Borrowing Constraints, Entrepreneurial Risks, and the Wealth Distribution in a Heterogeneous Agent Model

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Borrowing Constraints, Entrepreneurial Risks, and the Wealth Distribution in a Heterogeneous Agent Model

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Abstract

This paper deals with credit market imperfections and idiosyncratic risks in a two–sector heterogeneous agent dynamic general equilibrium model of occupational choice. We focus especially on the effects of tightening financial constraints on macroeconomic performance, entrepreneurial risk–taking, and social mobility. Contrary to many models in the literature, our comparative static results cover the entire range of borrowing constraints, from complete markets to a perfectly constrained economy. In our baseline model, we find substantial gains in output, welfare, and wealth equality associated with relaxing the constraints, but argue that it might also prove worthwhile to examine the marginal gains from credit market improvements. Interestingly, the amount of entrepreneurial activity and social mobility increases if borrowing constraints become more tight. These results can be attributed to the general equilibrium nature of our approach, where optimal firm sizes and the demand for credit are determined endogenously. The comparative static results on the entrepreneurship rate and social mobility respond sensitively to a change in income persistence.

Keywords: DSGE model, wealth distribution, occupational choice, borrowing constraints
JEL classification: C68, D3 , D8, D9, G0, J24

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1 Introduction

This paper examines the effects of credit market imperfections and idiosyncratic risks on occupational choice, macroeconomic performance, as well as on the income and wealth distribution. Our analysis contributes to recent literature on dynamic stochastic heterogeneous agent general equilibrium models concerned with risk and distributional dynamics, for instance, Quadrini (2000), Meh (2005), Boháček (2006, 2007) and Cagetti and De Nardi (2006*a,b,c*).

We develop a model which combines the features of a Huggett (1993) / Aiyagari (1994)–type economy with occupational choice under risk à la Kihlstrom and Laffont (1979) and Kanbur (1979*a,b*), and the two–sector approach of Romer (1990), but without endogenous growth. In each period of time, the risk–averse agents choose between between two alternative occupations. They either set up an enterprise in the intermediate goods industry which is characterized by monopolistic competition.[1] Or, they supply their labor endowment to the production of a final good in a perfectly competitive market. Producers of the final good use capital and labor inputs, and differentiated varieties of the intermediate good. All households are subject to an income risk. Managerial ability and productivity as a worker follow independent random processes. Entrepreneurial activity is rewarded with a higher expected income. Similar to Lucas (1978), there is no aggregate risk.

The economic performance in the intermediate goods industry crucially depends on two factors: uncertainty and credit constraints. Business owners face an firm–specific productivity shock, and there are no markets available for pooling the idiosyncratic risks. Physical capital is the single input factor in the intermediate goods industry. Entrepreneurs maximize their profits if their business operates at the optimal firm size. For an individual wealth too small to maintain the optimal firm size, the firm–owner would want to borrow the remaining amount on the credit market, where he might be subject to financial constraints. If the entrepreneur is wealthy enough, he operates his business at the profit–maximizing level and supplies the rest of his wealth to the capital market. Contrary to many models in the literature the two–sector general equilibrium approach allows us to endogenously determine optimal firm sizes and credit constraints, and we do not have to fall back on fixed investment projects (or entry costs respectively) in order to analyze the effects of credit market frictions. There is no further portfolio choice in our framework. To this end, our approach draws a simple picture of the empirical result, stated by Heaton and Lucas (2000), that the entrepreneurial households' business wealth on average constitutes a relevant fraction of their total wealth.

Capital accumulation plays a twofold role in the context outlined above: On the one hand, it endows individuals with the wealth necessary to set–up and op-

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[1]See also Clemens (2006*a,b*) for entrepreneurial risk–taking in a general equilibrium context.

erate a firm. On the other hand, buffer–stock saving provides a self–insurance on intertemporal markets against the non–diversifiable income risk. Accordingly, we find that wealthier households are more likely to be members of the entrepreneurial class than poorer ones and there is a marked concentration of wealth in the hands of entrepreneurs which is consistent with recent empirical findings (cf. Quadrini, 1999; Holtz-Eakin *et al.*, 1994a). Upward mobility of entrepreneurs in our model is primarily accumulation driven. The riskiness of entrepreneurial incomes looses its importance for occupational choice once the household's income share generated from profits declines relative to his capital income. Nevertheless, in accordance with Hamilton (2000), many entrepreneurs of our model enter and persist in business despite the fact that they have lower initial earnings than average wage incomes.

We are especially interested in the question of how tightening financial constraints affects the macroeconomic general equilibrium regarding aggregate output, the sectoral allocation of capital and labor, factor prices, the income and wealth distribution, occupational choice as well as the between–group mobility of households. Our comparative static analysis covers the entire range of borrowing constraints, from complete markets to a perfectly constrained economy. This is a novel approach since many models of the literature consider a fixed equity–to–loan ratio, or rest with a comparison of complete vs. a specific incomplete market, or they focus on the no–credit market scenario. We find that increasing the degree of constraint is accompanied by substantial losses in aggregate output, consumption, wealth holdings, and welfare, while wealth inequality increases.

Reviewing the empirical evidence, there is a strong support for the hypothesis that borrowing constraints are an impediment to entering entrepreneurship; see Evans and Leighton (1989), Evans and Jovanovic (1989), Holtz-Eakin *et al.* (1994b), Blanchflower and Oswald (1998), Moskowitz and Vissing-Jørgensen (2002), as well as Desai *et al.* (2003).

Gentry and Hubbard (2004) point out that external financing has important implications for individual investment and saving. This evidence is challenged by Hurst and Lusardi (2004), who find that the likelihood of entering entrepreneurship relative to initial wealth is flat over a large range of the wealth distribution and increasing only for higher wealth levels of workers.

The general equilibrium nature of our approach generates surprising and almost counter–intuitive results regarding the impact of credit constraints on occupational choice under risk. If the idiosyncratic risks are serially correlated, more households choose the entrepreneurial profession in the constrained compared to the unconstrained economy which is accompanied by a reduction in the average firm size, both results contradicting findings reported in Cagetti and De Nardi (2006a). Wealth inequality does not necessarily decline if we relax borrowing constraints. Additionally, we observe an increase in between–group mobility, if credit constraints become more binding. Workers and entrepreneurs with high individual productivity

tend to remain in their present occupation, whereas low productivity individuals are more likely to switch between professions.

These results reverse completely, if we consider *iid* shocks to individual productivity. In this case, credit constraints actually are an impediment to entrepreneurship. Only the wealthy workers tend to switch between occupations and between–group mobility drops down sharply for an increase in the tightness of credit constraints. Regarding the functional distribution of income, we find that credit constraints have a redistributive effect by raising the profit income share at the cost of capital incomes. The results indicate that the stochastic nature of the underlying idiosyncratic shocks also plays an important role for the explanation of the general equilibrium effects of financial constraints and credit market imperfections.

Recent contributions in this area of research suffer from several shortcomings which our approach aims to overcome. In Quadrini (2000), occupational choice and the level of entrepreneurship is (more or less) entirely governed by the underlying productivity shocks. Li (2002) and Boháček (2006) discuss economies with a single sector of production which does not allow for factor movements between industries and therefore neglects factor substitution. In our model, producers of the intermediate and the final good are subject to competition, especially with respect to capital demand. Our approach does not have fixed entry costs (in terms of discrete investment projects) of entrepreneurship as in Ghatak *et al.* (2001), Fernández-Villaverde *et al.* (2003) or Clementi and Hopenhayn (2006) . Instead, we have an endogenously determined optimal firm size and no discontinuities in individual credit demand. Occupational choice, entrepreneurial activity and performance crucially depend on monopoly profits, market shares and relative factor scarcity in the two sectors of production. Also different to Cagetti and De Nardi (2006*a*) or Kitao (2008), the entrepreneurs of our economy are essential for aggregate output. We will show that the interdependence of sectors is important for the general equilibrium results on occupational choice, between–group mobility and the income and wealth distribution, and contributes to the explanation of the sometimes counter–intuitive effects of borrowing constraints outlined above. To this end, the present paper is an extended version of Clemens and Heinemann (2006) and Clemens (2008), where we focus on the relation between entrepreneurial risk–taking and growth but do not consider financial constraints.

The paper is organized as follows: Section 2 develops the two–sector model. We describe the equilibrium associated with a stationary earnings and wealth distribution. Because the formal structure of the model does not allow for analytical solutions, we perform numerical simulations of a calibrated model in order to examine the general equilibrium effects of an increase in the tightness of credit constraints. Section 3 gives information on the calibration procedure and related empirical evidence. Section 4 discusses the simulation results. Section 5 concludes. Technical details are relegated to the Appendix.

2 The Model

2.1 Overview

We consider a neoclassical growth model with two sectors of production. Drawing from Quadrini (2000) and Romer (1990), we consider a corporate sector with perfectly competitive large firms who hire capital and labor services and use an intermediate good in order to produce a homogeneous output which can be consumed or invested respectively. The intermediate goods industry (noncorporate sector) consists of a large number of small firms operating under the regime of monopolistic competition. Each firm in this sector is owned and managed by an entrepreneur. Both sectors of production are essential.

Market activity in the intermediate goods industry is constrained. In order to run the business at the profit–maximizing firm size, entrepreneurs either possess sufficient wealth of their own, or they need to compensate for their lack of equity by borrowing on the credit market, where they might be subject to borrowing constraints. The two–sector setting allows us to endogenously relate financial constraints to individual characteristics and overall market activity.

The economy is populated by a continuum $[0,1]$ of infinitely–lived households, each endowed with one unit of labor. In each period of time, individuals follow their occupation predetermined from the previous period and make a decision regarding their future profession, which is either to become producers of the intermediate good or to supply their labor services to the production of the final good. Labor efficiency as well as entrepreneurial productivity are idiosyncratic random variables. Regarding the associated income risk, we assume that wage incomes are less risky than profit incomes. There is no aggregate risk.

With respect to the timing of events, we assume that individual occupational choice takes place before the resolution of uncertainty. Once the draw of nature has occurred, entrepreneurs as well as workers in the final goods sector know their individual productivity. Those monopolists, who now discover their own wealth being too low to operate at the optimal firms size, will express their capital demand on the credit market, probably become subject to credit–constraints, and then start production. After labor and profit income is realized, the households decide on how much to consume and to invest. There is no capital income risk and no risk of production in the corporate sector.

2.2 Final Goods Sector

The representative firm of the final goods sector produces a homogeneous good Y using capital K_F, labor L, and varieties of an intermediate good $x(j), j \in [0, \lambda]$ as inputs. Production in this sector takes place under perfect competition and the price

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of Y is normalized to unity. The production function is of the generalized CES–form[2]

$$Y = \left(K_F^\gamma L^{1-\gamma}\right)^{1-\alpha} \int_0^\lambda x(j)^\alpha \, \mathrm{d}j, \qquad 0 < \alpha < 1, \quad 0 < \gamma < 1. \tag{1}$$

Each type of intermediate good employed in the production of the final good is identified with one monopolistic producer in the intermediate goods sector. Consequently, the number of different types is identical with the population share λ of entrepreneurs in the population. The number of entrepreneurs is determined endogenously through occupational choices of the agents, which will be described below. Additive–separability of (1) in intermediate goods ensures that the marginal product of input j is independent of the quantity employed of $j' \neq j$. Intermediate goods are close but not perfect substitutes in production.

The profit of the representative firm in the final goods sector, π_F, is given in each period by

$$\pi_F = Y - wL - (r+\delta)K_F - \int_0^\lambda p(j)\,x(j)\,\mathrm{d}j, \tag{2}$$

where $p(j)$ denotes the price of intermediate good j. We further assume physical capital to depreciate over time at the constant rate δ, such that the interest factor is given by $R = 1 + r - \delta$. Optimization yields the profit maximizing factor demands consistent with marginal productivity theory

$$K_F = (1-\alpha)\gamma \frac{Y}{r+\delta}, \tag{3}$$

$$L = (1-\alpha)(1-\gamma)\frac{Y}{w} \tag{4}$$

$$x(j) = K_F^\gamma L^{1-\gamma}\left(\frac{\alpha}{p(j)}\right)^{1/(1-\alpha)}. \tag{5}$$

The monopolistic producer of intermediate good $x(j)$ faces the isoelastic demand function (5), where the direct price elasticity of demand is given by $-1/(1-\alpha)$. Condition (4) describes aggregate labor demand in efficiency units. Equation (3) is the final good sector demand for capital services.

