



**Trust and Adaptive Learning in Implicit
Contracts**

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FEMM Working Paper No. 17, July 2008

F E M M

Faculty of Economics and Management Magdeburg

Working Paper Series

Trust and Adaptive Learning in Implicit Contracts*

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This draft: 16.07.2008

Abstract

We study effects of trust in implicit contracts. Trust changes whenever the principal honors or dishonors an implicit contract. Usually a higher discount rate lowers the value of trade in an agency. We show that a sufficiently high level of (ex ante) trust can offset this effect. Strategies of principals representing unique equilibria are endogenously derived given different levels of agents' bounded rationality. These strategies mirror a subset of the class of trigger strategies which is exogenously entered into previous implicit contracting models. Therefore our results offer some justification for using that conventional approach. Implications for performance evaluation are discussed.

Keywords: trust, implicit contracts, bounded rationality, adaptive learning, trigger strategies, game theory

JEL code: D8, D81, M12, M 52

*We thank Holger Asseburg, Oliver Fabel, Alfred Luhmer, Barbara Schöndube-Pirchegger, Jack Stecher, and audiences at Bonn, Milan, and Rotterdam for helpful comments.

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1 Introduction

Over the past decade, the issue of trust in business transactions has gained increasing attention by researchers.¹ It is widely undisputed that trust affects many transactions and increases their value added.² Relational contracts often rely on trust. This is especially true in a knowledge based economy (Adler, 2001). Here, services delivered to or received from trade partners often require "innovation and knowledge inputs" which render complete contracts prohibitive if not infeasible, and trust - in receiving high quality inputs - then performs much better as a "contract" device than explicit and verifiable contract terms. Besides this interorganizational trust, intra-organizational or interpersonal trust is important, too. Firms, when contracting with their employees, often find it convenient to describe the job requirements in vague terms or they expect employees to provide additional input beyond contracted levels if that is necessary. Employees, in turn, expect firms to reward them for their flexibility and additional performance. As such both parties, employer and employee, expect the other party to behave cooperatively in the absence of binding agreements.

The problem to verify performance (and sign binding contracts) often shows up in incentive contracting with employees. The assumption of verifiable performance used to be standard but as Prendergast (1999, p. 57) notes, "most people don't work in jobs like these". Subjective - non-verifiable - performance evaluation then represents the method of choice. To make subjective performance evaluation schemes and processes work it is essential that those who are being evaluated and receive compensation contingent on evaluation trust the process (Baker, 1990, p. 55).

Analytical research on subjective performance evaluation has focused on self-enforcing contracts. Employers were assumed to comply with the implicit part of the contract, i.e. the bonus payment based on subjective performance evaluation, as long as it is beneficial with regard to future payoffs (e.g., Baker et al., 1994; Bull, 1987 ; Levin, 2003; MacLeod/Malcomson, 1988; Pearce/Stacchetti, 1998). When non-compliance was observed once, the agency could generate only the worst possible surplus in all future periods. This grim trigger strategy was therefore exogenous to the models. Also, trust could not play a role as contracting parties were completely rational and therefore able to foresee contract fulfillment. In this paper, we extend the aforementioned models and include trust as an integral part of the agency. This requires a departure from the full rationality assumption.

In the standard model of implicit contracting where rational players mutually anticipate their equilibrium strategies trust is either perfect or there is no trust at all. In a dynamic relationship,

¹Various journals published special issues on "trust", e.g. Academy of Management Review (1998), Journal of Economic Behavior and Organization (2004), or Organization Science (2003).

²See, for example, the review of empirical findings in Dirks/Ferrin (2001).

however, trust should evolve depending on past decisions.³ We develop a model where the agent uses the number of previous contract fulfillments to update beliefs on the probability that the contract will be fulfilled in the next period. To establish a model where an evolution of trust is consistent with equilibrium behavior, we use a bounded rationality approach. We assume that there are many different types of principals in the market: non-strategic types who always fulfill the implicit contract and strategic types who only fulfill a contract if it is optimal with regard to future payoffs. Furthermore, principals have different lifetimes leading to different strategies (for the strategic ones). The agent does neither know the type of his principal nor her lifetime. We assume that he is not able to determine sequential equilibria of the complete equilibrium path and to draw consistent probability assessments therefrom.⁴ As Camerer/Weigelt (1988, p. 2) suggest, while it is plausible to assume principals (firms) are able to compute these sequential equilibria, possibly by the help of consultants, agents (employees) are less likely to calculate them. Hence we model agents as players whose subjective probability assessments for contract fulfillment in a given period only depend on observed fulfillment decisions.⁵ This probability increases in the number of fulfillment decisions and decreases in the number of contract breaches. Trust, measured by the agent's subjective probability assessment that the contract will be honored in the next period, crucially influences gains of trade because it determines costs of inducing a given effort level.⁶ The core of the paper is that the history of play is the single source of information the agent uses to determine the principal's trustworthiness.

Related to our paper is the literature on reputation in repeated games (Mailath/Samuelson, 2006), specifically non-zero-sum games with incomplete information (Aumann/Maschler, 1995; Forges, 1992). Solutions to these games usually constitute a set of a possibly large number of equilibria and there is no reliable prediction which will prevail.⁷ Put differently, various trigger strategies can sustain an equilibrium (Friedman, 1971). To single out an equilibrium an assumption concerning a particular punishment has to be made (e.g., Green/Porter, 1984),

³See also Gürtler (2006).

⁴The critical discussion on the rationality assumption can be traced back at least to Simon (1955, p. 99), who called for a "fairly drastic revision" of the concept of "economic man"; this critique was (presumably not incidentally) published only one year after Savage's "Foundations of Statistics" (1954), which is still the standard reference for rational choice.

In Psychology, decision heuristics are considered the solution to the "tradeoff [that] exists between cognitive effort and judgmental accuracy" (Pitz/Sachs, 1984, p. 152) for boundedly rational individuals. Herrnstein/Prelec (1991) conclude that optimality in individuals' decisions given different alternatives is the exception, not the rule. It clearly follows that conventional rationality must be limited. For a discussion of limited rationality and its impact on individual choice, see, for example, the reviews by Camerer (1995) and Conlisk (1996), and, specifically on decisions in a dynamic context, Bone et al. (2003), Camerer/Johnson (2004), and Herrnstein (1991).

⁵Psychological research suggests that if one is unable to calculate exact probabilities and strategies, observed behavior is often the best predictor.

⁶That trust can increase performance has been shown by Zak/Knack (2001). In their model, however, agents were allowed to spend resources on verifying information and thus preventing to be (possibly) cheated.

⁷See also Kreps (1990), p. 102f.

or a certain level of "irrationality" is needed (Fudenberg/Maskin, 1986). While the cited work considers simultaneous moves by players, our model is based on sequential play. It is therefore most closely (although not closely) related to reputation models in financial contracting (e.g. Boot et al., 1993; John/Nachman, 1985). Boot et al. (1993) analyze reputation formation that may give highly-profitable principals (H principals in their formal diction) an advantage over less profitable principals (L principals) when sequentially offering possibly different types of financial contracts. In their three-period model with fully rational agents a separating or a pooling equilibrium may obtain. In either case, discretionary contracts are offered only in period 1 (and enforceable ones thereafter). Consequently, no strategy profile for L-principals spanning multiple periods is derived. Our model derives such a strategy profile in a pooling equilibrium over the agency's course of T periods. As already reasoned above, a departure from the full rationality assumption is necessary. One could therefore argue that our results relate more to employer-employee relations while the analysis in Boot et al. (1993) probably better predicts outcomes for games played between firms.

