hours of work to keep monthly earning

almost unaffected. Similar evidence is reported for Sweden (see, e.g., Jacobson and Ohlsson 2000 or Skans 2004).

### Employment effect

In the light of the trade unions' potential wage response, the overall impact of a reduction in working time on employment is ambiguous *a priori*. Formally, it is given by

$$\frac{dl}{dh} = \frac{\partial l}{\partial h} + \frac{dw}{dh} \frac{\partial l}{\partial w}.$$

As is shown in Appendix 6, we are able to write the overall employment effect of a joint cut in the hours of work, taking into account that all trade unions react by adjusting their wage rate as follows:

$$\frac{dl}{dh} = \left( \frac{\partial NB(w)}{\partial w} \right)^{-1} \cdot \left( \frac{\partial \varepsilon_{l,w}}{\partial h} h - \frac{\partial \varepsilon_{l,w}}{\partial w} w \frac{\varepsilon_{l,h}}{\varepsilon_{l,w}} + \varepsilon_{l,h} - \varepsilon_{l,w} \right) l. \quad (32)$$

When discussing the direction of the total employment effect, it is again necessary to interpret the terms in brackets of equation (32). This time, the change of the labor demand elasticity with respect to the wage rate becomes important. If it becomes more elastic, *ceteris paribus*, either due to the initial joint reduction in the working time ( $\partial \varepsilon_{l,w} / \partial h > 0$ ) or the subsequent joint increase in the wage rate ( $\partial \varepsilon_{l,w} / \partial w < 0$ ), the wage rate becomes a more costly instrument for the unions and is therefore not used to go all the way back to the initial equilibrium. This, in turn, contributes to a positive overall employment effect when working time is reduced. A second effect is relevant in terms of overall employment. Even if we fully abstract from the changes in the labor demand elasticity, there is, in general, a favorable employment effect since  $\varepsilon_{l,w} \leq \varepsilon_{l,h}$ ; see the last two terms in brackets of equation (32).

In the light of the ambiguity of the overall employment effect so far, the term in brackets in equation (32) can be written as  $-\alpha \varepsilon_{l,w} (2 + F''' L / F'')$ . Interestingly, from (16) we already know that we must have  $2 + F''' l h^\alpha / F'' > 0$  in a symmetric Nash equilibrium. As  $\partial NB(w) / \partial w < 0$  and  $\varepsilon_{l,w} < 0$ , the whole expression in (32) then becomes non-positive. The only possibility of a zero employment effect is the extreme case of  $\alpha = 0$ . In this scenario, the trade unions will claim sufficiently higher wages in response to the reduced working time such that any employment effect is fully washed away. The explanation runs as follows. For  $\alpha = 0$ , we know from equations (5) and (9) that  $\varepsilon_{l,w} = \varepsilon_{l,h}$  and  $(\partial \varepsilon_{l,w} / \partial w) \cdot w = (\partial \varepsilon_{l,h} / \partial h) \cdot h$  so that the wage rate amounts to a perfect mimicry of the working hours in restoring the initial Nash equilibrium. Otherwise, i.e. for  $\alpha > 0$ , a favorable total employment effect remains since the wage rate is not fully used to go back to the initial employment level. In fact, the second-order condition of the unions' decision problem with respect

to the wage rate is sufficient to ensure that the adjustment of the wage rate is small enough not to compensate the effect of the working time reduction.

### **Welfare effect**

For the repercussion of each union's welfare, we might expect a similar pattern as in the preceding subsection in the sense that the sign of the employment effect also determines the direction of the welfare effect. This can be confirmed by inspecting

$$\begin{aligned} \frac{dV}{dh} &= \frac{\partial V}{\partial h} + \frac{dw}{dh} \frac{\partial V}{\partial w} \\ &= \left( \frac{\partial NB(w)}{\partial w} \right)^{-1} \cdot \left( \frac{\partial \varepsilon_{l,w}}{\partial w} w \frac{\varepsilon_{l,h}}{\varepsilon_{l,w}} - \frac{\partial \varepsilon_{l,w}}{\partial h} h + \varepsilon_{l,w} - \varepsilon_{l,h} \right) \frac{whl}{\varepsilon_{l,w}} \frac{l}{m}. \end{aligned}$$