2.3 Intermediate Goods Sector

The intermediate goods sector consists of the population fraction λ of entrepreneurs who self–employ their labor endowment by operating a monopolistic firm. Each monopolist produces a single variety j of the differentiated intermediate good by

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[2]All macroeconomic variables are time–dependent. For notational convenience, we will drop the explicit time–notation unless necessary.

employing capital from own wealth and borrowed resources according to the identical constant returns to scale technology of the form

$$x(j) = \theta(i)_e k(i) \ . \tag{6}$$

Firm owners are heterogeneous in terms of their talent as entrepreneurs. They differ with respect to the realization of an idiosyncratic productivity shock $\theta(i)_e$ which is assumed to be non–diversifiable and uncorrelated across firms. We will give more details on the properties of the shock below. Entrepreneurs hire capital after the draw of nature has occurred. The firm problem essentially is a static one. Under perfect competition of the capital market, the producer treats the rental rate to capital as exogenously given and maximizes his profit

$$\pi(k(i), \theta(i)_e) = p(j) x(j) - (r + \delta) k(i) \ . \tag{7}$$

Utilizing the demand function for intermediate good type–i, (5), and the production technology (6), the optimal firm decision can be expressed in terms of the optimal firm size $k(i)^*$ as a function of capital input, which is given by:

$$k(i)^* = L (\theta(i)_e)^{\frac{\alpha}{1-\alpha}} \left(\frac{\gamma w}{(1-\gamma)(r+\delta)} \right)^{\gamma} \ . \tag{8}$$

Because capital demand takes place after the draw of nature has occurred, there is no individual capital risk and no under–employment of input factors. The optimal firm size increases with random individual productivity $\theta(i)_e$, such that more productive business owners demand more capital on the capital market. Labor input in efficiency units determines the optimal firm size by means of the demand function for intermediate good type j. Aggregate employment is a weighted average and depends on the size of the labor force $1 - \lambda$, i.e. the population fraction of agents choosing the occupation of a worker, and the idiosyncratic shock on labor productivity θ_w. The larger the labor force $1 - \lambda$, the higher—*ceteris paribus*—will be aggregate employment L. This goes along with fewer monopolists in the intermediate goods industry, less competition, and a larger market share, as measured by the optimal firm size.

2.4 Incomes and Equilibrium Income Shares in the Unconstrained Economy

Households derive income from three sources: labor income, capital income and monopolistic profits. The technology parameters α and γ determine the division of aggregate income among the three income sources in the absence of financial constraints on entrepreneurial activity. According to marginal productivity theory, we obtain from (1) a labor share of $(1-\alpha)(1-\gamma)$ and a capital share of $(1-\alpha)\gamma$. The remaining income share α accrues to the two types of income generated in the intermediate goods sector, and splits on profits with $\alpha(1-\alpha)$ and capital income with α^2, respectively, such that the economy–wide capital share amounts to $(1-\alpha)\gamma + \alpha^2$.

2.5 Capital Market and Financial Constraints

Firms of the final goods sector and the intermediate goods industry differ with respect to access to financial markets. While the first are not constrained in their financing, the latter face greater difficulties in diversifying the risk from their entrepreneurial activities and, moreover, are subject to borrowing constraints. Entrepreneurs of the intermediate goods industry, who are wealth–constrained in operating their business at the optimal size (8), seek external financing from financial intermediaries. The credit market is imperfect with respect to lenders not being able to enforce loan–repayment due to limited commitment of borrowers (cf. Banerjee and Newman, 1993). In order not to default on loan contracts, borrowing amounts are limited, and individual wealth acts as collateral. We do not explicitly model financial intermediaries and assume that there is no difference between borrowing and lending rates.

In case of default, the financial intermediator is able to seize a fraction of the borrowers gross capital income $(1+r)a(i)$. Alternatively, one could assume the entrepreneur's profit income to act as collateral. The major difference between the two approaches is that, in the first case, borrowing amounts are entirely determined by the debtors individual wealth $a(i)$, whereas in the second, they also depend on his entrepreneurial talent $\theta(i)_e$, which might be private information. We will discuss the consequences of the second formulation in a separate treatment below.

The creditor will lend to the borrower only the amount consistent with the borrower's incentive compatibility constraint, such that it is in the borrower's interest to repay the loan and there is no credit default in equilibrium.

Let $k(i) = a(i) + b(i)$ be the firm size an entrepreneur is able to operate at from own wealth $a(i)$ and borrowed resources $b(i)$. This operating capital $k(i)$ is not necessarily equal to the optimal firm size $k(i)^*$ determined in (8). An entrepreneur with individual wealth $a(i)$ lower than $k(i)^*$ would want to borrow the amount $k(i)^* - a(i)$. In case of $k(i) < k(i)^*$ the firm faces a borrowing constraint. Incentive compatibility requires that it is never optimal for the borrower to default, that is

$$\pi(i) + (1+r)a(i) \geqslant \pi(i) + b(i)(1+r) + (1-\phi)(1+r)a(i)$$
$$b(i) \leq \phi a(i) \, . \tag{9}$$

The borrowing amount is limited such that the maximum possible loan is proportional to the borrowers individual wealth $a(i)$. The parameter ϕ is a measure for the extent to which a lender can use the borrower's wealth income as collateral. Credit constraints become less tight with rising ϕ and vanish for large ϕ. The limiting cases consequently reflect the two cases of either complete enforceability ($\phi \to \infty$) or no enforceability ($\phi = 0$), such that in the first case the borrower is considered solvent, whereas in the second one he is not. The sensitivity of results with respect to changes ϕ constitutes the major part of our numerical analysis later on.

Summing up, the operating firm size $k(i)$ of entrepreneur i with productivity $\theta(i)_e$ and wealth $a(i)$ can be written as:

$$k(i) = k\left(\theta(i)_e, a(i)\right) = \min\left[a(i), k(i)^*\right] + \min\left[\phi a(i), k(i)^* - \min\left[a(i), k(i)^*\right]\right]. \quad (10)$$

The first term on the RHS of (10) reflects the size of a firm not seeking external financing, where the business owner simply rests with his own wealth. The second term describes the amount an entrepreneur with wealth $a(i)$ will actually borrow. The subsequent numerical analysis shows that the high–productivity entrepreneurs are more likely to be constrained than the low–productivity ones, because the optimal firm size and henceforth the capital demand increase in the productivity shock.

An entrepreneur, whose individual wealth exceeds the level needed to operate his business at the optimal firm size will lend the amount $a(i) - k(i)^*$ on the capital market at the equilibrium interest rate. The supply side of the capital market altogether consists of those entrepreneurs whose wealth exceeds their individual optimal firm size and of workers, who supply their savings. On the demand side we have the credit–constrained entrepreneurs and firms from the final goods industry. From this follows immediately that the size of the intermediate goods industry relative to the final goods sector essentially depends on occupational choice and individual wealth accumulation, both determined endogenously in equilibrium.

2.6 Idiosyncratic Risks

In each period of time, workers are endowed with one unit of raw labor and are subject to an idiosyncratic shock θ_w affecting labor supply in efficiency units, and exposing each of them to an uninsurable income risk. For simplicity, we assume that labor productivity θ_w evolves according to a first–order Markov process with $h = 1, \ldots, m$ states, and $\theta_{w,h} > 0$. The transition matrix associated with the Markov process is P_w.

Entrepreneurial productivity θ_e also evolves according to a first–order Markov process with $h = 1, \ldots, m$ different states $\theta_{e,1}, \ldots, \theta_{e,m}$; $\theta_{e,h} > 0$, and transition probability P_e. Since agents can either be workers or entrepreneurs, it is possible to identify the occupational status of an agent with his productivity in the respective occupation. We assume worker productivities to be more evenly distributed than managerial skills, such that profit incomes in general are more risky than wage incomes. As is well–known from the literature, entrepreneurs on average are compensated with a positive income differential (*aka* 'risk premium') for bearing the production risk.

By modeling two distinct random processes for workers and entrepreneurs, we take into account that the two professions demand different talents, for instance specific managerial skills. We assume the processes θ_w and θ_e to be uncorrelated, such that for an individual the conditional expectation of entrepreneurial productivity is

independent of the labor efficiency, if employed as a worker.[3] A high productivity as a worker in the present does not necessarily indicate an equivalently high future productivity as an entrepreneur, if the individual should decide to switch between occupations in the next period. The associated probabilities are summarized in a $m \times m$ transition matrices $P_{n,n'}$ describing the transition from productivity state $\theta_{n,h}$ to state $\theta_{n',h'}$ for $h, h' = 1, \ldots, m$, $n = e, w$ and $n \neq n'$.

We consider two different specifications regarding the Markov processes for entrepreneurial talent and worker efficiency respectively. Shocks of the first setting are serially correlated, thus introducing a certain persistence in individual income processes. Currently highly productive workers and entrepreneurs are more likely to be highly productive in the future. The individual is able to infer from his present productivity how his future productivity in the same occupation will be. Shocks of the second setting are *iid*. Although empirically not supported, when confronted with the data (cf. Guvenen, 2007, and references therein), the second setting allows us to illustrate the role intertemporal income persistence has for occupational choice and social mobility.

2.7 Intertemporal Decision and Occupational Choice

Each household i has preferences over consumption and maximizes discounted expected lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t U[c_t(i)] \qquad 0 < \beta < 1 .$$

E_0 is the expectation operator conditional on information at date 0 and β is the discount factor. Individuals are assumed to be identical with respect to their preferences regarding momentary consumption $c(i)$ which are described by constant relative risk aversion

$$U[c(i)] = \begin{cases} \dfrac{c(i)^{1-\rho}}{1-\rho} & \text{for } \rho > 0, \rho \neq 1 \\ \ln c(i) & \text{for } \rho = 1 , \end{cases}$$

where ρ denotes the Arrow/Pratt measure of relative risk aversion.

In each period, the single household is endowed with a unit of raw labor and—in addition to his intertemporal decision—makes a choice on his future occupation, which is either to become a self–employed producer of an intermediate good in the monopolistically competitive market or to supply his labor services in efficiency units inelastically to the production of the final good. Occupational choice, once made in a certain period, is irreversible.

Let $V^w(a(i), \theta(i)_{w,h})$ denote the optimal value function of an agent currently being a worker with wealth $a(i)$, who is in productivity state $\theta_{w,h}$, $h = 1, \ldots, m$. If he decides

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[3]The analysis of correlated skill processes is left for future research.

to remain a worker, his productivity evolves according to the transition matrix P_w of the underlying Markov process with states $\theta_{w,1}, \ldots, \theta_{w,m}$. If, instead, he decides to become an entrepreneur in the following period, his next period productivity θ'_e is determined by the transition matrix $P_{w,e}$. For analytical convenience, individual asset holdings are bounded from below, the lowest possible wealth level set to $\underline{a} = 0$.

The associated maximized value function for a typical individual currently being a worker is given by

$$V^w\left(a(i), \theta(i)_{w,h}\right) =$$
$$\max_{c(i) \geqslant 0, a(i)' \geqslant \underline{a}} \left\{ U[c(i)] + \beta \max_{\xi' \in \{0,1\}} \left\{ \mathrm{E}\left[V^w\left(a(i)', \theta(i)'_{w,h}\right) | \theta(i)_{w,h}\right], \mathrm{E}\left[V^e\left(a(i)', \theta(i)'_{e,h}\right)\right] \right\} \right\}$$

$$\text{s.t.} \quad a(i)' = (1+r)\,a(i) + \theta(i)_{w,h}\,w - c(i)\,.$$
$$\tag{11}$$

ξ is a boolean variable which takes on the values 0 or 1, depending on whether or not the agent decides to switch between occupations. r and w denote the equilibrium returns to capital and labor in efficiency units, which are constant over time for a stationary distribution of wealth and occupational statuses over agents. The optimal decision associated with the problem (11) is described by the two decision rules for individual asset holdings $a(i)'_w = a_w\left(a(i), \theta(i)_{w,h}\right)$ and the future professional state $\xi(i)'_w = \xi_w\left(a(i), \theta(i)_{w,h}\right)$.