The contribution of our paper is threefold. First, we include trust and – associated with it – (reasonably) bounded rationality of agents in implicit contracts. Several important results emerge from this analysis. Our results show that a sufficiently high level of ex ante trust can offset detrimental effects of a high discount rate. As results do not depend qualitatively on the ex ante "distribution" of trust, they are generalizable in the sense that circumstances with quite different levels of ex ante trust lead to the same strategy patterns being optimal. For a sufficiently low discount rate principals will honor the implicit contract (if at all) at the beginning of an agency and dishonor it towards the end. The option of principals to switch back and forth between fulfillment and nonfulfillment is not exercised in equilibrium. This outcome mirrors the grim trigger strategy which is usually exogenously entered into implicit contracting models to sustain an equilibrium. Our paper can therefore provide a rationale and some justification for the popular use of that approach. However, the equivalence depends on full memory of the agent in our trust setting. In case the agent is not able to fully recall the entire history of play, alternating strategies can become optimal, i.e. the principal switches back and forth between fulfillment and nonfulfillment. Depending on how many periods of play the agent recalls, we can endogenously derive strategy profiles that mirror different trigger strategies (that are also exogenous to previous models). An optimal and a critical (or minimal) level of trust in the agency will be identified as well.

Second, our results contribute to game theory by deriving strategy profiles as unique incomplete-information equilibria in two-player sequential move games. The specific equilibrium depends on how bounded rational the agent is, i.e. whether he is able to recall the entire history of play or only a limited number of periods. As Fudenberg/Maskin (1986) demonstrate for (n -person) simultaneous move games that the level of "irrationality" determines a specific equilibrium, we do so for two-person sequential move games.

Third, since trust has a moderating effect on performance - that is, trust provides the condition for higher performance - we also contribute to the relatively undeveloped literature (Dirks/Ferrin, 2001, p. 451) in that respect.

Finally, our research has implications for performance evaluation practices which we discuss in detail.

From here on we proceed as follows. As there are up to now only very few papers that formally link trust to agency⁸, we begin with a short section on trust and on how it might be linked to economic transactions. It includes a brief separation of trust from reputation. The following section will then introduce the model. Section 4 contains equilibrium analysis. An extension of the model, limited recall by the agent, is presented in section 5. In section 6 we interpret and apply our findings. The final section concludes.

2 Trust in transactions

Trust is "something" that is present in many real-life situations although one may not very often be aware of it.⁹ For example, you ask a local in a foreign city to show you the way to a famous sight. Can you trust him (or her)? Will the prescribed way lead you to the sight or will you end up in a neighborhood that you would not like to have visited otherwise? Many other examples of trust quickly come to mind; you expect food you bought to have been produced in a hygienic way; or, you expect the supplier to deliver in time and in good quality. What unites these (and other) examples is the fact that an "expectation concerning the behavior of others" is involved, which is a common understanding of trust.¹⁰ Luhmann (1979) argues that trust is linked with complexity reduction or predictability.¹¹ Consequently, trust allows for a specific conjecture of others' behavior. Following Luhmann (1989), the trustor - as the subject of trust - puts trust in the trustee - the object of trust - by offering a favour in advance, a 'Vorleistung' (Luhmann, 1989, p. 23), to him, and the trustor then expects the trustee to not behave opportunistically. Without such a 'Vorleistung' as a form of trust, however, many (mutually beneficial) social or economic interactions would simply not take place.¹² The mutual benefit - besides complexity

⁸Rosen (1992, p. 187) links trust to agency, and Jones (1995) considers trust as a solution to agency problems. Casadesus-Masanell (2004) formally models trust in a one-period agency, while Zak/Knack (2001) relate trust between principals and agents to economic growth.

⁹This section draws from different chapters in Nooteboom (2002). For a comprehensive elaboration on trust in an economic context, see Nooteboom (2002) and the references therein; Hosmer (1995) and James (2002) review the concept of trust and trustworthiness, and Bachmann (2001) analyzes the role of trust in co-ordinating organizational relationships, including a discussion of various notions and theories of trust.

¹⁰Nooteboom, 2002, p. 6.

¹¹Other accepted theories besides Luhmann's Systems Theory are Coleman's (1990) Rational Choice approach and Giddens' Structuration Theory (1976, 1984).

¹²See Bachmann (2001), p. 342.

reduction - offers a convincing argument why we trust.¹³ Nevertheless, trust remains a risky engagement (Luhmann, 1979) for both parties. As such, following Bachmann (2001, p. 342f.), trust reduces risk but at the same time produces risk. If the trustee behaves opportunistically it may lead to substantial losses on the side of the trustor. And one could argue that this is exactly the risk one would like to minimize.

Trust as a mean to reduce uncertainty and complexity may come in two different forms: personal trust and system trust; and as Zucker (1986) argues, both forms are needed given the complexity of today's world. Personal trust is confidence in individuals, whereas system trust refers to confidence in abstract systems.¹⁴ Given these two forms of trust and the above considerations, a definition of trust can be given: Trust in institutions or persons means to accept that they may take advantage of you though you expect them not to do so. A question that inevitable arises, though, is, (1) how can trust be established or 'produced'? And (2) how, if at all, can it be calculated? To begin with the first question, observed behavior like loyalty and commitment in face-to-face contacts is very likely to establish personal trust; or think of internet reputation systems like ebay or amazon that work through feedback by buyers (Resnik/Zeckhauser, 2002). System trust can be built up by enforcing rules or certain standards. Sydow (1998, p. 54) coined the term 'trust-sensitive management', meaning that managers acting on behalf of the organization take into account how their actions and decisions affect trust that individual employees have in the organization.¹⁵ As such, system trust can be easily related to accounting and performance measurement. According to Kwon (2005), accounting may be perceived as consisting of two stages: (1) aggregating underlying detailed data into "summary statistics" and (2) mapping summary statistics into accounting reports.¹⁶ Either of the two stages can lead to accounting noise, i.e. summary statistics do not (accurately) reflect disaggregate data, or accounting reports are prone to errors. Trust could then be defined as the expectation of low accounting noise.¹⁷

To turn to the second question, there has been and still is a debate as to whether trust, since it entails risk, can be identified as a (subjective) probability. Dasgupta (1988), Kreps et al.

¹³Johnson (1750) offers an early and distinct answer to that question: 'It is happier to be sometimes cheated than not to trust'. (Samuel Johnson (1750), "The Rambler"; quoted from Nooteboom (2002), p. 2.)

¹⁴Zucker (1986) refers to personal trust as process-based trust, and uses institutional-based trust as the term for what is called system trust in Luhmann's (1979) or Giddens' (1990) terminology.

¹⁵See also Nooteboom (2002), pp. 84ff.

¹⁶Cf. Kwon (2005), fn. 9.

¹⁷See Busco et al. (2006) for an analysis of how a management accounting system can act as a source of trust (in a situation of change). Johansson/Baldvinsdottir (2003) provide evidence that performance evaluation is based on trust and the production and reproduction of performance-evaluation routines.

For purposes here, we are not concerned about problems that arise when different perceptions between the organization (principal) and the employee (agent) are present. MacLeod (2003) analyzes such a situation in the context of a non-availability of verifiable performance measures, whereas Mitusch (2006) deals with the principal's ability to produce "hard facts", i.e. verifiable performance measures.