### **Repercussion on profit income**

Finally, we can again analyze whether profit income is increased or not following a collective reduction in working time with wage autonomy left to each single trade union. We arrive at:

$$\frac{d[X - whl]}{dh} = -wl \left[ (1 - \alpha) + \frac{dw}{dh} \frac{h}{w} \right].$$

Recalling the reference case of full coordination [see equation (19)], profits remain unchanged when the wage rate is not adjusted and employment and working time are perfect substitutes ( $\alpha = 1$ ). However, if the wage rate remains at discretion of unions, the special case of perfect substitutes implies that recipients of profit income are worse-off if unions agree to jointly reduce working time. To see this, note that equation (8) implies that  $\varepsilon_{l,w}$  remains constant for  $\alpha = 1$  and Appendix 5 shows that this is sufficient to infer that  $dw/dh < 0$ .

## **4 Concluding remarks**

When individual decision-making imposes external effects on others, the resulting equilibrium will be inefficient. It is well known that cooperating on the externality generating activity can make all participants better off. In our framework, we apply this idea to decentralized trade union behavior, where the externality runs through each small union's contribution to the overall (un)employment rate. However, each union has two possibilities to impose the external effect - by choosing its wage rate and the hours of work, respectively. An internalization agreement should therefore cover both of them. If only a partial agreement on one instrument is possible, the outcome of such coordination is less clear-cut. In general, we can conclude that they are less effective. For very special cases, they have no impact at all or are even

counterproductive. A constant marginal disutility of labor can contribute to such extreme cases. Together with employment and working time being perfect substitutes, partial wage cooperation is unable to affect employment and welfare. Given that employment and the hours of work are less than perfectly substitutable, joint (partial) wage moderation even lowers employment if the labor demand becomes less elastic with respect to the working hours.

It is important to note that the only coordination agreement considered in this contribution is among individual trade unions. On the one hand, this approach might be a suitable starting point as it puts a lower bound what can be expected from coordination given that it is not possible to attract other participants in negotiations (government, firms). Thus, in our setting, the best outcome can only entail eliminating externalities between trade unions. On the other hand, of course, even stronger effects on employment and welfare are possible if other parties effectively joint the agreement. In the spirit of McDonald and Solow (1981), both parties can commit to policies that deviate from their individual optimum but make both parties together better off.

What is the policy relevance of this contribution? Basically, our analysis has only shown that institutional arrangement matter for the effectiveness of agreements among private agents. If trade unions can influence the employment level in their respective sector by using more than one instrument and the employment level imposes external effects on members of other unions, then any cooperation must include all these instruments that affect employment. If this is not possible, coordination is most likely to be less effective or may even be doomed to fail. This is what policy makers should be interested in. In the light of the (potential) inability of private institutions to correct for an external effect, assigning this internalization to the government will, in principle, perform better. The results therefore support government interventions which make unions be aware of the full costs of their behavior.



# Appendix

## 1. Fully centralized wage setting

A countrywide monopoly trade union would maximize

$$\max_{w,h} V = l(w, h) [wh - e(h)] + [m - l(w, h)] \frac{l(w, h)}{m} [wh - e(h)].$$

The first-order conditions are

$$\left(2 - \frac{l}{m}\right) lh + 2 \frac{\partial l}{\partial w} [wh - e(h)] \left(1 - \frac{l}{m}\right) = 0$$

and

$$l [w - e'(h)] \left(2 - \frac{l}{m}\right) + 2 \frac{\partial l}{\partial h} [wh - e(h)] \left(1 - \frac{l}{m}\right) = 0,$$

which, again, imply  $l < m$  and  $w > e'(h)$ . However, rewriting both conditions shows that the wage rate and the hours of work are now chosen to attain a higher employment level as is the case with decentralized trade unions. To see this, we express both conditions as

$$-\frac{l}{m}lh + 2 \left\{ lh + \frac{\partial l}{\partial w} [wh - e(h)] \left(1 - \frac{l}{m}\right) \right\} = 0$$

and

$$-\frac{l}{m}l[w - e'(h)] + 2 \left\{ l[w - e'(h)] + \frac{\partial l}{\partial h} [wh - e(h)] \left(1 - \frac{l}{m}\right) \right\} = 0.$$

The terms in curly brackets are equivalent to the first-order conditions in the case of decentralized decision problem. Since in both equations an additional negative term enters, a central union perceives the marginal net benefit from increasing  $w$  and  $h$ , respectively, to be lower and will therefore choose lower levels of both the wage rate and the hours of work. In turn, this implies a higher employment level in the country.