Let $V^e(a(i), \theta(i)_{e,h})$ denote the maximized value function of an entrepreneur with wealth $a(i)$ in productivity state $\theta(i)_{e,h}$, who faces a decision problem similar to those of a worker. If he decides to remain an entrepreneur, his productivity evolves according to the transition matrix P_e of the underlying Markov process with states $\theta_{e,1}, \ldots, \theta_{e,m}$. If, instead, he decides to switch between occupations by becoming a worker in the next period, his future productivity θ'_w is determined by the transition matrix $P_{e,w}$. With $k(\theta(i)_{e,h})^*$ denoting the optimal firm size, the intertemporal problem of an entrepreneur currently in productivity state $\theta(i)_{e,h}$, can be written as

$$V^e(a(i), \theta(i)_{e,h}) =$$
$$\max_{c(i) \geqslant 0, a(i)' \geqslant \underline{a}} \left\{ U[c(i)] + \beta \max_{\xi' \in \{0,1\}} \left\{ \mathrm{E}\left[V^e\left(a(i)', \theta(i)'_{e,h}\right) | \theta(i)_{e,h}\right], \mathrm{E}\left[V^w\left(a(i)', \theta(i)'_{w,h}\right)\right] \right\} \right\}$$

$$\text{s.t.} \quad a(i)' = (1+r)\,a(i) + \pi(k(i), \theta(i)_{e,h}) - c(i)$$

$$k(i) = \min\left[a(i), k(\theta(i)_{e,h})^*\right] + \min\left[\phi a(i), k(\theta(i)_{e,h})^* - \min\left[a(i), k(\theta(i)_{e,h})^*\right]\right]$$

$$\pi(\theta(i)_{e,h}, k(i)) = p(x(i))\,x(\theta(i)_{e,h}, k(i)) - (r+\delta)\,k(i)$$
$$\tag{12}$$

Again, ξ is a boolean variable, indicating the agent's decision on leaving or remaining in his present occupation. The optimal decision is described by the decision rules

for individual asset holdings $a(i)'_e = a_e(a(i), \theta(i)_{e,h})$ and the future professional state $\xi(i)'_e = \xi_e(a(i), \theta(i)_{e,h})$.[4]

In general, our model generates the same implications for individual savings and wealth accumulation under risk as, for instance, discussed in Aiyagari (1994) or Huggett (1996). Similar to Quadrini (2000) we additionally consider occupational choice. Consequently, wealth accumulation plays a two–fold role: On the one hand, the shocks to worker efficiency and entrepreneurial productivity generate an income risk which households respond to with buffer–stock saving. On the other hand, higher wealth levels protect entrepreneurs against the danger of being subject to financial constraints. In terms of Sandmo (1970) there is only an income but no capital risk in our model, such that the share of risky incomes in total household income declines with growing wealth. Accordingly, the importance of risky profits providing negative incentives towards entrepreneurship fades for high levels of wealth.

2.8 Stationary Recursive Equilibrium

A stationary recursive competitive general equilibrium is an allocation, where equilibrium prices generate a distribution of wealth and occupations over agents which is consistent with these prices given the exogenous process for the idiosyncratic shocks and the agents' optimal decision rules.

Let K_F, L and $x(j)^D$ denote the demands of capital, effective labor and intermediate goods in the final goods sector. We obtain aggregate labor supply by summing up individual labor supplies in efficiency units over the population fraction $1 - \lambda$ of workers. Let, furthermore, $q_h, h = 1, \ldots, m$ denote probabilities of states $\theta_{w,h}$ in the equilibrium distribution of labor productivities. The stationary recursive equilibrium is a set of value functions $V^w(a, \theta_w)$, $V^e(a, \theta_e)$, decision rules $a_w(a; \theta_w)$, $\xi_w(a; \theta_w)$ and $a_e(a; \theta_e)$, $\xi_e(a; \theta_e)$, prices $w, r, p(j)$ and a distribution $\lambda, 1 - \lambda$ of households over occupations such that:

(i) the decision rules $a_w(a; \theta_w)$, $\xi_w(a; \theta_w)$ and $a_e(a; \theta_e)$, $\xi_e(a; \theta_e)$ solve the workers' and entrepreneurs' problems (11) and (12) at prices $w, r, p(j)$,

(ii) the aggregate demands of consumption, labor, capital and intermediate goods are the aggregation of individual demands. Factor and commodity markets clear at constant prices $w, r, p(j)$, where factor inputs are paid according to

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[4]Note that the value functions (11) and (12) may not be concave because of the boolean variable ξ, indicating binary choice between occupations. Similar to Fernández-Villaverde *et al.* (2003), we would like to stress that the dynamic programming algorithm underlying our computational modeling does not require concavity but monotonicity to converge to the true value function; see also Boháček (2007, fn. 4).

their marginal product:

$$Y = C + \delta K$$

$$\int_0^1 k(i)\,\mathrm{d}i \equiv K = K_F + \int_0^\lambda k(i)\,\mathrm{d}i$$

$$\int_\lambda^1 \sum_{h=1}^m q_h \theta_{w,h}\,\mathrm{d}i = L$$

$$x(j)^S = x(j)^D\ ,$$

(iii) the stationary distribution $\Gamma(\lambda, a, P_e, P_w, P_{e,w}, P_{w,e})$ of agents over individual wealth holdings, occupations and associated productivities is the fixed point of the law of motion which is consistent with the individual decision rules and equilibrium prices. The distribution $\lambda, 1-\lambda$ of agents over occupations is time–invariant.

The decision rules for workers, $a_w(a, s_w)$, $\xi_w(a, s_w)$, and entrepreneurs, $a_e(a, s_e)$, $\xi_e(a, s_e)$, together with the stochastic processes for individual labor productivity and entrepreneurial productivity, determine the stationary distribution Γ at equilibrium prices w, r. The stationary distribution Γ governs the entrepreneurship rate (i.e. the mass of firms in the intermediate goods sector), the efficiency units of labor supplied by workers, capital demand of the intermediate goods sector, and the aggregate capital supply, the latter equaling the mean of individual wealth holdings. Once the entrepreneurship rate λ is derived, this together with the stationary distribution of entrepreneurial productivities determines the supply of intermediate goods.

3   Calibration

The model is calibrated to match standard macro data from OECD countries. Table 1 summarizes the parameterization of the model. Regarding preferences, we set the discount factor β and the coefficient of relative risk aversion ρ according to estimates from the literature, in order to generate equilibrium interest rates on safe assets consistent with empirical findings (cf. Mehra and Prescott, 1985; Obstfeld, 1994). The parameters of production technology, α and γ, are chosen such as to generate an equilibrium labor income share of 0.63 which matches empirical observations e.g. for the U.S. economy (King and Rebelo, 1999). The corresponding capital and profit income shares of the frictionless economy ($\phi \rightarrow \infty$) are 0.16 and 0.21. PSID data report a income share for entrepreneurs of around 22%. The depreciation rate is fixed at 6%, which also is a standard choice in the literature.

The steady state of the simulated economy by and large replicates the Gini coefficient of wealth inequality in the range of 0.55 to 0.75 usually observed for OECD countries. Introducing occupational choice into Aiyagari (1994)–type models of

Table 1: Parameters of the baseline model

α	γ	δ	β	ρ	ϕ
0.3	0.1	0.06	0.95	2.0	$0 \leftrightarrow \infty$

uninsurable shocks and borrowing constraints, improves the prediction of wealth inequality, especially in the upper tail of the distribution (cf. Quadrini, 2000).

We consider an entrepreneur as someone, who owns and operates a small business, and who is willing to take risks, to be innovative, and to exploit profit opportunities (Knight, 1921; Schumpeter, 1930; Kirzner, 1973). Definitions of self–employment and entrepreneurial activity differ widely across countries.[5] According to the OECD, self–employment encompasses *"…those jobs, where the remuneration is directly dependent upon the profits derived from the goods and services produced. The incumbents make the operational decisions affecting the enterprise, or delegate such decisions while retaining responsibility for the welfare of the enterprise."* (OECD, 2000, Ch. 5, p. 191). Our model generates self–employment business ownership rates around 20%, which is somewhat more at the upper range of values for OECD countries (including owner–managers), matching countries like New Zealand (20.8%), Italy (24.8%), or Spain (18.3%); see also the annual Global Entrepreneurship Monitor (e.g. GEM 2005, Minniti *et al.*) for data on total entrepreneurial activity.

Entry rates into entrepreneurship equal exit rates in the stationary recursive equilibrium. Our model was calibrated to generate entry rates around 15%, which is higher than the rates reported by Evans (1987) for the U.S. and also in the upper range of empirically plausible values for OECD countries (cf. Vale, 2006; Aghion *et al.*, 2007).

The fraction of aggregate capital employed in the (corporate) final goods sector strongly depends on two factors: First, on the strictness of financial constraints effective in the intermediate goods industry, which we vary over the entire domain from perfect markets ($\phi \to \infty$) to a complete absence of credit markets ($\phi = 0$), and second, on the degree of persistence of the idiosyncratic shocks. Consequently, our results cover a wide range for the percentage of capital inputs in the final goods sector from 43% to over 60%, the latter being consistent with U.S. data reported by Quadrini (2000). The capital–to–output ratio of the simulated economy ranges around values of 2.

To take account of empirically observed income persistence, we assume that the processes for labor efficiency θ_w and entrepreneurial productivity θ_e are lognormal with normalized mean $\ln\theta_w \sim \mathcal{N}\left(-\sigma_w^2/2, \sigma_w^2\right)$, $\ln\theta_e \sim \mathcal{N}\left(-\sigma_e^2/2, \sigma_e^2\right)$ and $AR(1)$ of

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[5]Often, the agricultural sector is excluded from the computation of entrepreneurship rates.

the general form:

$$\ln \theta'_w = (p_w - 1)\frac{\sigma_w^2}{2} + p_w \ln \theta_w + \sigma_w \sqrt{1 - p_w^2}\, \varepsilon\,, \tag{13}$$

$$\ln \theta'_e = (p_e - 1)\frac{\sigma_e^2}{2} + p_e \ln \theta_e + \sigma_e \sqrt{1 - p_e^2}\, \varepsilon\,, \tag{14}$$

where $\varepsilon \sim \mathcal{N}(0,1)$. The process (13) was parameterized following Aiyagari (1994). With respect to the process (14) we assume an identical serial correlation, but choose a larger variance in order to reproduce the higher risk associated with entrepreneurial activity. Table 2 presents the parameter values underlying the stochastic processes.

Table 2: Parameters of the stochastic processes

σ_w	p_w	σ_e	p_e
0.2	0.6	1.5	0.6

The processes are approximated with a five–state Markov chain by using the method described in Tauchen (1986). The transition matrices for individuals who decide to switch occupations are derived from the stationary distributions of the respective Markov processes. The probability for a worker (entrepreneur) of ending up in a specific state of entrepreneurial (worker) productivity $\theta_{e,h}$ ($\theta_{w,h}$) is given by the stationary (unconditional) probabilities of this state. The algorithm for finding the equilibrium consists of three nested loops, starting from an initial guess on factor prices w, r and employment L, then iterating until markets are cleared and the conditions of a stationary recursive equilibrium are met.

4   Results

Our baseline model is a model of income and earnings persistence. We investigate the effects of financial constraints on (a) inequality and the distribution of wealth, (b) on output, factor prices, and the factor income distribution, and (c) on occupational choice and social mobility.

A common finding for models with credit market imperfections is that the properties of the equilibrium often respond non–monotonically to parameter changes. If we look at the literature, we find models assessing the effects of credit market imperfections by assuming no credit market at all. Other approaches compare imperfect to perfect markets. As Matsuyama (2007, p. 3) points out, there is no reason to believe that, first, the effects of an imperfect market equal those of no credit market, and second, the effects of improving credit markets are similar to those of

completely eliminating market imperfections. Instead of discussing only a single case by assuming a predetermined magnitude of financial constraint, we vary the tightness of constraints in our simulations to cover the range from no credit market ($\phi = 0$) to a perfect market ($\phi \to \infty$).[6] Although the value of ϕ is fixed exogenously, the credit demand as well as the amount of rationing is determined endogenously and depends on firm specific factors, such as the optimal business size (8), individual wealth, equilibrium factor prices and the realization of the ability shock.

Regarding the comparative static results, we find that the properties of the equilibrium respond sensitive to a change in serial correlation. We contrast the baseline model, where processes are serially correlated, with the case of serially uncorrelated shocks and find striking differences with regard to the equilibrium entrepreneurship rate and mobility between occupations.

Our analysis proceeds as follows: We first investigate to what extent our model is able to replicate empirical evidence on wealth distributions. We then examine how the presence of credit constraints affects the key macroeconomic variables, such as aggregate output, average firm size, factor prices and factor income shares as well as individual incomes, household wealth and the degree of inequality, the latter measured by the Gini coefficient. In a next step, we analyze mobility between occupations. The comparative static analysis concludes with the discussion of ability–related borrowing constraints. We also contrast the baseline model with the case of uncorrelated shocks.[7]

4.1 Results for the Baseline Model

Wealth distribution Figure 1a shows the distribution of wealth over individuals for the two limiting cases of an unconstrained economy ($\phi \to \infty$) versus fully absent markets for loans ($\phi = 0$). As can be seen, the presence of financial constraints tends to reduce the mass of very wealthy individuals. Moreover, the distribution becomes more concentrated at lower wealth levels in the case of no credit markets. Figure 1b, displaying the wealth distribution in a logarithmic scale, brings out more visibly the differences between the two cases, especially for the domain of very low wealth levels. A major consequence of borrowing constraints in the underlying model is that the fraction of individuals, for whom the bottom threshold ($a(i) \geqslant \underline{a} = 0$) actually becomes binding, rises from about 2% of the population to a value around 5%.