(1982), Mayer et al. (1995), and Mui et al. (2002) are examples of advocates in this respect. Williamson (1993) and Nootboom (2002, p. 40) have opposing views on this matter; the latter objects trust as a subjective probability because it can become one, implying certainty; but trust is related to *uncertainty*. He prefers to speak of trust as an "expectation", leaving room for "residual uncertainty about agency and unforeseeable contingencies" (Nootboom, 2002, p. 41). For purposes of analytical research, e.g. in agency or contract theory, the distinction between probability and expectation seems to be semantic or philosophical. What will qualitatively matter for (our) modeling work is that with trust increasing between contract partners, higher-stake transactions become possible, which leads back to trust allowing to predict the behavior of contracting partners. In this vein, reputation is to be understood as a source of trust and trustworthiness.¹⁸

3 The model

We consider a long-term agency relationship of T periods with spot (one period) contracts. The agent's action in period t is $e_t \in [0, 1]$, at cost $c(e_t) = e_t^2/2$. Actions are observable but not verifiable. The principal's expected gross outcome (before compensation) in period t is e_t . An interpretation of this outcome is as follows: Let gross output in each period be a random variable $x \in \{0, 1\}$ independently distributed over time, then agent effort e_t can be considered as the probability that $x = 1$ is achieved in period t . Expected gross outcome of period t then amounts to $E(x|e_t) = 1 \cdot e_t + 0 \cdot (1 - e_t) = e_t$.

At the end of every period $t, t = 1, 2, \dots, T$, the agent receives compensation W_t , consisting of a fixed component s_t (which is guaranteed by the contract) and a possible bonus payment $b_t = v_t e_t$ given observation e_t ¹⁹ which is contingent on the principal's decision to pay, i.e. to comply with the implicit contract or not.²⁰ The bonus equals a share of the expected output in period t with sharing rate v_t . Both parties are risk neutral and the agent's reservation utility is zero in each period.

Trust is modeled as the probability - from the agent's point of view - that the principal honors the implicit contract at the end of period t . We assume that there are different types of principals in

¹⁸See Nootboom (2002), p. 68f.

¹⁹Formally, the contract of period t can be thought of as consisting of an explicit fixed payment s_t , an effort level e_t and an implicit bonus b_t . If the agent has performed the pre-specified level of effort in period t , he is eligible for bonus payment. Instead of including the desired effort level explicitly into the contract we let the agent's effort being induced via the bonus function $b_t = v_t e_t$, depending on the observed effort level e_t . As both parties are risk-neutral the principal can induce every desired level e_t between 0 and 1 via the bonus function at the same cost as with directly writing this level into the contract. Consequently, there is no loss of generality in using this approach.

²⁰Given that the outcome of the agency is binary in each period, the optimal compensation contract can be expressed by a linear (bonus) contract without loss of generality. (See Christensen/Feltham, 2005, p. 153.)

the market: honest (non-strategic) principals who always pay the implicit bonus and dishonest (strategic) principals who only pay the bonus if it is optimal with respect to future payoffs. Furthermore, principals have different lifetimes leading to possibly different fulfillment strategies of dishonest types. The agent does not know whether he is working for a strategic principal or for a non-strategic one, nor does he know his principal's lifetime. We assume that the agent is bounded rational in the sense that he is not able to determine sequential equilibria of a dynamic relationship facing different types of principals with different lifetimes and to draw consistent probability beliefs therefrom. Instead, by entering the agency relation, the agent has some initial trust. Specifically, the agent knows a distribution g of the probability γ that the contract is honored in the first period. The distribution $g(\gamma)$ is a beta distribution with parameters α and β , $\alpha, \beta > 0$, and mean (initial trust) $E(\gamma) = \frac{\alpha}{\alpha + \beta}$. To prevent that the agent assigns nonfulfillment of a contract unambiguously to a strategic principal, principals may pretend to have been hit by a financial shock or other unforeseeable contingencies. In line with Englmaier/Segal (2006) we assume that the agent cannot verify whether or not such a shock has occurred. We analyze a relationship between the agent and a strategic principal who lives for T periods.

While the agent knows the principal's single period profit function which allows him to calculate the single period optimal contract, he is not able to strategically infer (or predict) the behavior of different types of principals unlike principals reveal their types by offering different contracts.²¹ (In fact, assuming full rationality would lead to the infeasibility of a "trust solution" in our model.) When the agent believes his firm is of the strategic type, trust is completely lost and he will never perform any productive effort.

Trust – the probability assessment that the contract will be fulfilled – is exclusively determined via Bayesian updating based on the history of fulfillment decisions. Let $\theta_t \in \{0, 1\}$ indicate nonfulfillment ($\theta_t = 0$) or fulfillment ($\theta_t = 1$) of the implicit agreement in period t . Then the expected probability at the beginning of period t that the bonus is paid at the end of the period given a history $\theta^{t-1} = (\theta_1, \theta_2, \dots, \theta_{t-1})$ is given by²²

$$E(\gamma | \theta^{t-1}) = \frac{\alpha + \sum_{i=1}^{t-1} \theta_i}{\alpha + \beta + t - 1}. \quad (1)$$

The trust dynamics assumed by (1) have been observed in experimental studies (Jonker et al., 2004) and field studies (Butler, 1983).²³

²¹There are a number of experimental studies showing that individuals fail to correctly apply backward induction (see, e.g., Binmore et al., 2002; Johnson et al., 2002; or Rapoport, 1997), or do not plan ahead (Bone et al., 2003; Hey/Knoll, 2007).

²²Note that, from an agent's point of view, the principal's decision to honor or dishonor the implicit part of the contract is a draw from a Bernoulli distribution with γ as the unknown parameter. As this parameter has a beta distribution with (α, β) , the draw can be used to update the agent's probability assessment of γ (DeGroot (1970), p. 160).

²³For further experimental evidence on reputation and trust formation, see Camerer/Weigelt (1988),

At this point, a note on the functional assumption of the beta distribution seems appropriate. The beta distribution is rich in the sense that it includes quite different bell-shaped, unimodal, and bimodal forms (with modes at zero and/or one). Therefore, virtually any *ex ante* assessment of trust can be mapped. Updating in light of implicit bonus payments or not occurs technically by applying Bayes' rule, so beliefs are consistent in that respect. But the updating process is intuitively appealing, too, which it should be in light of the bounded rationality assumption. Starting with an initial assessment the agent views every fulfillment decision as a sign that the principal is of the "good type", and every nonfulfillment as a sign for the "bad type". In that respect trust evolves according to a simple adaptive learning process.

The intuitive functional form allows for closed-form solutions and will be helpful to derive tractable results if more structure is added to the model. One possible extension could be trust-enhancing measures taken by the principal. Following Lukas (2005, p. 197), a measure m would impact distributional parameters as follows:

$$\begin{aligned}\alpha_m &= \alpha + m \cdot \lambda \\ \beta_m &= \beta + (1 - \lambda) \cdot m,\end{aligned}$$

where $\lambda, 0 \leq \lambda \leq 1$, is the principal's choice parameter indicating how effectively the measure is implemented. The mean obtains as

$$E(\gamma | m, \lambda) = \frac{\alpha + m \cdot \lambda}{\alpha + m \cdot \lambda + \beta + (1 - \lambda) \cdot m} = \frac{\alpha + m \cdot \lambda}{\alpha + \beta + m}.$$

As an example, think of m as a code of ethics that is adopted. If employees observe the firm and its managers live up to the code ($\lambda \rightarrow 1$), trust increases (as $\alpha_m > \alpha$ and $\beta_m = \beta$ lead to an increasing mean of the distribution). Conversely, non-compliance with the code ($\lambda \rightarrow 0$) leads to lower trust.