## 2. The sign of $dh/dw$

Recalling equation (22) in the text, i.e.

$$\frac{dh}{dw} = -\frac{\partial NB(h)}{\partial w} \cdot \left(\frac{\partial NB(h)}{\partial h}\right)^{-1},$$

we know that stability of the Nash equilibrium requires  $\partial NB(h)/\partial h < 0$ . Hence,

$$\text{sign} \left\{ \frac{dh}{dw} \right\} = \text{sign} \left\{ \frac{\partial NB(h)}{\partial w} \right\}.$$

We have

$$\frac{\partial NB(h)}{\partial w} = h + \varepsilon_{l,h} h \left(1 - \frac{l}{m}\right) - \varepsilon_{l,w} \varepsilon_{l,h} \frac{l}{m} \frac{wh - e(h)}{w} + \frac{\partial \varepsilon_{l,h}}{\partial w} [wh - e(h)] \left(1 - \frac{l}{m}\right),$$

which is rewritten as

$$\frac{\partial NB(h)}{\partial w} = h(1 + \varepsilon_{l,h}) - \frac{l}{m} \varepsilon_{l,h} h \left[1 + \varepsilon_{l,w} \frac{wh - e(h)}{wh}\right] + \frac{\partial \varepsilon_{l,h}}{\partial w} [wh - e(h)] \left(1 - \frac{l}{m}\right).$$

As stated in the text,  $\varepsilon_{l,h} \leq -1$ . From equation (13), we know that

$$\varepsilon_{l,w} \frac{wh - e(h)}{wh} = - \left(1 - \frac{l}{m}\right)^{-1},$$

i.e. since  $0 < (1 - l/m) < 1$ , we have  $\varepsilon_{l,w} [wh - e(h)]/wh < -1$ . Hence, for  $\partial \varepsilon_{l,h}/\partial w \leq 0$  we can unambiguously sign

$$\frac{\partial NB(h)}{\partial w} < 0,$$

so that a joint wage cut increases the hours of work.

### 3. Derivation of equation (26): The employment effect of a partial agreement with respect to the wage rate

The overall employment effect is given by

$$\frac{dl}{dw} = \frac{\partial l}{\partial w} + \frac{dh}{dw} \frac{\partial l}{\partial h}.$$

Inserting in the corresponding expression for  $dh/dw$ , this becomes

$$\frac{dl}{dw} = \frac{1}{\partial NB(h)/\partial h} \left[ \frac{\partial NB(h)}{\partial h} \frac{\partial l}{\partial w} - \frac{\partial NB(h)}{\partial w} \frac{\partial l}{\partial h} \right].$$

Plugging in (23) and (24) and using (15) as given in the text, the expression in brackets is written as follows

$$\left[ \left( \frac{\partial \varepsilon_{l,h}}{\partial h} \frac{\varepsilon_{l,w}}{w} - \frac{\partial \varepsilon_{l,h}}{\partial w} \frac{\varepsilon_{l,h}}{h} \right) (wh - e(h)) \left(1 - \frac{l}{m}\right) - \varepsilon_{l,w} \frac{h}{w} e'' \right] l.$$

Using equation (13), i.e.

$$[wh - e(h)] \left(1 - \frac{l}{m}\right) = - \frac{wh}{\varepsilon_{l,w}}, \quad (33)$$

this can be rewritten as

$$\left( \frac{\partial \varepsilon_{l,h}}{\partial w} w \frac{\varepsilon_{l,h}}{\varepsilon_{l,w}} - \frac{\partial \varepsilon_{l,h}}{\partial h} h - \frac{h}{w} \varepsilon_{l,w} e'' \right) l.$$