Figure 2 shows the stationary distribution of wealth for the two limiting cases $\phi \to \infty$ and $\phi = 0$ differentiated with respect to the two occupational classes. In gen-

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[6]Tables 3 and 4 only display selected cases, with perfect markets ($\phi \to \infty$), no credit markets ($\phi = 0$) and the case, where the lower bound for the equity–loan–ratio is one half of the operating capital ($\phi = 1$). The upper limit is approximated with $\phi = 1000$ in our numerical simulations. Figures 4, 6, and 7 cover a wider range of values for ϕ from our simulations.

[7]Throughout the discussion of results we will refer to 'optimal' levels as those of the unconstrained economy, thereby blinding out risk and imperfect competition as additional sources of inefficiency.
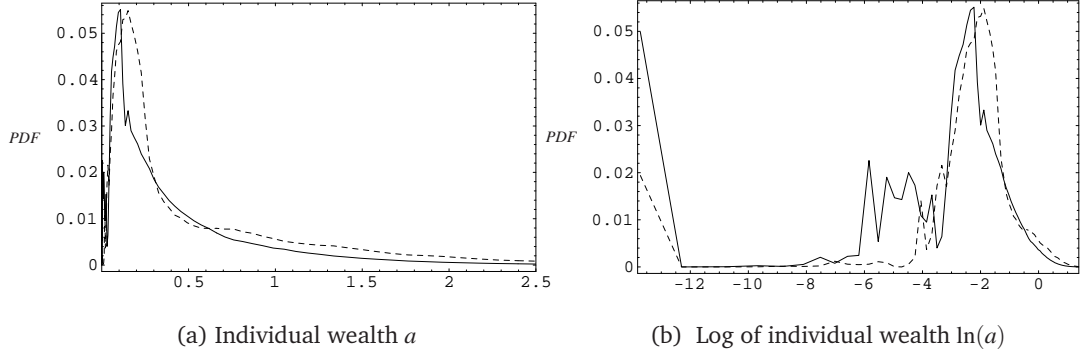
16

(a) Individual wealth a (b) Log of individual wealth $\ln(a)$

Figure 1: Wealth distribution in the baseline model, $\phi \to \infty$ (dashed) and $\phi = 0$ (solid)

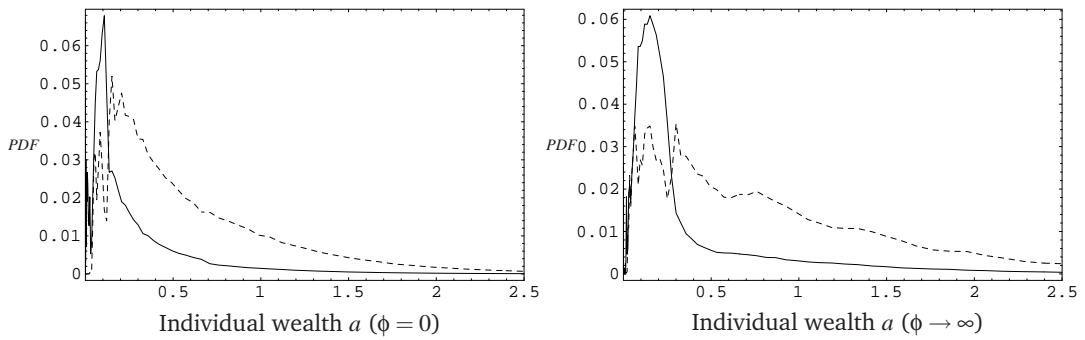Individual wealth a ($\phi = 0$)   Individual wealth a ($\phi \to \infty$)

Figure 2: Wealth distribution in the baseline model for workers (solid) and entrepreneurs (dashed) with $\phi \to \infty$ and $\phi = 0$

eral, our model produces wealth distributions similar to those reported in the literature for heterogeneous agent models with entrepreneurial activity (see Quadrini, 1999, 2000; Cagetti and De Nardi, 2006a). We observe that workers are more concentrated at lower wealth levels, and there exists a significant mass of wealthy entrepreneurs but also a comparably large share of poorer ones. This is in line with empirical findings by Gentry and Hubbard (2004); Hamilton (2000) as well as with related theoretical contributions (cf. Boháček, 2006, 2007). Relaxing credit constraints significantly increases the mass of entrepreneurs in the upper tail of the distribution, but also leads towards an outward shift of the worker PDF of wealth, increasing mean and modal worker wealth levels.

Figure 3 shows the cumulative distribution of firm sizes in the intermediate goods sector for three distinct values of the parameter ϕ which indicates the tightness of financial constraints. Each entrepreneur is able to operate his business at the optimal firm size (8) in the perfect market case ($\phi \to \infty$). Consequently, we observe
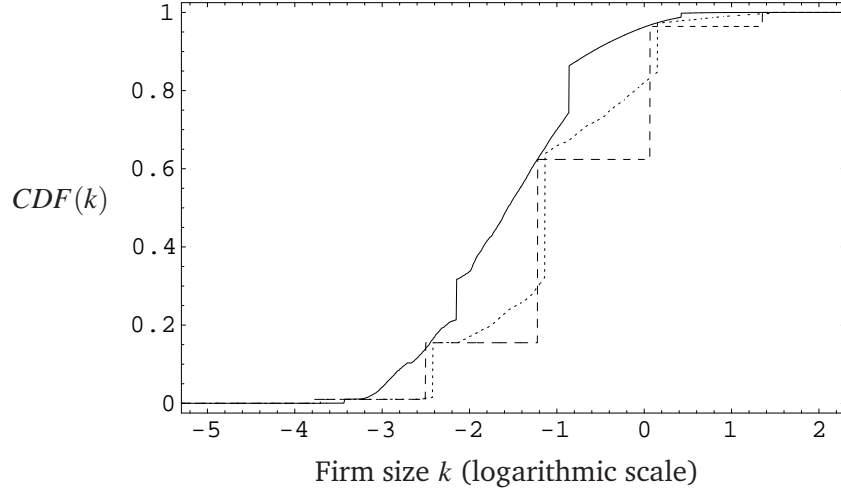
Figure 3: CDF of firm size in the baseline model with $\phi \to \infty$ (dashed), $\phi = 1.0$ (dotted) and $\phi = 0$ (solid)

a stepwise CDF, each step corresponding to the optimal firm size associated with one out of the five underlying possible productivity states $\theta_{e,h}$.

Consider next the case $\phi = 1$, where entrepreneurs are able to acquire external financing up to maximum sum equal to their own wealth. Here, the operating firm size is bounded from above to twice the amount of individual wealth, which need not be the optimal firm size, especially, if the firm owner is highly productive. Recall at this point that the optimal firm size is endogenously determined; besides idiosyncratic random productivity also depending on factor prices, which in turn are determined by aggregate market activities and occupational choice in the general equilibrium.

The first observation is that the optimal firm sizes rise slightly for each possible state of entrepreneurial talent $\theta_{e,h}$. This increase in firm sizes can be ascribed to the factor price effect. Borrowing constraints prevent the efficient allocation of capital among sectors such that too much capital is employed in the production of the final good. This is associated with a decline in the real interest rate, which in turn raises the optimal firm size in the intermediate sector for each state of productivity.

The second, major observation in the credit–constrained economy is that there is a positive mass of entrepreneurs between each two subsequent steps of optimal firm sizes, and the distribution is more concentrated at smaller firm sizes. Constraints become binding for many entrepreneurs, who now have to operate their enterprise at a suboptimally low scale. Non–surprisingly, this effect is aggravated, if we reduce the availability of external financing to naught. For $\phi = 0$, steps in the CDF almost vanish, which means that more business owners are subject to constraints. The

Table 3: Simulation results — baseline model and modified credit constraint

		Tightness of constraints			
		$\phi \rightarrow \infty$	$\phi = 1.0$		$\phi = 0$
			baseline model	modified constraint	
entrepreneurship rate	(%)	0.230	0.247	0.243	0.250
⌀ firm size	total	0.652	0.440	0.504	0.296
⌀ credit rationing	total	0.000	0.330	0.205	0.637
⌀ profits	total	0.134	0.124	0.130	0.114
final	Y	0.147	0.130	0.139	0.111
goods	K_F (%)	0.438	0.521	0.492	0.612
sector	K_F	0.116	0.118	0.119	0.116
	L_F	0.820	0.796	0.805	0.786
factor	w	0.113	0.103	0.109	0.089
prices	r	0.028	0.017	0.022	0.007
	$w/(r+\delta)$	1.277	1.337	1.326	1.335
factor	labor	0.630	0.630	0.630	0.630
income	capital	0.160	0.134	0.142	0.114
shares	profits	0.210	0.236	0.228	0.256
⌀ wealth	total	0.266	0.227	0.241	0.190
	workers	0.203	0.167	0.180	0.128
	entrepreneurs	0.477	0.410	0.431	0.378
⌀ income	workers	0.138	0.121	0.131	0.102
	entrepreneurs	0.176	0.156	0.166	0.139
risk premium		0.118	0.141	0.126	0.218
⌀ consumption		0.131	0.117	0.125	0.100
welfare		-8.024	-8.966	-8.397	-10.504
wealth	total	0.559	0.556	0.557	0.599
inequality	workers	0.506	0.528	0.511	0.588
(Gini)	entrepreneurs	0.546	0.488	0.525	0.478
mobility		0.143	0.154	0.152	0.156

optimal levels of firm sizes for the different states of productivity rise even further, due to the factor price effect. In numbers, if we compare the unconstrained with the completely constrained economy, businesses in the entrepreneurial sector on average operate at 32% of their respective optimal firm size.

Macroeconomic effects Table 3 and Figure 4 summarize the results for the macroe-conomic key variables of the calibrated baseline model. The general picture reflects the outcome one would expect from credit market improvements. Aggregate output Y, consumption, aggregate wealth holdings a, factor prices r,w and incomes as well as welfare increase if we relax borrowing constraints.
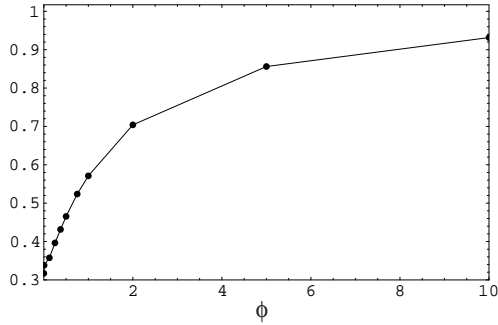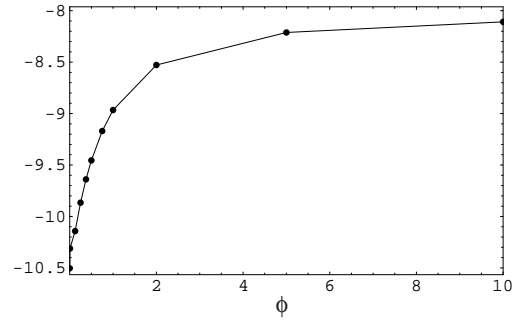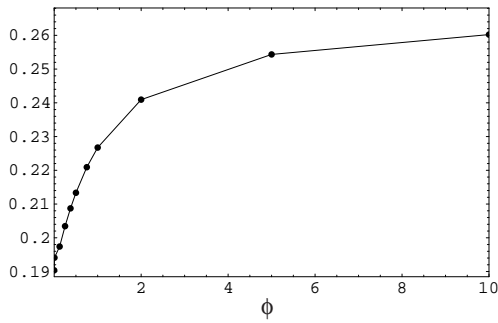
(a) % of optimal average firmsize
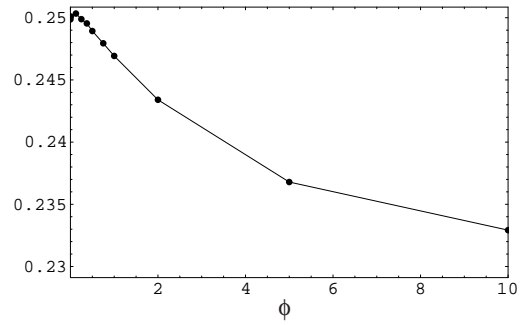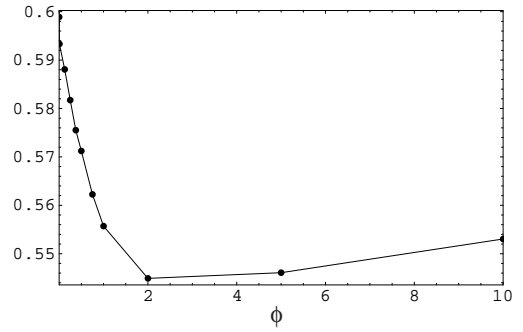
(b) Entrepreneurship rate
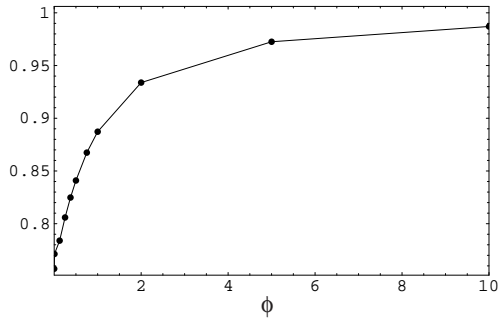
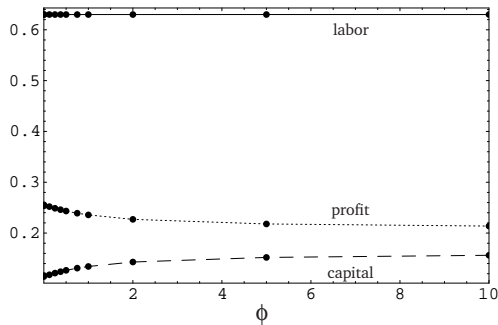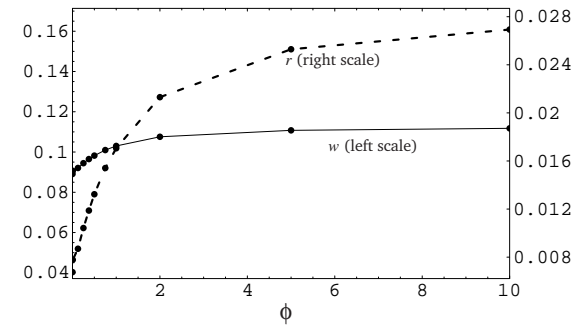(c) Y relative to $\phi \to \infty$