4 Equilibrium analysis

4.1 Beliefs and strategies

At the beginning of every contracting period each type of principal offers a linear contract W_t . As the agent knows the principal's one shot problems, he is also able to determine the optimal current period contract given the principal is willing to pay the bonus. Denote b_t^* the optimal bonus for period t for a principal who is willing to pay the bonus, consistent with probability assessment $E(\gamma | \theta^{t-1})$ by the agent. Notice that in our model all types of principals are equally productive in each period they live. As periods are independent in terms of stochastic output, the optimal bonus b_t^* is independent of a principal's lifetime. To sustain our equilibrium we

Kagel/Roth (1995), or Mayhew (2001).

impose the following out-of-equilibrium belief for the agent: If a bonus $b_t \neq b_t^*$ is offered to the agent he assumes that this offer comes from a strategic principal. This belief is plausible for a bounded rational agent who presumes that an honest type will not deviate from her optimal bonus b_t^* . Given this belief a bonus offer $b_t \neq b_t^*$ implies that trust becomes zero and all future gains from the agency are lost. Hence, in equilibrium all types of principals will offer a bonus b_t^* in period t . We determine the equilibrium bonus in the next subsection.

4.2 The single-period problem

To determine the solution to the principal's multi-period decision problem, we consider first a single arbitrary period of the relationship between principal and agent. From the above section we know that the equilibrium contract offer of each type of principal is equal to the optimal contract that would be offered by a principal who is willing to honor the implicit contract. The corresponding optimization problem is given by²⁴:

$$\max_{s,v} (e - ve - s) \quad (2)$$

s.t.

$$s + E(\gamma)ve - c(e) \geq 0 \quad (3)$$

$$e = \arg \max_{\hat{e}} s + E(\gamma)v\hat{e} - c(\hat{e}) \quad (4)$$

The principal maximizes her expected output net of the agent's expected compensation, $E(e - b - s) = (e - ve - s)$, taking into account that the contract must be individually rational (3) and incentive compatible (4). The agent's objective function is expected compensation, $E(s + b) = s + E(\gamma)ve$, less cost of effort, $c(e)$. Incentive compatibility condition (4) can also be written as

$$e = E(\gamma)v.$$

It is obvious that the level of trust impacts the effort choice; higher levels of trust allow to implement higher effort levels for a given bonus rate v .

With a binding participation constraint, the principal's optimization program simplifies to

$$\max_v vE(\gamma)(1 - v + vE(\gamma)) - \frac{1}{2}v^2E(\gamma)^2.$$

Optimal bonus rate v^* and induced action e^* are given by

$$v^* = \frac{c'(e^*)}{E(\gamma)} = \frac{1}{2 - E(\gamma)}, \quad (5)$$

$$e^* = \frac{E(\gamma)}{2 - E(\gamma)}. \quad (6)$$

²⁴As all period are alike except for induced trust we dropped the time indices.

If trust was perfect, $E(\gamma) = 1$, (5) and (6) would lead to

$$e^* = v^* = b^* = 1$$

and a benchmark surplus of $S^H = \frac{1}{2}$ when the principal honors (H) the contract.

With trust being imperfect, $E(\gamma) < 1$, inefficiency results. When the principal honors the contract, her expected surplus as a function of induced trust $E(\gamma)$ is given by

$$S^H = [e^* (1 - v^* + v^* E(\gamma)) - c(e^*)] = \frac{1}{2} \frac{E(\gamma)}{(2 - E(\gamma))} < \frac{1}{2}, \quad (7)$$

while the agent always receives his reservation utility in equilibrium. Imperfect trust does not restrict the set of implementable actions. The principal could always set $v = 1/E(\gamma)$ to induce first-best incentives. However, implementing the first best action is too costly for the principal if $E(\gamma) < 1$. Similar to Gurtler (2006), the agent weights his bonus with the probability of contract fulfillment but the principal has to bear the full cost of compensation. Hence, the optimal bonus b^* solves a trade-off between incentives and (imperfect) trust: The lower the level of trust the higher the required bonus to induce a given level of effort and - at the same time - the higher the cost of imperfect trust (in terms of expected compensation) resulting from the distortion in the agent's incentives. According to (7) this cost is given by $e(v - e) = vE(\gamma)(v - vE(\gamma))$. With perfect trust, $e = v$, this term vanishes, but with imperfect trust this cost is increasing in the bonus rate v . Inefficiency here results already from limited trust.

If the principal dishonors (D) the implicit contract, the resulting surplus is

$$S^D = v^* (1 + v^* E(\gamma)) - c(e^*) = \frac{E(\gamma) (4 - E(\gamma))}{2 (E(\gamma) - 2)^2} > S^H. \quad (8)$$

4.3 The multi-period problem

In the multi-period problem let $\theta^t = \{\theta_1, \dots, \theta_t\}$ be the history of play up to the beginning of period $(t + 1)$ determined by the sequence of past bonus payment decisions. The principal must strategically decide whether to fulfill the contract or not. Furthermore let $\gamma_{t-1}(\theta^{t-1}) \equiv E[\gamma | \theta^{t-1}]$ be the expected probability (from the agent's perspective given his adaptive learning process) that the bonus will be paid in period t given history θ^{t-1} . Define $S_t(\theta^{t-1})$ as the principal's surplus in period t given θ^{t-1} . According to surplus values defined in (7) and (8) we have

$$S_t^H(\theta^{t-1}) = \frac{1}{2} \frac{\gamma_{t-1}(\theta^{t-1})}{(2 - \gamma_{t-1}(\theta^{t-1}))} \quad (9)$$

and

$$S_t^D(\theta^{t-1}) = \frac{\gamma_{t-1}(\theta^{t-1}) (4 - \gamma_{t-1}(\theta^{t-1}))}{2 (\gamma_{t-1}(\theta^{t-1}) - 2)^2}. \quad (10)$$

A principal's strategy $\boldsymbol{\theta} = (\theta_1, \theta_2, \dots, \theta_T)$ specifies for each period t an optimal decision θ_t . The principal's ex ante expected payoff of period t playing strategy $\boldsymbol{\theta}$ equals $S_t(\boldsymbol{\theta})$. Here $S_t(\cdot) \in \{S_t^H(\cdot), S_t^D(\cdot)\}$ indicates the principal's payoff from the agency in period t given strategy $\boldsymbol{\theta}$.

Because nonfulfillment is optimal in period T , $\theta_T = 0$, the principal's non-renegeing constraint of period $t = 1, \dots, (T - 1)$ is given by

$$S_t^H + \sum_{i=t+1}^T \delta^{i-t} [S_i(\theta^{i-1*})] \geq S_t^D + \sum_{i=t+1}^T \delta^{i-t} [S_i(\tilde{\theta}^{i-1*})] \quad (11)$$

where $\delta^i = \frac{1}{(1+r)^i}$ is the discount factor of an agency with interest rate $r \geq 0$. Notice that the optimal decision in period t requires anticipating optimal future decisions. Hence, the set of $(T - 1)$ non-renegeing constraints is a result of a dynamic optimization problem. To clarify that non-renegeing constraints must be evaluated at optimal future decisions we write θ^{i-1*} and $\tilde{\theta}^{i-1*}$, respectively. θ^{i-1*} captures the history up to period $i > t$, given the implicit bonus has been paid in period t and $\tilde{\theta}^{i-1*}$ is the analogon if not. The non-renegeing constraint (11) can be rewritten as

$$\sum_{i=t+1}^T \delta^{i-t} [S_i(\theta^{i-1*}) - S_i(\tilde{\theta}^{i-1*})] \geq S_t^D - S_t^H \quad (12)$$

with $S_t^D - S_t^H = \frac{\gamma_{t-1}(\theta^{t-1})}{(\gamma_{t-1}(\theta^{t-1}) - 2)^2}$.

The principal's optimal strategy $\boldsymbol{\theta}^*$ is the solution to the following dynamic optimization problem: Beginning with the last period the principal determines in every period his optimal decision $\theta_t^*(\theta^{t-1})$ as a function of the history θ^{t-1} by maximizing period t 's contribution plus optimal contributions of future periods,

$$P_{t+1} = \sum_{i=t+1}^T \delta^{i-t} [S_i(\theta^{i-1*})].$$

We call P_t the principal's value function of period t .