#### 4. The welfare effect of a partial agreement with respect to the wage rate

We have

$$\begin{aligned}\frac{dV}{dw} &= \frac{\partial V}{\partial w} + \frac{dh}{dw} \frac{\partial V}{\partial h} \\ &= \left( \frac{\partial NB(h)}{\partial h} \right)^{-1} \frac{l}{m} \left\{ -h \cdot \frac{\partial NB(h)}{\partial h} + [w - e'(h)] \cdot \frac{\partial NB(h)}{\partial w} \right\} l,\end{aligned}$$

where, after using (23), (24) and (15), the term in curly brackets can be written as

$$\left( \frac{\varepsilon_{l,h}}{\varepsilon_{l,w}} \frac{\partial \varepsilon_{l,h}}{\partial w} w - \frac{\partial \varepsilon_{l,h}}{\partial h} h \right) [wh - e(h)] \left( 1 - \frac{l}{m} \right) + h h e''.$$

Recalling equation (33), this expression reads

$$\left( \frac{\partial \varepsilon_{l,h}}{\partial h} h - \frac{\varepsilon_{l,h}}{\varepsilon_{l,w}} \frac{\partial \varepsilon_{l,h}}{\partial w} w + \frac{h}{w} \varepsilon_{l,w} e'' \right) \frac{wh}{\varepsilon_{l,w}}.$$

#### 5. The sign of $dw/dh$

As has been set out in the text, the sign of  $\partial NB(w)/\partial h$  also determines the direction of the adjustment of the wage rate following a joint reduction in the hours of work.

Rewriting this expression yields

$$\begin{aligned}\frac{\partial NB(w)}{\partial h} &= w + \varepsilon_{l,w}(w - e') \left( 1 - \frac{l}{m} \right) - \frac{wh - e(h)}{h} \left( 1 - \frac{l}{m} \right) \varepsilon_{l,h} \varepsilon_{l,w} \\ &\quad - \frac{wh - e(h)}{h} \varepsilon_{l,h} \varepsilon_{l,w} + \frac{\partial \varepsilon_{l,w}}{\partial h} [wh - e(h)] \left( 1 - \frac{l}{m} \right).\end{aligned}$$

Using equation (13), the third term in the upper line is equivalent to

$$-\frac{wh - e(h)}{h} \left( 1 - \frac{l}{m} \right) \varepsilon_{l,h} \varepsilon_{l,w} = \varepsilon_{l,h} w,$$

so that we are able to write

$$\begin{aligned}\frac{\partial NB(w)}{\partial h} &= w(1 + \varepsilon_{l,h}) + \varepsilon_{l,w}(w - e') \left( 1 - \frac{l}{m} \right) - \frac{wh - e(h)}{h} \varepsilon_{l,h} \varepsilon_{l,w} \\ &\quad + \frac{\partial \varepsilon_{l,w}}{\partial h} [wh - e(h)] \left( 1 - \frac{l}{m} \right),\end{aligned}$$

where all term in the upper line are negative in sign. Thus, if  $\partial \varepsilon_{l,w}/\partial h \leq 0$ , a joint reduction in  $h$  will induce trade unions to unambiguously raise the wage as a response.

**6. Derivation of equation (32): The employment effect of a partial agreement with respect to the working hours**

We have,

$$\begin{aligned}\frac{dl}{dh} &= \frac{\partial l}{\partial h} + \frac{dw}{dh} \frac{\partial l}{\partial w}, \\ &= \frac{1}{\partial NB(w)/\partial w} \left[ \frac{\partial l}{\partial h} \frac{\partial NB(w)}{\partial w} - \frac{\partial l}{\partial w} \frac{\partial NB(w)}{\partial h} \right]\end{aligned}\quad (34)$$

where the term in brackets can be written as

$$l \left[ \varepsilon_{l,h} - \varepsilon_{l,w} + \left( \varepsilon_{l,h} \frac{\partial \varepsilon_{l,w}}{\partial w} - \varepsilon_{l,w} \frac{\partial \varepsilon_{l,h}}{\partial h} \right) \frac{wh - e(h)}{w} \left( 1 - \frac{l}{m} \right) \right].$$

Again, making use of equation (33), this can be simplified to

$$l \left[ \varepsilon_{l,h} - \varepsilon_{l,w} + \left( \varepsilon_{l,w} \frac{\partial \varepsilon_{l,w}}{\partial h} - \varepsilon_{l,h} \frac{\partial \varepsilon_{l,w}}{\partial w} \right) \frac{h}{\varepsilon_{l,w}} \right].$$

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