(d) Welfare

(e) Wealth holdings

(f) Gini of total wealth

(g) Functional distribution of income

(h) Factor prices

Figure 4: Macroeconomic effects of a change in ϕ, baseline model

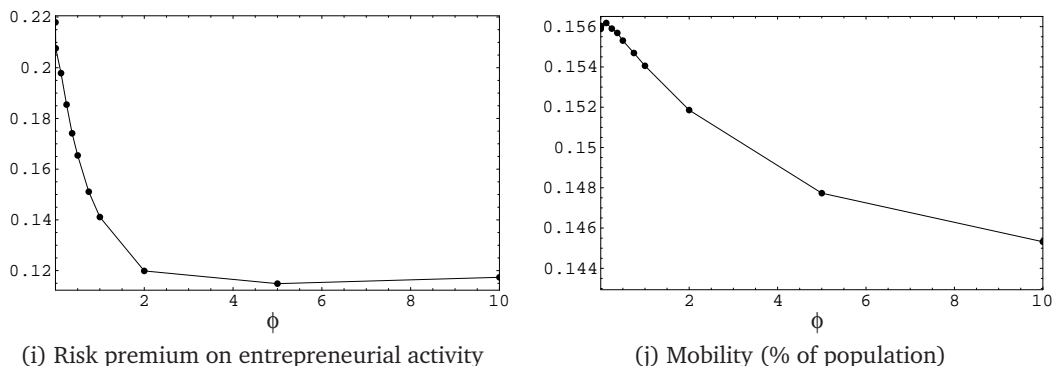(i) Risk premium on entrepreneurial activity   (j) Mobility (% of population)

Figure 4: Macroeconomic effects of a change in φ, baseline model (cont.)

Figure 4 shows that, except for wealth inequality (which we will refer to later), the response of the macroeconomic variables to a change in φ is monotonous. The overall loss in output of a perfectly constrained compared to a frictionless economy lies at about 25%, and wealth holdings only make up to 72% of their optimal level. Increasing financial constraints goes along with a substantial drop in economic performance. Average consumption declines by 24% and the associated welfare loss of the simulated model amounts to 30%.

We also see from Figure 4 that the response of output, wealth, factor prices, and welfare to a change in φ is concave. The marginal gains of improving credit markets are much higher for small values of φ, especially in the range of loan–to–equity ratios from $0 < \phi < 2$, which is the empirically plausible domain. This interval accounts for more than two–thirds of the overall output loss associated with financial constraints.

Given the general equilibrium nature of the underlying model, one would expect several adjustments to take place following a reduction in external financing as borrowing constraints become more tight. If there is only limited or no capital demand from the intermediate goods industry, we observe a capital–relocation effect between sectors. More capital is employed in the final goods industry. This amounts to shifting about 17% of the aggregate capital stock from the intermediate to the final goods sector over the entire range $0 \leqslant \phi < \infty$. The average excess demand for capital in the intermediate goods industry amounts to more than twice the average firm size.

With diminishing marginal returns, the equilibrium interest rate r, and accordingly the factor price for capital $r + \delta$, decline in both sectors of the economy. Recalling that entrepreneurial households receive income from two sources, profits and capital incomes, the income share reflecting the user costs of capital declines for any given level of individual wealth, whereas the profit share rises. Altogether, we

observe a shift in the functional income distribution from capital to profit incomes of 4.6 p.p. over the entire domain of ϕ.[8] The additional employment of capital c.p. raises labor productivity. The factor–price ratio $w/(r+\delta)$ increases, but only by a small scale of 4.4%, because this effect is partly offset by a reduction in intermediate good inputs, the latter reducing labor productivity.

The presence of credit constraints not necessarily implies that only those agents choose to become an entrepreneur, who have sufficient own wealth and borrowed resources to operate their business at the optimal firm size k^*. These are the only firms which actually maximize their profits, whereas the constrained entrepreneurs are forced to operate at suboptimally small business sizes. Consequently, the average firm size in the intermediate goods industry decreases substantially as financial constraints become more tight, and highly productive entrepreneurs are more affected by the constraints than those with a low θ_e. Figure 4a shows that in a completely constrained economy the average firm size only amounts to around 32% of its optimal size.

Most strikingly, this result is also partly due to the fact that the entrepreneurship rate increases by almost 2 p.p. for smaller values of ϕ. Instead of less competition in the intermediate goods industry, as one might have expected, we observe an increase in the number of firms in the constrained economy. This, however, comes at the cost of smaller market shares and lower average profits (–15%).

A higher rate of entrepreneurship as a consequence of tightening borrowing constraints is to some extent a counter–intuitive result and can be traced back to the general equilibrium nature of our approach. Credit constraints are only one out of several determinants of occupational choice. The competition for capital between the final and intermediate goods sector determines the equilibrium interest rate, the firm size and expected profits of the monopolistic enterprises. The expected premium of entrepreneurial incomes over wages, too, affects the individual decision on the future occupation. Figure 4i shows that the expected income differential attains its largest value in the perfectly constrained economy, then dropping sharply by more than 46% for an increase in ϕ.

Households continuously decide between two lotteries and possess (at least subjective) knowledge regarding the stochastic properties of the underlying shocks. If shocks are serially correlated, a low–productivity worker is aware of the fact that being also lowly productive in the future is a more probable outcome than otherwise. Consequently, he might be inclined to take his chances with entrepreneurship, knowing that his current productivity as a worker is not related to his future productivity as a business owner.[9]

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[8]See 2.4 for a short remark on the equilibrium factor shares of the frictionless economy.

[9]Relaxing this assumption is left for future research; see also Cagetti and De Nardi (2006*a*, Appendix) on this issue.

Table 4: Mobility in the baseline model: Individual probability of a switch in occupations with respect to current productivity state and wealth quintile

		Tightness of constraints					Tightness of constraints			
	Productivity	$\phi \to \infty$	$\phi = 1$	$\phi = 1^a$	$\phi = 0$	Quintile	$\phi \to \infty$	$\phi = 1$	$\phi = 1^a$	$\phi = 0$
workers	$\theta_{w,1}$	1	0.51	0.73	0.37	1.	0.15	0.06	0.12	0
	$\theta_{w,2}$	0.92	0.70	0.85	0.53	2.	0.14	0.15	0.14	0.13
	$\theta_{w,3}$	0.10	0.18	0.14	0.21	3.	0.12	0.13	0.12	0.13
	$\theta_{w,4}$	0	0	0	0.02	4.	0.12	0.32	0.21	0.46
	$\theta_{w,5}$	0	0	0	0	5.	0.58	0.59	0.59	0.65
entre-preneurs	$\theta_{e,1}$	1	1	1	1	1.	0.76	0.76	0.76	0.76
	$\theta_{e,2}$	1	1	1	1	2.	0.76	0.74	0.74	0.76
	$\theta_{e,3}$	1	1	1	0.99	3.	0.64	0.68	0.66	0.67
	$\theta_{e,4}$	0	0	0	0	4.	0.58	0.65	0.62	0.67
	$\theta_{e,5}$	0	0	0	0	5.	0.56	0.54	0.54	0.55

[a]Column depicts probabilities of the model with modified constraints, see Section 4.2.

Regarding the wealth distribution, we first observe a sharp decline in the Gini coefficient for a rise in ϕ from 0.60 to 0.54, which is then followed by a gradual increase in overall wealth inequality back to a value of 0.56. This non–monotonic behavior of total wealth inequality can be explained, if we look at the within–group inequality for workers and entrepreneurs respectively. Table 3 shows that wealth becomes more unevenly distributed among workers, whereas wealth inequality among entrepreneurs declines.[10]

Mobility Next, we are interested in the mobility between occupations taking place under the stationary distribution. Table 3 and Figure 4j show that around 15% of the population switch between occupations in each period. We even observe over-all mobility to increase by 8% if credit constraints become more tight. While this change in overall mobility might seem small from a quantitative perspective, it is nevertheless remarkable, since it indicates that credit constraints not only increase the entrepreneurship rate of the economy but also the fluctuation between occupations.

Table 4 presents the quantitative results for each of the five productivity states $\theta_{e,h}$, $\theta_{w,h}$ and with respect to wealth quintiles. A more detailed look at the mobility patterns shown in Table 4 reveals that generally workers and entrepreneurs who exhibit a low productivity ($\theta_1 - \theta_3$) in their current profession decide to switch between professions, whereas the high productivity individuals (θ_4, θ_5) stay put. For instance, all workers in the unconstrained economy, who currently are in the low-est labor productivity state, decide to take their chances with entrepreneurship in

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[10]Notice, that the Gini coefficient does not allow for a simple decomposition of total inequality into inequality within and between subgroups.

the next period. However, the probability of status change responds sensitive to the degree of credit availability. For $\phi = 0$, the probability drops down by almost two–thirds. As a general result we find that the likelihood of workers to start a business and become self–employed in the next period decreases in productivity and in constrained access to external financing. More productive workers show a larger persistence in their present occupation.

Regarding mobility, the picture is even more striking for individuals, who currently are low productivity entrepreneurs. (Almost) All entrepreneurs finding themselves in the lowest three productivity states change their occupation. They, (almost) with certainty will exit the market to seek employment as a worker in the next period. This results holds irrespective of the degree of constraint. Summing up, mobility over occupations in our model is confined to agents who are not successful in their current professions.

The intuition is as outlined before: Serially correlated shocks provide agents with a signal regarding future productivity. Since we assumed the processes for labor efficiency and entrepreneurial ability to be uncorrelated, a worker can infer from a low productivity today a probably low labor efficiency tomorrow, but this not necessarily indicates a equally low future ability as entrepreneur, which is given by the unconditional probability of states.

Table 4 also shows how the mobility over occupations depends on individual wealth. Conditional on the given occupation and the tightness of credit constraints, the values in the table represent the probability for a change of occupation for each quintile of the wealth distribution. As can be seen, the probability for a worker to become an entrepreneur increases in wealth, whereas the opposite is true for entrepreneurs. The general mobility pattern is robust over different levels of ϕ. The result is, however, not quite surprising given the fact that agents are risk averse and that profit income is more risky than labor income.

While the effects of credit constraints on the mobility patterns for entrepreneurs are only small in scale, we observe significant effects on mobility patterns for workers. More tight credit constraints strikingly decrease the probabilities of becoming an entrepreneur for poor workers, while the corresponding probabilities for rich workers (especially for those in the fourth quintile) increase. New entrepreneurs are mainly recruited among the group of wealthy workers.