Equivalently, define

$$\begin{aligned} \mathcal{S}(\boldsymbol{\theta}) &\equiv P_1(\boldsymbol{\theta}) = \sum_{t=1}^T [S_t(\boldsymbol{\theta})] \delta^{t-1} \\ &= \sum_{t=1}^T S_t^H(\theta^{t-1}) \theta_t \delta^{t-1} + \sum_{t=1}^T S_t^D(\theta^{t-1}) (1 - \theta_t) \delta^{t-1} \end{aligned} \quad (13)$$

as expected ex ante surplus given strategy $\boldsymbol{\theta}$. $\mathcal{S}(\boldsymbol{\theta})$ is decreasing in r . Then

$$\boldsymbol{\theta}^* \in \underset{\boldsymbol{\theta}}{\operatorname{argmax}} \mathcal{S}(\boldsymbol{\theta}).$$

Notice that the equilibrium path of fulfillment or nonfulfillment decisions is not trivial in this model. Although we assume a stationary production process, periods differ in the level of trust

and this level determines the cost of inducing a given effort. The principal's optimal dynamic bonus payment strategy solves the following trade-off: If she does not pay the implicit bonus today she earns surplus S^D instead of S^H , but she also damages trust such that future payoffs are getting lower. In addition, with an increasing discount rate the value of future surpluses decreases.

To answer the question whether a higher level of initial trust can compensate for a higher discount rate, observe that both expected surplus from fulfillment, S_t^H , and from nonfulfillment, S_t^D , increase in the expected value of γ , i.e. in the level of trust:

$$\frac{dS_t^H(\theta^{t-1})}{d\gamma_{t-1}} = \frac{1}{(2 - \gamma_{t-1})^2} > 0 \quad (14)$$

$$\frac{dS_t^D(\theta^{t-1})}{d\gamma_{t-1}} = \frac{4}{(2 - \gamma_{t-1})^3} > 0 \quad (15)$$

Since (14) and (15) are positive in every period given any history of play, a higher ex ante level of trust leads to a positive affine transformation of (13) and thus increases the value of trade.

Proposition 1 (a) *With an increasing interest rate r , fulfillment of implicit contracts becomes (weakly) less attractive. The expected equilibrium surplus is decreasing in r .*

(b) *A higher level of ex ante trust offsets, at least partially, the detrimental effect of a higher discount rate and increases the value of trade in the agency.*

Proof. See appendix. ■

If it is optimal for the principal to fulfill the implicit contract in some period, trust builds up according to the agent's adaptive learning process which enables higher future surpluses. If the discount rate r is increasing, however, the value of future surpluses decreases such that building up trust becomes less attractive. Hence, the number of fulfillments up to some period t is weakly decreasing in r as the optimal level of trust induced in period t is weakly decreasing in r (part (a) of the proposition). One immediate conclusion from this proposition is that if complete nonfulfillment is optimal for some discount rate r it must be optimal for all discount rates higher than r . Part (b) of proposition 1 accords with intuition as a higher level of trust ceteris paribus increases the value of trade in the agency.

Now the strategy θ will be analyzed in depth to derive general patterns of decisions. We seek to characterize strategies as alternating or non-alternating strategies first.

Definition 1 (Alternating strategy) *A strategy θ is called alternating, if it features more than one switch between fulfillment and nonfulfillment during the agency.*

Definition 2 (Non-alternating strategy) *A strategy θ is called non-alternating, if it features exactly one switch between fulfillment and nonfulfillment during the agency (or vice versa).*

We know that for every history θ^{t-1} , $S_t^D(\theta^{t-1}) > S_t^H(\theta^{t-1})$. We further know from (15) and (14) that both S_t^H and S_t^D are increasing functions of γ_{t-1} , the level of trust generated up to period t . The level of trust in period t , γ_{t-1} , is increasing in the number of fulfillments up to period t (see (1)).

Definition 3 (*Strict strategy*) A strategy θ is called strict, if it features the same decision in every period.

Writing payoffs in terms of induced trust γ_{t-1} at the beginning of period t transforms (13) into $\mathcal{S}^D(\theta) = \sum_{t=1}^T S_t^D(\gamma_{t-1}^0)\delta^{t-1}$, for strict nonfulfillment, $\theta = \mathbf{0}$, and $\mathcal{S}^H(\theta) = \sum_{t=1}^T S_t^H(\gamma_{t-1}^1)\delta^{t-1}$, for strict fulfillment, $\theta = \mathbf{1}$, where γ_{t-1}^θ indicates the level of trust at the beginning of period t contingent on θ . Given (14) and (15), notice $\mathcal{S}^D(\theta)$ is a strictly monotone increasing concave function of t and $\mathcal{S}^H(\theta)$ is a strictly monotone increasing convex function of t . This leads to the following conclusion.

Conclusion 1 *There exists a threshold value \tilde{T} of the length of the contracting relationship T such that strict fulfillment dominates strict nonfulfillment, if $T > \tilde{T}$.*

Now consider a relationship of T periods and assume that the principal's strategy exhibits fulfillment from period $t = 1, \dots, t'$; and nonfulfillment from $t = t' + 1, \dots, T$, with corresponding surplus $\mathcal{S} = \sum_{t=1}^{t'} S_t^H(\gamma_{t-1}^1)\delta^{t-1} + \sum_{t=t'+1}^T S_t^D(\gamma_{t-1}^0)\delta^{t-1}$. Next consider a variation such that the principal chooses nonfulfillment in periods $t = 1, \dots, (T - t')$; and fulfillment from $t = (T - t' + 1), \dots, T$, with surplus $\mathcal{S}' = \sum_{t=0}^{T-t'} S_t^D(\gamma_{t-1}^0)\delta^{t-1} + \sum_{t=T-t'+1}^T S_t^H(\gamma_{t-1}^1)\delta^{t-1}$. Compared to the previous strategy the total number of fulfillments and nonfulfillments is identical but timing differs: Under the first strategy there is fulfillment for the first t' periods whereas under the second strategy there is fulfillment for the last t' periods. Assume no discounting, $\delta = 1$. Then timing of fulfillment decisions is material only with regard to the induced level of trust. As S_t^D and S_t^H are increasing functions of γ_{t-1} , it is always optimal to build up trust first, i.e. $\mathcal{S} > \mathcal{S}'$: The surplus from the i th fulfillment under \mathcal{S} is higher than the i th fulfillment under \mathcal{S}' for all t' -fulfillments, and the surplus of the j th nonfulfillment under \mathcal{S} is higher than the surplus from the j th nonfulfillment in \mathcal{S}' for all $(T - t')$ -nonfulfillments. We summarize our findings in proposition 2.