4.2 An Alternative Formulation of Borrowing Constraints

The borrowing of the baseline model were assumed to be entirely related to individual wealth. We will now discuss a separate treatment, where the maximum loan also depends on the entrepreneur's individual productivity. In case of default, the lender is able to seize the fraction ϕ of the borrower's gross income $(1+r)a(i) + \pi(\theta(i)_e, a(i)+b(i))$ of period t. The associated incentive compatibility
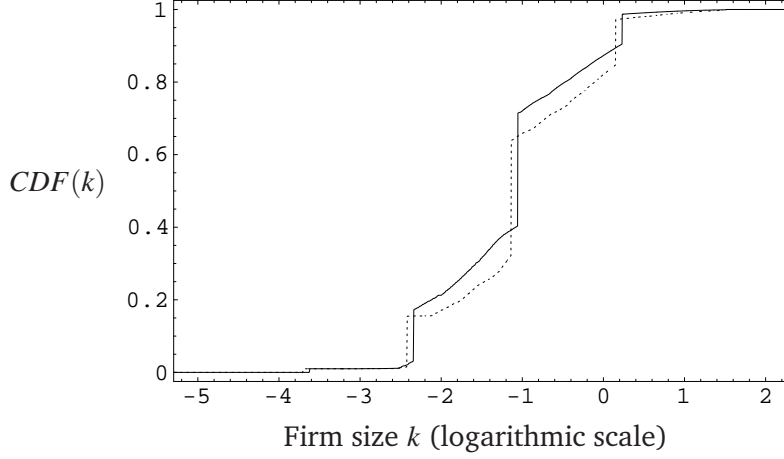
Figure 5: CDF of firm size with $\phi = 1$ for the baseline model (solid) and the model with modified credit constraint (dashed)

constraint making sure that it is never optimal for the borrower of going into default becomes:

$$\pi(\theta(i)_e, a(i) + b(i)) + (1+r)a(i) \geqslant (1-\phi)\left[\pi(\theta(i)_e, a(i) + b(i)) + (1+r)a(i)\right] + (1+r)b(i)$$

$$\Longleftrightarrow \quad \frac{b(i)}{a(i)} \leq \phi + \phi \frac{\pi(\theta(i)_e, a(i) + b(i))}{(1+r)a(i)} \tag{15}$$

The upper bound for the debt–to–equity ratio now also depends on the entrepreneur's profitability. It increases with a higher realization $\theta(i)_e$ of the idiosyncratic productivity shock.

Figure 5 shows the resulting cumulative distribution function of firmsizes for the modified model and compares it with the baseline setting for the case of $\phi = 1$. A comparison between eqs. (9) and (15) shows that including profit incomes into the collateral raises the debt–to–equity ratio by the amount of the second term on the RHS of equation (15).

Obviously, the model modification does not alter the general picture of how financial constraints affect the size distribution of firms. We observe a greater number of larger firms. The optimal firm sizes (indicated by the steps in the CDF) decrease slightly for each state of entrepreneurial talent $\theta_{e,h}$. This, again, can be explained with the factor price effect. If being more productive compensates for a lack of wealth, we expect the overall credit supply to the intermediate goods industry to be larger than in the baseline model. Less capital is employed in the final good sector, and the real interest rate rises. The larger user cost of capital explain the decrease in the optimal firm size in the noncorporate sector for each state of productivity.

(a) % of optimal firmsize

(b) entrepreneurship rate

(c) Y relative to $\phi \to \infty$

(d) welfare

(e) Gini of total wealth
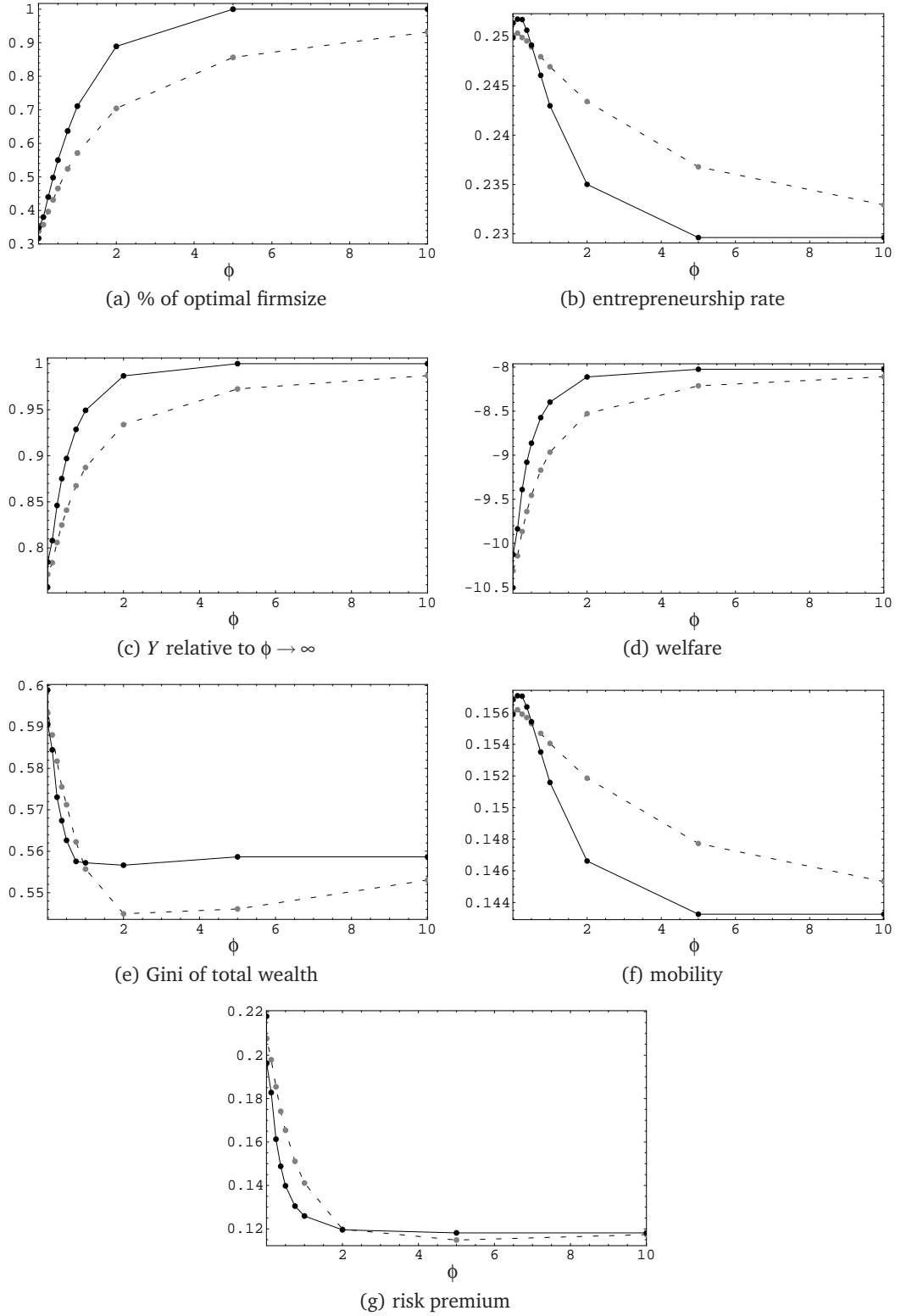
(f) mobility

(g) risk premium

Figure 6: Properties of the model with modified credit constraint (solid) for different levels of ϕ compared to the baseline model (dashed)

The last column of Table 3 presents the results from the numerical simulation of the modified model for $\phi = 1$. Naturally, there should be no differences in results for the two limiting cases of perfect ($\phi \to \infty$) or no credit market ($\phi = 0$) respectively. Figure 6 compares the baseline to the modified setting. We find the major results of our analysis preserved.

The significant effect is one in magnitude. Compared to the baseline model, the maximum loan is positively related to individual entrepreneurial productivity, which is effective for any given value of $0 < \phi < \infty$. This constitutes a credit market improvement, because now also relatively poor but highly productive agents are eligible for external financing. If we consider the case of $\phi = 1$, the loss in aggregate output compared to the perfect market scenario now only amounts to 5%, versus 11.5% of the baseline model. Qualitatively similar results can be observed for average wealth, individual incomes, consumption and factor prices, which exceed their corresponding values of the baseline economy. Wealth inequality is slightly larger under the modified borrowing constraint, which can mainly be ascribed to a more uneven distribution among firm owners.

The fraction of capital employed in the intermediate goods industry increases by roughly 3 p.p. Except for very small values of ϕ, the entrepreneurship rate of the modified model is smaller than in the baseline economy (see the above mentioned general equilibrium effects), but average firm sizes and profits are larger. The average firm size attains around 77% (vs. 67%) of its respective optimal value, and the average excess demand for capital in the intermediate goods industry only amounts to 40% (vs. 75%) of average business size.

Altogether, we observe that the economy, where the maximum loan also depends on individual productivity, responds more sensitive to changes in ϕ, the marginal gains of relaxing borrowing constraints being larger than in the baseline model.

Regarding mobility, we also find results qualitatively similar to the original model. Mobility is decreasing if borrowing constraints are relaxed, and—except for very small values of ϕ—the overall mobility between occupations is lower under the modified borrowing constraint. Table 4 shows how the probabilities of switching between occupations with respect to productivity states and wealth quintiles are affected, if credit availability also depends on individual productivity. The probability of switching occupations is higher for workers of the two lowest productivity states $(\theta_{w,1}, \theta_{w,2})$ over the entire range of ϕ, which follows directly from (15), where productivity unambiguously has a positive effect on the debt–to–equity ratio. Those entrepreneurs, who are members of the third and fourth quintile are less likely to switch, which reflects that ability compensates for lack of wealth.

Table 5: Simulation results with *iid* shocks

		Tightness of constraints		
		$\phi \to \infty$	$\phi = 1.0$	$\phi = 0$
entrepreneurship rate	(%)	0.245	0.244	0.240
∅ firm size	total	0.376	0.355	0.327
∅ credit rationing	total	0.000	0.045	0.129
∅ profits	total	0.093	0.093	0.094
final	Y	0.108	0.105	0.100
goods	K_F (%)	0.438	0.456	0.489
sector	K_F	0.072	0.073	0.075
	L_F	0.758	0.759	0.763
factor	w	0.090	0.087	0.082
prices	r	0.045	0.041	0.033
	$w/(r+\delta)$	0.849	0.861	0.887
factor	labor	0.630	0.630	0.630
income	capital	0.160	0.154	0.143
shares	profits	0.210	0.216	0.227
∅ wealth	total	0.164	0.159	0.154
	workers	0.054	0.053	0.039
	entrepreneurs	0.502	0.450	0.517
∅ income	workers	0.096	0.093	0.086
	entrepreneurs	0.146	0.143	0.142
risk premium		0.030	0.065	0.140
∅ consumption		0.098	0.095	0.091
welfare		-10.420	-10.713	-11.336
wealth	total	0.637	0.631	0.676
inequality	workers	0.323	0.362	0.414
(Gini)	entrepreneurs	0.379	0.311	0.246
mobility		0.025	0.014	0.004

4.3 The Model with *IID* Shocks

Although income and earnings persistence is the relevant environment from an empirical point of view, we now confront the results from the baseline model with the case of *iid* shocks. The major purpose of this exercise is to demonstrate how sensitive the model responds to a change in serial correlation, especially, if it comes to the implications for occupational choice and social mobility.