Proposition 2 *Given the interest rate r is sufficiently low, fulfillment of the implicit part of the contract will be observed - if at all - at the beginning of the relationship (and correspondingly: nonfulfillment of the implicit part of the contract will be observed at end of the relationship).*

Putting conclusion (1) and proposition (2) together, either a non-alternating strategy starting with fulfillment or a strict nonfulfillment strategy will be observed. Knowing that in any period

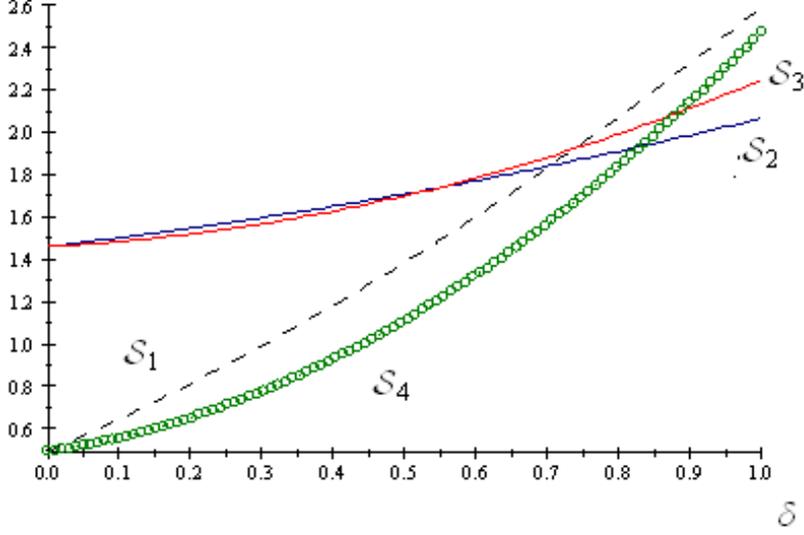


Figure 1: Optimal strategies contingent on δ

τ given any history $\theta^{\tau-1}$, $S_{\tau}^D(\theta^{\tau-1}) > S_{\tau}^H(\theta^{\tau-1})$, the strategy "strict fulfillment" over a contract length of $\bar{\tau}$ periods is dominated by the strategy $(\theta_1 = 1, \theta_2 = 1, \dots, \theta_{\bar{\tau}-1} = 1, \theta_{\bar{\tau}} = 0)$. (The latter simply means that strategic principals will renege in the last period.) By backward induction, we find the threshold period where - given the non-renegeing constraint for period 1 is fulfilled - the principal switches from fulfillment to nonfulfillment.

What if the interest rate is not sufficiently low? Here the principal faces a trade-off between building up trust and the time value of money: As $S_t^D(\gamma_{t-1}) > S_t^H(\gamma_{t-1})$ there might be an incentive to dishonor contracts in early periods if the discount rate is high. Consider the extreme case $\delta \rightarrow 0$. Then only first period payoffs matter (in our model discounting begins in period 2). We obtain $\mathcal{S} = S_1^H(\gamma_0)\theta_1 + S_1^D(\gamma_0)(1 - \theta_1)$, which is maximized by setting $\theta_1 = 0$. To clarify the trade-off, consider the following example:

Example 1 Assume a three-period relationship with strategies

$\{\theta_1 = (1, 0, 0), \theta_2 = (0, 0, 0), \theta_3 = (0, 1, 0), \text{ and } \theta_4 = (1, 1, 0)\}$, assuming $\alpha = 1$ and $\beta = 1/100$. Figure 1 plots the strategy dependent surplus levels ($\mathcal{S}_i \equiv \mathcal{S}(\theta_i)$) against δ . With infinitely high discounting ($\delta \rightarrow 0$) strategies θ_2 and θ_3 are optimal, because both dishonor the agreement in period 1 (which will not be discounted). Increasing δ from the origin, initially \mathcal{S}_2 rises stronger than \mathcal{S}_3 as the second period payoff under θ_2 , $S_2^D(\gamma_1)$, is higher than under θ_3 , $S_2^H(\gamma_1)$, and the effect of fulfilling the second period contract on the third period payoff under θ_3 does not preponderate because with δ relatively small the third period payoff is of little value. As δ is increasing further the effect reverses. Now the advantage of having built up trust in period 2 becomes dominant and strategy θ_3 yields a higher surplus than θ_2 . If the discount rate r becomes

sufficiently small, $\delta \rightarrow 1$, the advantage of early trust building appears and θ_1 dominates θ_3 . (Depending on the numerical values of the example θ_4 will never be optimal here). What becomes obvious from this example is that with r being sufficiently high alternating strategies may be optimal.

If the time value of money is irrelevant marginal gains from fulfillment in terms of trust are highest at the beginning of the agency and marginal losses due to nonfulfillment are lowest at the end of the agency. Hence, if it is optimal to build up trust during the agency the principal will do so at the beginning of the relationship because she can harvest high trust over the longest period of time. To provide some more intuition for the result note that the agent's belief about the principal's trustworthiness rests on only a few observations at the beginning of the agency but on many observations later in the agency. Therefore, the principal's decisions early in the agency have a stronger impact on the agent's belief than the ones later in the agency. This feature of the agent's adaptive learning process (which is consistent with Bayesian inference) creates effective incentives for trust formation through early fulfillments.²⁵ Once the principal decides not to fulfill the contract in a period it must be optimal to dishonor in all subsequent periods. As such the strategy pattern which is endogenous equilibrium behavior in our model resembles the grim-trigger strategy that exogenously enters into previous implicit contracting models to sustain an equilibrium.

5 Limited recall by the agent

In this section we investigate if and how the principal's optimal decisions change when the agent is not able to recall an arbitrarily long sequence of play. Technically, the level of trust that determines the value of the agency in any given period τ will now depend upon a history of τ^F periods prior to it, where F denotes forgetfulness. To cut-off the history of play at a certain point is a simplification of exponential or power forgetting functions proposed by psychologists²⁶, or can be interpreted as the well-known recency bias in decision analysis.²⁷ The crucial change for the principal - compared to the situation in the previous section - is that the decision in a given period t influences future periods only up to period $t + \tau^F$, and repercussions from previous decisions are limited to periods $t - \tau^F, \dots, t - 1$. For instance assume $\tau^F = 3$ and $\alpha = \beta = 1$. Then

²⁵The same effect occurs if a sample from a normal distribution is used to update the mean of a normally distributed variable: The higher the variance of the prior distribution, the stronger the impact of the sample on the posterior mean.

²⁶See Wixted (2004) for a review of - and for non-psychologists an introduction into - the topic.

²⁷Basu/Waymire (2006) and Basu et al. (2007) show that recordkeeping - e.g. as required by modern accounting systems- enhances trust and therefore enables complex economics transactions. The need for recordkeeping to support memory by the human brain sustains our assumption of limited recall in the first place. As in practice, however, recordkeeping in an employer-employee relationship will not be observed, we do not consider formal recordkeeping as a device to support a more effective recall by the agent.

starting with initial trust of $\gamma_0 = \frac{\alpha}{\alpha+\beta} = \frac{1}{2}$ the evolution of trust assuming decision sequence $(0, 1, 0, 0)$ is given by $\gamma_1 = \frac{\alpha}{\alpha+\beta+1} = \frac{1}{3}$, $\gamma_2 = \frac{\alpha+1}{\alpha+\beta+1+1} = \frac{1}{2}$, and $\gamma_3 = \frac{1}{1+1} = \frac{1}{2}$, where at the beginning of the fourth period the agent does not recall initial trust but his adaptive learning rests only on the last three periods $(0, 1, 0)$.

Whenever the principal does not want to pay the implicit bonus τ^F - times in a row trust will be completely destroyed and the agent will choose no effort in the following period.²⁸ But what about a non bonus payment following a period with an effort level of zero? Here we have to clarify what actually constitutes "nonfulfillment". In the word's literal sense, a non bonus payment subsequent to zero effort cannot be nonfulfillment because payment accords with promise to pay. However, we consider a non bonus payment following zero effort as a period where no implicit agreement could be established; it constitutes nonfulfillment. Hence, the game is effectively over after τ^F nonfulfillments in a row. In the period following a series of τ^F breaches of contract the agent chooses no effort and accordingly receives no bonus payment from the principal. This period is interpreted as a "no implicit contract period" by the agent such that he does not perform effort in the subsequent period and consequently receives no bonus. This procedure repeats until the end of the relationship.

We consider first the extreme case of $\tau^F = 1$ and then proceed to $\tau^F \geq 2$.