Table 5 shows the results from our numerical simulations, and Figure 7 compares the baseline setting to the model with *iid* shocks. We observe that the qualitative results for aggregate output, wealth holdings, factor prices, consumption, and welfare closely resemble the baseline model, although the effects differ in magnitude.
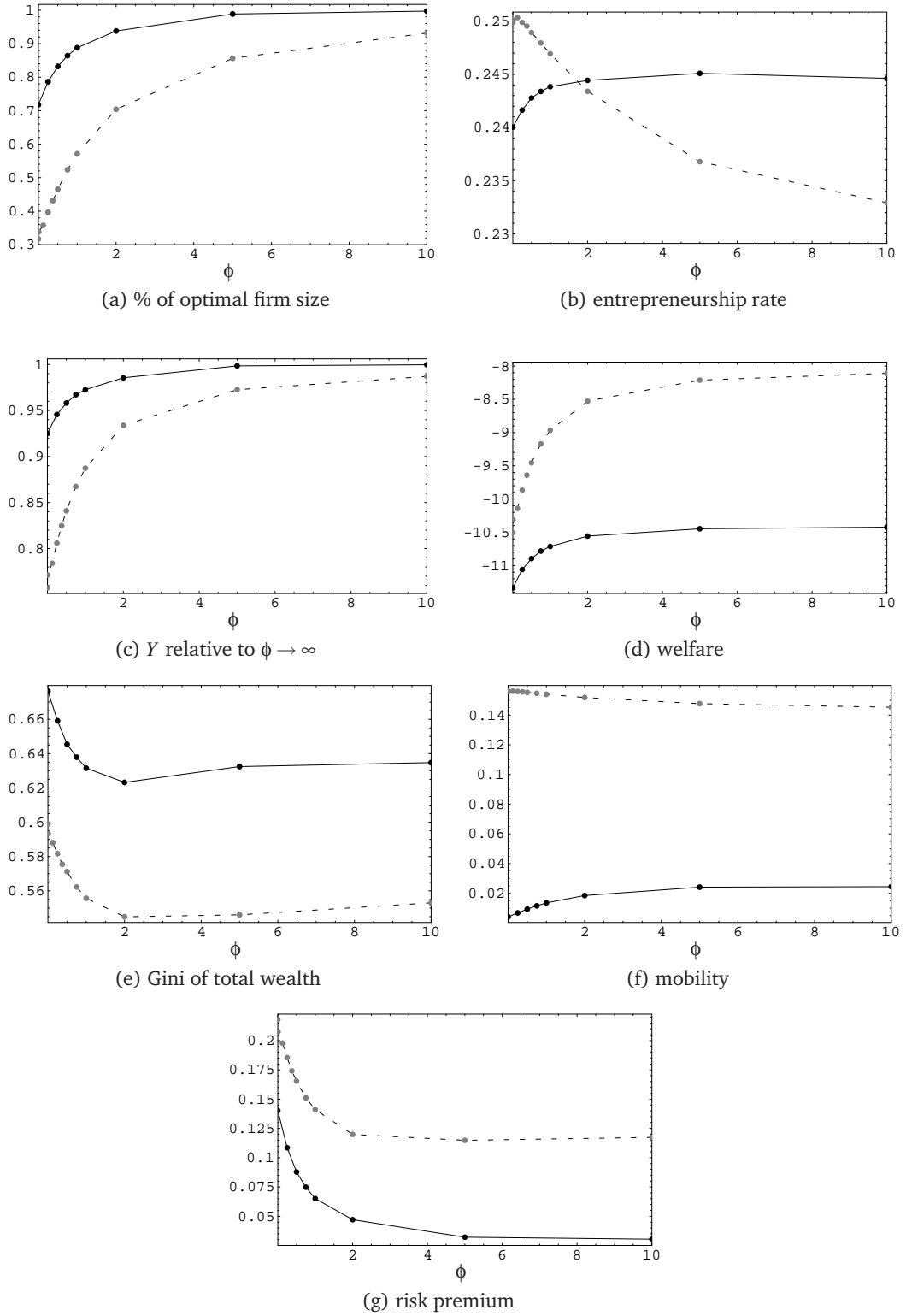
(a) % of optimal firm size

(b) entrepreneurship rate

(c) *Y* relative to $\phi \to \infty$

(d) welfare

(e) Gini of total wealth

(f) mobility

(g) risk premium

Figure 7: Properties of the baseline model (dashed) for different levels of ϕ compared with the model with *iid* shocks (solid)

Table 6: Mobility and *iid* shocks: Individual probability of a switch in occupations with respect to current productivity state and wealth quintile

		Tightness of credit constraints				Tightness of credit constraints		
	Productivity	$\phi \to \infty$	$\phi = 1.0$	$\phi = 0$	Quintile	$\phi \to \infty$	$\phi = 1.0$	$\phi = 0$
workers	$\theta_{w,1}$	0	0	0	1.	0	0	0
	$\theta_{w,2}$	0	0	0	2.	0.001	0	0
	$\theta_{w,3}$	0	0	0	3.	0.012	0.004	0
	$\theta_{w,4}$	0.125	0.067	0.019	4.	0.141	0.081	0.026
	$\theta_{w,5}$	0.489	0.288	0.111	5.	0	0	0
entre- preneurs	$\theta_{e,1}$	0.205	0.144	0.057	1.	0	0	0
	$\theta_{e,2}$	0.181	0.124	0.047	2.	0	0	0
	$\theta_{e,3}$	0.109	0.050	0.012	3.	0	0	0
	$\theta_{e,4}$	0	0	0	4.	0.554	0.314	0.104
	$\theta_{e,5}$	0	0	0	5.	0.0003	0	0

The overall output loss of the completely constrained economy vs. perfect capital markets only amounts to 7.5%, compared to almost 25% in the baseline model. Only 5% (vs. 17%) of aggregate capital is relocated towards the final goods sector, and the corresponding shift in the functional income distribution between profit and capital income shares is rather small (1.7 vs. 4.6 p.p.). The same is true for consumption (–10%) and wealth holdings (–6.2%), if constraints become more tight. Altogether, we observe that the economy with *iid* shocks responds less sensitive to changes in credit availability.

We find substantial differences between the two settings, if it comes to the intermediate goods industry. The rate of entrepreneurship rises (vs. decline) if we relax borrowing constraints in the model of serially uncorrelated shocks, and we observe a strikingly different pattern of mobility; see Figure 7b and Table 6.

While the average firm size in the intermediate goods industry behaves similar to the baseline model for a reduction in credit availability, the response of firm profits is of opposite sign. The average firm size declines by 13% (vs. 68%) for $\phi \to 0$, but profits rise by 1.8% (vs. –15%). Although the change in the entrepreneurship rate is rather small in scale, amounting only to 0.005 p.p., the effect from market exits is large enough to increase average profits of those agents remaining in the industry.

Table 6 summarizes our results on between–group mobility in a stationary equilibrium for the model with *iid* shocks. Irrespective of the degree of credit availability, we find that switches between occupations can only be observed for the highly productive workers ($\theta_{w,4}, \theta_{w,5}$), earning the highest wages, and the low–productivity entrepreneurs ($\theta_{e,1} - \theta_{e,3}$), earning the lowest profit incomes. Low and average productivity workers as well as the highly productive entrepreneurs never change their occupation. These results are in accordance with the economic intuition that earnings
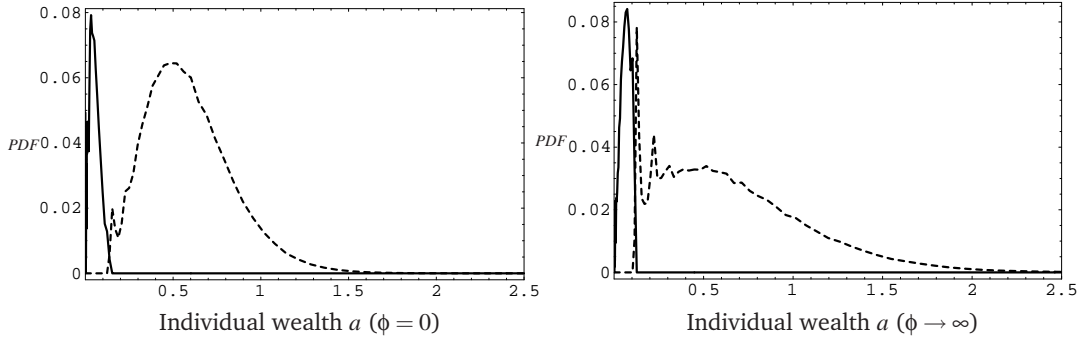
Figure 8: Wealth distribution in the baseline model for workers (solid) and entrepreneurs (dashed) with $\phi \to \infty$ and $\phi = 0$

advantages translate into higher individual wealth, the latter being an important determinant of entrepreneurship, especially in the presence of credit constraints.

However, overall mobility drops down sharply in the model with *iid* shocks compared with the baseline model. Whereas in the first setting on average 15% of the population changed their occupation in the stationary equilibrium, this figure goes down to 0.4% – 2.4% over the entire range of ϕ.

If we look at the probability of a change in occupation for members of wealth quintiles, we have to bear in mind that the wealth distribution of the economy with uncorrelated shocks is almost completely segregated with respect to occupational classes. Workers possess little wealth, and all rich households are entrepreneurs. The nature of the underlying shocks is crucial for the between–group equilibrium wealth distribution, which becomes obvious by comparing Figure 8 to Figure 2 of the baseline model.

Referring to Table 6, we observe a zero probability for a change of occupation for entrepreneurs of the lower three wealth quintiles as well for workers of the topmost quintile because none of them is a member of the respective wealth group. For workers, the probability of a status change is increasing in wealth and in credit availability. However, this altogether takes place at a very small scale, with only a 2.5% chance for workers of the 4th quintile to become an entrepreneur in the next period in the completely constrained economy and a 14% chance in the unconstrained one.

To understand mobility of entrepreneurs, we have to consider several factors. The future occupation is determined (a) by the present level of wealth, (b) the current draw of productivity governing present income, consumption and saving, (c) the choice between two lotteries with unconditional probabilities governing future income, consumption and saving, where the lottery over worker efficiencies is less risky than the lottery over entrepreneurial productivities, and (d) the expected

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market equilibrium of the next period, determining factor prices and factor income differentials.

Entrepreneurs of the 4th wealth quintile possess wealth amounts close to the critical level which separates future entrepreneurs from future workers.[11] If a member of this wealth class receives a bad productivity shock today, his wealth might not be large enough for self–employment to prove worthwhile, especially if becoming a worker in the next period is the safer option and the expected income differential is comparably small. Table 5 and Figure 7g show that the risk premium on entrepreneurial activity becomes very small if we relax borrowing constraints. This altogether explains the comparably large probability for a market exit of entrepreneurs in the 4th wealth quintile. If hit by a bad productivity shock in the present, even some business owners of the topmost wealth quintile switch occupations in the next period. Generally, the probability of a change in occupation declines for an increase in the tightness of financial constraints, which can be traced back to the substantial increase in the risk premium on entrepreneurial activity and rising average profits.

5 Concluding Remarks

In this paper, we examined the effects of borrowing constraints and idiosyncratic risks on macroeconomic performance, wealth inequality, and social mobility in a two–sector heterogeneous agent dynamic general equilibrium model. Workers and firm owners are subject to idiosyncratic (serially correlated) shocks. Entrepreneurship in the intermediate (noncorporate) goods industry is the riskier occupation. Our comparative static results cover the entire range of borrowing constraints, from complete markets to a perfectly constrained economy.

The stationary wealth distribution generated in the model is consistent with empirical findings. Entrepreneurial households own a substantial share of household wealth and their share increases throughout the wealth distribution.

Independent of the persistence of the idiosyncratic shocks, we find that tightening financial constraints is accompanied by substantial losses in aggregate output, consumption, wealth holdings, and welfare, while wealth inequality increases. The response of the macroeconomic variables to a change in credit availability is monotonous and concave, indicating, that it is a worthwhile question to explore in more detail the marginal gains from credit market improvement, which at this point is left for future research. To the extent firms of the intermediate goods industry are barred from participation in the credit market, more capital is employed in the final (corporate) goods sector. The associated decline in the interest rate causes a shift in the functional income distribution towards profit incomes.

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[11]This, too, is endogenously determined in the occupational choice equilibrium.

The general equilibrium context of our model, where optimal firm sizes and the demand for credit are determined endogenously, gives rise to interesting implications regarding the change in the entrepreneurship rate and in social mobility as we vary the degree of credit availability in the noncorporate sector. We find that more individuals choose the entrepreneurial profession in the presence of credit constraints compared to the unconstrained economy, and that mobility between occupations increases, too. Workers and entrepreneurs with high individual productivity tend to remain in their present occupation, whereas low productivity individuals are more likely to switch between professions. Regarding exit and entry rates into entrepreneurship, we find that higher persistence of shocks generally increases between–group mobility.

These results reverse strikingly, if we assume *iid* shocks, thus indicating that the nature of the underlying shocks plays an important role for the general equilibrium effects. The comparative static results on the entrepreneurship rate and social mobility respond sensitively to a change in income persistence.

There are many important issues this paper does not address. The model lacks a fully micro.-founded formulation of credit constraints and a more detailed modeling of financial intermediation. Also, testing the robustness of results with respect to attitudes towards risk is left for future research. So far, we assume worker efficiency and entrepreneurial ability to be uncorrelated, which can also be questioned, but it is difficult to measure such correlation in the data (cf. Cagetti and De Nardi, 2006*a*). Last, by simply stating the changes in the Gini coefficient, our results on inequality are still highly aggregated and should, in a next step, be decomposed in order to find out how good our calibration results on wealth concentration match distributional data.

References

Aghion, Philippe, Fally, Thibault, and Scarpetta, Stefano (2007), Credit Constraints as a Barrier to the Entry and Post–entry Growth of Firms, *Economic Policy*, October 2007, 731–779.

Aiyagari, S. Rao (1994), Uninsured Idiosyncratic Risk and Aggregate Saving, *Quarterly Journal of Economics*, **109** (3), 659–684.

Banerjee, Abhijit V. and Newman, Andrew F. (1993), Occupational Choice and the Process of Development, *Journal of Political Economy*, **101**, 274–298.

Blanchflower, David G. and Oswald, Andrew J. (1998), What Makes an Entrepreneur?, *Journal of Labor Economics*, **16** (1), 26–60.

Boháček, Radim (2006), Financial Constraints and Entrepreneurial Investment, *Journal of Monetary Economics*, **53**, 2195–2212.

— (2007), Financial Intermediation with Credit Constrained Agents, *Journal of Macroeconomics*, **29**, 741–759.