5.1 One-period recall by the agent

If the agent recalls only one period of play, her projection for the upcoming period will be that the principal will repeat her decision from the previous period. Formally we obtain

$$\gamma_{t-1} = Prob(\theta_t = 1 | \theta_{t-1}) = \begin{cases} 1 & \text{if } \theta_{t-1} = 1 \\ 0 & \text{if } \theta_{t-1} = 0 \end{cases}, \quad (16)$$

and

$$(1 - \gamma_{t-1}) = Prob(\theta_t = 0 | \theta_{t-1}) = \begin{cases} 0 & \text{if } \theta_{t-1} = 1 \\ 1 & \text{if } \theta_{t-1} = 0 \end{cases}. \quad (17)$$

At first sight, (16) and (17) seem to render alternating strategies infeasible. The following proposition confirms the conjecture and describes the principal's optimal strategy.

Proposition 3 *Let $\tau^F = 1$, then*

(a) given r is sufficiently low, the principal's optimal strategy is to fulfill the implicit part of the contract in all but the final period,

(b) if r is sufficiently large the principal chooses $\theta_1 = 0$.

²⁸Ittner et al. (2003) provide an empirical example where individual balanced scorecards were removed because employees did not trust the scorecard measures anymore after supervisors ignored a number of them or attached different weights to them from quarter to quarter.

Short recall on the side of the agent approximates the cooperative solution very closely if the discount rate is sufficiently low. Strategic principals are forced to fulfill the implicit part of the contract as nonfulfillment once prevents "cooperative play" in all future periods. As such, the optimal strategy is endogenous and resembles the grim-trigger strategy - defect once and cooperative play is ruled out for all future periods - that is usually exogenous to implicit contracting models.

5.2 Multi-period recall by the agent

In this section we assume the agent is able to recall the sequence of play for τ^F periods, $2 \leq \tau^F \ll T$. Limited recall has two fundamental effects. First, if principals defect τ^F -times in a row they never regain a chance to build up trust, that is, they realize zero surpluses for all periods to come. Second, a decision in a given period t influences future periods only up to period $t + \tau^F$. Hence, trust reducing nonfulfillment decisions will eventually be cleared from the agent's memory offering a chance for principals to not-fulfill again.

>From previous sections it is straightforward that if the discount rate is very high the principal decides not to fulfill the first period contract and subsequent periods are immaterial. For r sufficiently small it can never be optimal to dishonor the contract τ^F -times in a row if there are subsequent periods to play. To single out candidates for optimality and intuitively approach a (possibly) unique sequential equilibrium, let the optimal decision for some period \hat{t} be $\theta_{\hat{t}} = 0$. Moving on period by period, the principal will fulfill the contract - at the latest in period $t = \hat{t} + \tau^F - 1$ - because another nonfulfillment would reduce *all* future surpluses from trade to zero. The principal has to trade-off surpluses from repeated nonfulfillments against those arising from fulfillment based on ceteris paribus higher trust levels. In other words it could be optimal to either frequently switch back and forth between fulfillment and nonfulfillment or to insert only few nonfulfillments into the (otherwise) strict fulfillment strategy.

Now assume there is a time span during which principals dishonor the implicit contract d times starting from full trust $\gamma_{\hat{t}-1} = 1$, $d \in \{0, 1, \dots, \tau^F - 1\}$. By induction, if it is optimal to dishonor implicit contracts in periods $t = \hat{t}, \hat{t} + 1, \dots, \hat{t} + (d - 1)$, it will again be optimal to do so in periods $t = \hat{t} + d + \tau^F, \hat{t} + d + \tau^F + 1, \dots, \hat{t} + d + \tau^F + d$, after full trust has been reestablished by fulfillment decisions in periods $t = \hat{t} + d, \hat{t} + d + 1, \dots, \hat{t} + d + \tau^F - 1$. Let us call the decision sequence $\theta^R(\tau^F, d)$ of τ^F fulfillments and d nonfulfillments a representative sequence.

Lemma 1 *Assume a repetition of the representative sequence $\theta^R(\tau^F, d)$, $d > 0$, is an element of the principal's optimal strategy: Then, given r is sufficiently small, the number of nonfulfillments d contained in $\theta^R(\tau^F, d)$ must be placed in a row, i.e. the representative sequence shows at most one change where the principal switches from fulfillment to nonfulfillment.*

Proof. See appendix. ■

If more than one nonfulfillment in the sequence is optimal, $d > 1$, all nonfulfillments must be placed in a row as otherwise a sequence of τ^F fulfillments is discontinued, destroying full trust and leading to a strictly lower payoff in the representative sequence. Lemma 1 indicates that switching back and forth between one fulfillment and one nonfulfillment cannot be optimal equilibrium behavior, just as it was not under perfect recall either. Thus the result holds even in light of the agent's limited recall. This is noteworthy because the latter prevents erosion of marginal gains from fulfillment due to a constant sample size of τ^F periods for trust formation. Until now we analyzed representative sequences assuming $d > 0$ being optimal, showing that it cannot be optimal to destroy full trust by playing alternating nonfulfillment strategies. But we neither proved optimality of $d > 0$ nor did we describe the optimal way to induce perfect trust from the beginning of the relationship. Furthermore, we did not care about the final periods of the relationship that are different to "regular" periods as no long-term effects need to be considered. The following proposition describes a principal's optimal strategy for a low discount rate.

Proposition 4 *Given r is sufficiently small. The principal's optimal strategy is to fulfill the contract in periods 1 to τ^F and subsequently dishonors the implicit contract $d > 0$ times; then the principal selects this representative sequence $\theta^R(\tau^F, d)$ as often as possible given there are at least τ^F periods remaining to harvest full trust. The final periods are subject to separate optimization.*

The principal's optimal strategy is to induce full trust just at the beginning of the relationship and then to absorb the benefits from perfect trust by not paying the implicit bonus d times in a row. This procedure will be repeated as long as possible, i.e. establishing full trust is optimal as long as there are at least τ^F periods to harvest full trust. The remaining periods exhibit a "last-round effect" for the principal - including a nonfulfillment in the last period- such that they are subject to separate optimization.

Definition 4 *(Optimal trust) The average level of trust in the representative sequence, $\frac{\tau^F}{d+\tau^F}$, is called the optimal level of trust in the agency.*

Definition 5 *(Critical trust level) The level of trust the principal will not go below, $\frac{\tau^F-d}{\tau^F}$, is called the critical level of trust in the agency.*

Proposition 5 *The number of nonfulfillments d contained in the representative sequence $\theta^R(\tau^F, d)$ is weakly increasing in τ^F , the number of periods the agent recalls.*

Corollary 1 *The optimal level of trust is generally lower than 1. Neither the optimal level of trust nor the critical level of trust is monotone in the number of periods the agent recalls. Both increase in τ^F with rollbacks whenever an additional nonfulfillment becomes optimal.*

Before a numerical example highlighting the results from propositions 4 and 5 will be presented, an intuitive explanation for the propositions and its proofs should be given. If full trust is based on only a few observations then nonfulfillment will destroy trust severely, whereas the adverse impact of nonfulfillment is less severe if full trust is based on a larger number of observations. For example, if $\tau^F = 2$, nonfulfillment (after full trust has been reestablished) reduces the level of trust from $2/2$ to just $1/2$; compare it with $\tau^F = 10$: Nonfulfillment after full trust has been reestablished lowers trust from $10/10$ to $9/10$. Therefore, additional nonfulfillment(s) may be optimal if the number of periods the agent recalls increases. The fact that the number of optimal nonfulfillments increases only weakly in the number of periods the agent recalls leads to non-monotonicity of the optimal and the critical level of trust, respectively, in the agency. Therefore, equal values of these levels can be reconciled with quite different degrees of the agent's bounded rationality, i.e. how able he is to recall sequences of play.