Cagetti, Marco and De Nardi, Mariacristina (2006*a*), Entrepreneurship, Frictions, and Wealth, *Journal of Political Economy*, **114** (5), 835–870.

— (2006*b*), Taxation, Entrepreneurship, and Wealth, Working Paper 2006–07, Federal Reserve Bank of Chicago, Chicago, IL.

— (2006*c*), Wealth Inequality: Data and Models, NBER Working Paper 12550, National Bureau of Economic Research, Cambridge, MA.

Clemens, Christiane (2006*a*), Monopolistic Competition and Entrepreneurial Risk–Taking, *Economics Letters*, **93** (1), 68–74.

— (2006*b*), Status Concerns and Occupational Choice under Uncertainty, *Advances in Theoretical Economics*, **6** (1), Article 4, BEPress.

— (2008), Imperfect Competition and Growth with Entrepreneurial Risk, *German Economic Review*, **9** (2), 179–205.

Clemens, Christiane and Heinemann, Maik (2006), On the Effects of Redistribution on Growth and Entrepreneurial Risk–Taking, *Journal of Economics*, **88** (2), 131–158.

Clementi, Gian Luca and Hopenhayn, Hugo A. (2006), A Theory of Financing Constraints and Firm Dynamics, *Quarterly Journal of Economics*, **121** (1), 229–265.

Desai, Mihir, Gompers, Paul, and Lerner, Josh (2003), Institutions, Capital Constraints and Entrepreneurial Firm Dynamics: Evidence from Europe, NBER Working Paper 10165, National Bureau of Economic Research, Cambridge, MA.

Evans, David S. (1987), Test of Alternative Theories of Firm Growth, *Journal of Political Economy*, **95** (4), 657–674.

Evans, David S. and Jovanovic, Boyan (1989), An Estimated Model of Entrepreneurial Choice under Liquidity Constraints, *Journal of Political Economy*, **97** (4), 808–827.

Evans, David S. and Leighton, Linda S. (1989), Some Empirical Aspects of Entrepreneurship, *American Economic Review*, **79** (3), 519–535.

Fernández-Villaverde, Jesús, Galdón-Sánchez, José Enrique, and Carranza, Luis (2003), Entrepreneurship, Financial Intermediation and Aggregate Activity.

Gentry, William M. and Hubbard, R. Glenn (2004), Entrepreneurship and Household Saving, *Advances in Economic Analysis and Policy*, **4** (1), Article 8.

Ghatak, Maitreesh, Morelli, Massimo, and Sjöstrom, Tomas (2001), Occupational Choice and Dynamic Incentives, *Review of Economic Studies*, **68**, 781–810.

Guvenen, Fatih (2007), An Empirical Investigation of Labor Income Processes, NBER Working Paper 13394, National Bureau of Economic Research, Cambridge, MA.

Hamilton, Barton H. (2000), Does Entrepreneurship Pay? An Empirical Analysis of the Returns to Self–Employment, *Journal of Political Economy*, **108** (3), 604–631.

Heaton, John and Lucas, Deborah (2000), Portfolio Choice and Asset Prices: The Importance of Entrepreneurial Risk, *Journal of Finance*, **55** (3), 1163–1198.

Holtz-Eakin, D., Joulfaian, D., and Rosen, Harvey S. (1994*a*), Entrepreneurial Decisions and Liquidity Constraints, *Rand Journal of Economics*, **25** (2), 334–347.

— (1994*b*), Sticking it Out: Entrepreneurial Survival and Liquidity Constraints, *Journal of Political Economy*, **102** (1), 53–75.

Huggett, Mark (1993), The Risk-Free Rate in Heterogeneous–Agent Incomplete–Insurance Economies, *Journal of Economic Dynamics and Control*, **17** (5–6), 953–969.

— (1996), Wealth Distribution in Life–Cycle Economies, *Journal of Monetary Economics*, **38**, 469–494.

Hurst, Erik and Lusardi, Annamaria (2004), Liquidity Constraints, Household Wealth, and Entrepreneurship, *Journal of Political Economy*, **112**, 319–347.

Kanbur, S. M. (1979*a*), Impatience, Information and Risk Taking in a General Equilibrium Model of Occupational Choice, *Review of Economic Studies*, **46**, 707–718.

— (1979*b*), Of Risk Taking and the Personal Distribution of Income, *Journal of Political Economy*, **87**, 769–797.

Kihlstrom, Richard E. and Laffont, Jean-Jacques (1979), A General Equilibrium Entrepreneurial Theory of Firm Formation based on Risk Aversion, *Journal of Political Economy*, **87**, 719–748.

King, Robert G. and Rebelo, Sergio T. (1999), Resuscitating Real Business Cycles, in: John B. Taylor and Michael Woodford (eds.), *Handbook of Macroeconomics*, North–Holland, Amsterdam, vol. Ib, pp. 927–1007.

Kirzner, I. (1973), *Competition and Entrepreneurship*, University of Chicago Press, Chicago.

Kitao, Sagiri (2008), Entrepreneurship, Taxation and Capital Investment, *Review of Economic Dynamics*, **11**, 44–69.

Knight, Frank (1921), *Risk, Uncertainty and Profit*, University of Chicago Press, (reprint) Chicago, 1979.

Li, Wenli (2002), Entrepreneurship and Government Subsidies: A General Equilibrium Analysis, *Journal of Economic Dynamics and Control*, **26**, 1815–1844.

Lucas, Robert E. (1978), On the Size Distribution of Business Firms, *Bell Journal of Economics*, **9**, 508–523.

Matsuyama, Kiminori (2007), Aggregate Implications of Credit Market Imperfections, NBER Working Paper 13209, National Bureau of Economic Research, Cambridge, MA.

Meh, Césaire A. (2005), Entrepreneurship, Wealth Inequality, and Taxation, *Review of Economic Dynamics*, **8**, 688–719.

Mehra, Rainish and Prescott, Edward C. (1985), The Equity Premium. A Puzzle, *Journal of Monetary Economics*, **15**, 145–161.

Minniti, Maria, Bygrave, William D., and Autio, Erkko (2005), Global Entrepreneurship Monitor. 2005 Executive Report.

Moskowitz, Tobias J. and Vissing-Jørgensen, Annette (2002), The Returns to Entrepreneurial Investment: A Private Equity Premium Puzzle, *American Economic Review*, **92** (4), 745–778.

Obstfeld, Maurice (1994), Evaluating Risky Consumption Paths: The Role of Intertemporal Substitutability, *European Economic Review*, **38**, 1471–1486.

OECD (2000), *OECD Employment Outlook 2000*, OECD Publishing, Paris.

Quadrini, Vincenzo (1999), The Importance of Entrepreneurship for Wealth Concentration and Mobility, *Review of Income and Wealth*, **45** (1), 1–19.

— (2000), Entrepreneurship, Saving and Social Mobility, *Review of Economic Dynamics*, **3** (1), 1–40.

Romer, Paul M. (1990), Endogenous Technological Change, *Journal of Political Economy*, **98** (5), S71–S102.

Sandmo, Agnar (1970), The Effect of Uncertainty on Savings Decisions, *Review of Economic Studies*, **37**, 353–360.

Schumpeter, Joseph (1930), *The Theory of Economic Development*, Oxford University Press, (reprint) Oxford, 1961.

Tauchen, George (1986), Finite State Markov-Chain Approximation to Univariate and Vector Autoregressions, *Economics Letters*, **20**, 177–181.

Vale, Steven (2006), The International Comparability of Business Start–Up Rates. Final Report, OECD Statistics Working Paper STD/DOC(2006)4, OECD, Paris.

A Computational Issues

The state space of wealth is approximated by a grid of N wealth levels a_n for $n = 1, \cdots, N$ with $a_1 = \underline{a}$ and $a_N = \bar{k}$. The macroeconomic equilibrium is recursively computed. We start with a initial guess on factor prices $\tilde{w}, \tilde{r}$, and the equilibrium level of employment in efficiency units $\tilde{L}$. Let $\mu = \{\tilde{w}, \tilde{r}, \tilde{L}\}$ denote the vector of the initial guesses. We obtain factor proportions in the final goods sector from this first solution trial. The underlying production technology implies $\tilde{K}_F = \tilde{L} \frac{\tilde{w}}{\tilde{r}+\delta} \frac{\gamma}{1-\gamma}$. Moreover, $F(\tilde{K}_F, \tilde{L})$ equals $\tilde{L} \left(\frac{\tilde{w}}{\tilde{r}+\delta} \frac{\gamma}{1-\gamma} \right)^\gamma$.

Let $k(a_n, s(j)_e)$ denote the firm size of an entrepreneur with productivity $s(j)_e$ and wealth a_n is able to operate at for a given degree of borrowing constraints. His profit is given by

$$\pi[a_n, s(j)_e | \mu] = \alpha \, (B\theta(i)_e \, k(a_n, s(j)_e))^\alpha \, \tilde{L} \left(\frac{\tilde{w}}{\tilde{r}+\delta} \frac{\gamma}{1-\gamma} \right)^\gamma - (\tilde{r}+\delta) \, k(a_n, s(j)_e) \, .$$

Let $a_w(a_n, s(j)_w | \mu)$ and $\xi_w(a_n, s(j)_w | \mu)$ as well as $a_e(a_n, s(j)_e | \mu)$ and $\xi_e(a_n, s(j)_e | \mu)$ denote the policy functions associated with the optimization problems (11) and (12) for the given initial guess on prices and employment. We characterize agents by their wealth holdings a_n, their occupational status ζ, where $\zeta = 1$ denotes a worker and $\zeta = 2$ an entrepreneur, and their current productivity state $s(j)_h$, $h = e, w$.

Knowing the policy functions and transition matrices for the underlying productivity shocks, we are able to compute the probability for an agent to have wealth a_n, occupational status ζ and productivity state $s(j)$. Let $\psi_{n,\zeta,s}(\mu)$ denote the respective probability for $n = 1, \ldots, N$, $\zeta = 1, 2$ and $s(j)_h = \theta_{h,j}$, $j = 1, \ldots, m$, $h = e, w$.

The probabilities $\psi_{n,\zeta,s}(\tilde{w}, \tilde{r}, \tilde{L})$ can be used to compute aggregate quantities. The aggregate capital stock (i.e. mean wealth holdings) can be determined as:

$$K(\mu) = \sum_{n=1}^{N} \sum_{\zeta=1}^{2} \sum_{j=1}^{m} \psi_{n,\zeta,s}(\mu) \, a_n$$

The entrepreneurship rate results as

$$\lambda(\mu) = \sum_{n=1}^{N} \sum_{j=1}^{m} \psi_{n,2,s}(\mu)$$

while labor supply in efficiency units is given by

$$L(\mu) = \sum_{n=1}^{N} \sum_{j=1}^{m} \psi_{n,1,s}(\mu) \, \theta_{w,j}$$

Capital demand of the intermediate goods sector can be computed as:

$$K_I^D(\mu) = \sum_{n=1}^{N} \sum_{j=1}^{m} \psi_{n,2,s}(\mu) \, k(a_n, s(j)_e)$$

The supply of capital to the final goods sector is given by $K_F^S(\mu) = K(\mu) - K_I^D(\mu)$. Employment L and capital input K_F in the final goods sector generate an aggregate output of

$$Y(K_F, L | \mu) = \left(K_F^\gamma, L^{1-\gamma} \right)^{1-\alpha} \sum_{n=1}^{N} \sum_{j=1}^{m} \psi_{n,2,s}(\mu) \, (B \, s(j)_e) \, k(a_n, s(j)_e))^\alpha$$

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The initial solution guess only represents an equilibrium if the following conditions must hold:

Labor supply in efficiency units must equal the initial guess $\tilde{L}$

$$L(\mu) = \tilde{L} \tag{i}$$

Labor demand and capital demand in the final goods sector equal their respective supplies:

$$L(\mu) = (1-\alpha)(1-\gamma)\frac{Y(K_F^S(\mu), L(\mu)\,|\,\mu)}{\tilde{w}} \tag{ii}$$

$$K_F^S(\mu) = (1-\alpha)\gamma\frac{Y(K_F^S(\mu), L(\mu)\,|\,\mu)}{\tilde{r}+\delta} \tag{iii}$$

The algorithm for finding the equilibrium values consists of three nested loops over $\tilde{L}$, $\tilde{w}$ and $\tilde{r}$. The first loop iteratively computes the value $\tilde{L}$ which meets condition (i) for given factor prices $\tilde{w}$ and $\tilde{r}$. Then, factor prices $\tilde{w}$ and $\tilde{r}$ are adjusted according to the resulting excess demands for labor and capital according to conditions (ii) and (iii). The whole procedure is repeated until the equilibrium conditions (i) to (iii) are satisfied, except or a tolerably small approximation error.

To implement the algorithm, we used the programming language C++. The underlying source code and the data are available from the authors upon request.