Example 2 *The following calculation exemplifies the results of propositions 4 and 5. Assume the agency lasts for 30 periods. Let the prior distribution of trust be given by $\alpha = \beta = 5$. Discounting is either irrelevant ($\delta = 1$), or sufficiently low ($\delta = 0.909$) to ensure that building up trust is potentially viable. Calculation of strategy contingent payoffs yields the following numbers:*

τ^F	strategy profile* ($\theta_1; \theta_2; \dots; \theta_{30}$)		$\mathcal{S}(\delta = 0.909)$	$\mathcal{S}(\delta = 1)$
2	1	$29 \times 1; 0$	4.11	15.24
	2	$2 \times 1; 13 \times \{0, 1\}; 1; 0$	3.08	9.46
	3	$10 \times \{2 \times 1; 0\}$	5.02	17.24
4	4	$27 \times 1; 3 \times 0$	3.82	14.95
	5	$4 \times 1; 12 \times \{0; 1\}; 3 \times 0$	3.36	9.86
	6	$6 \times \{4 \times 1; 0\}$	4.34	15.78
	7	$5 \times \{4 \times 1; 0; 0\}$	4.30	15.38
6	8	$27 \times 1; 3 \times 0$	3.55	14.77
	9	$6 \times 1; 11 \times \{0; 1\}; 0; 0$	3.37	10.36
	10	$4 \times \{6 \times 1; 0\}; 0; 0$	3.89	15.29
	11	$2 \times \{6 \times 1; 0; 0\}; 6 \times 1; 0; 4 \times 1; 3 \times 0$	3.92	14.57
	12	$2 \times \{6 \times 1; 0; 0\}; 6 \times 1; 5 \times 1; 3 \times 0$	3.91	15.02
	13	$2 \times \{6 \times 1; 3 \times 0\}; 9 \times 1; 3 \times 0$	3.88	14.60

Table 1: Strategies and payoffs under limited recall
(Optimal strategies in bold.)

* *Hint: A strategy profile reads as follows: $2 \times \{6 \times 1; 0; 0\}; 11 \times 1; 3 \times 0$ - play the (representative) sequence consisting of 6 fulfillments followed by 2 fulfillments twice; then play 11 fulfillments followed by 3 nonfulfillments.*

Table 1 compares strategy contingent payoffs if the history of play the agent recalls amounts to $\tau^F = 2$ (4;6) periods, respectively. Optimal strategies are highlighted in bold. They have three features in common: (1) at the beginning, full trust is established (which relates to part (a) of proposition 4); (2) the representative sequence is repeated as often as possible; (3) the final periods are subject to separate optimization. For $\tau^F = \{2; 4\}$, the optimal number of defections d within the representative sequence is 1. For example, strategy 7 having two defections within the representative sequence is inferior to strategy 6, the (optimal) strategy with one defection. If $\tau^F = 6$, d increases to 2. Taken together, d is weakly increasing in τ^F . Also note that the final periods of play - decisions noted after braces - may be subject to separate optimization. Specifically, compare strategies 11 and 12. With discounting given $\delta = 0.909$, strategy 11 is preferred to 12 showing a nonfulfillment after reestablishing full trust subsequent to completing the representative sequence twice. Without discounting, or, in terms of proposition 4, with sufficiently low discounting, strategy 12 starting with additional fulfillments and ending with nonfulfillments is superior to the alternating strategy in 11.

The optimal and critical level of trust, respectively, obtain as follows:

τ^F	1	2	3	4	5	6
optimal trust	1	2/3	3/4	4/5	5/6	3/4
critical trust	1	1/2	2/3	3/4	4/5	5/8

Table 2: Optimal trust and critical trust

The optimal alternating strategies under limited recall of at least two periods of play, $\tau^F \geq 2$, correspond to trigger strategies like "defect $d + 1$ times in a row and cooperative play is ruled out for all future periods". As d is weakly increasing in τ^F , the optimal strategies form a subset of trigger strategies that is usually treated as exogenous in implicit contract models. Our model derives these strategies endogenously.

6 Interpretation, application and extensions

The results in this paper that have empirical support or suggest empirical research can be summarized as follows:

- There is an optimal level of trust and achieving or maintaining full trust is generally not optimal. However, there also exists a critical trust level that the principal (firm) will not go below. Different degrees of bounded rationality can be reconciled with identical levels of optimal and critical trust, respectively.
- Equal trust measures (optimal trust, critical trust) can be reconciled with different degrees of the agent's bounded rationality.

- Improving performance measurement systems (PMS) can improve performance via trust as moderator.

One could interpret the average level of trust prevalent in exchange relationships as optimal trust (Wicks et al., 1999). We demonstrate that such an optimal level of trust exists if agents or employees are unable to recall entire histories of play (section 5). Trust dynamics as suggested by the optimality of alternating strategies in the model have been observed in the laboratory (Jonker et al., 2004), i.e. trust can be rebuilt. Besides forgetfulness limiting the number of periods which determine trust relevant for a particular period, one could also think of trust as a replaceable intangible asset with finite life of τ^F -periods. The optimal trust level depends on situational parameters and is given by $\frac{\tau^F}{\tau^F+d} < 1$, indicating that achieving and maintaining maximum trust is not in every case viable. A prominent example is (or used to be) the practice of lifetime employment by Japanese firms which analysts believe to represent a too heavy financial burden.²⁹ To the other end, there exists a critical level of trust that firms optimally do not go below. Since the number of nonfulfillments only weakly increases in the agent's recall capabilities, the critical level of trust tends to increase in the number of periods the agent recalls. If – in empirical research – the latter is proxied by tenure in the firm, then longer tenure translates into higher trust levels which in turn increase gains from trade. Or, in other words, subjective incentives become more effective. As such our model provides an analytical rationale for Gibbs et al.'s (2004) empirical finding that subjective incentives can be more effective when trust – proxied by tenure – is higher.

In recent papers Basu/Waymire (2006) and Basu/Dickhaut et al. (2007) emphasize the benefits of recordkeeping in complex economic transactions. Basu/Dickhaut et al. (2007) show in a complex multi-period trust game experiment that recordkeeping leads to higher trust and induces higher reciprocity. If recordkeeping is possible (in the experiment) the trustor (investor) better recalls past transactions and trust formation by higher reciprocity of the trustee shows. In contrast to these papers we demonstrate that limited recall by the agent may lead to improved transactions. As the agent refuses cooperation for all future if he only recalls breaches of contract, shorter recall by the agent forces the principal to reciprocate more often.³⁰

In our model, trust has a moderating effect on performance as trust "affects how one assesses the future behavior of another party ... and how one interprets past (or present) actions of the other party".³¹ Bonus payments increase trust which in turn leads to higher performance. Such trust in the principal's willingness to pay can also be identified as trust in the performance measurement system (PMS) or system trust (Luhmann, 1979), i.e. the likelihood that (perceived) individual performance and performance appraisal coincide. For that purpose, $E(\gamma)$, the proba-

²⁹Cf. Wicks et al. (1999), p. 101.

³⁰Cf. optimal trust for $\tau^F = \{4, 6\}$ in table 2.

³¹Dirks/Ferrin (2001), p. 456.

bility that the principal pays the bonus at the end of the period would have to be reinterpreted as the level of trust the agent puts into the performance evaluation process. In line with empirical evidence (Folger/Konovsky, 1989), the importance of transparent and fair performance appraisals has been pointed out by several authors, e.g. Campbell et al. (1998), Baker (1990), or Milkovich/Newman (2002, p. 302). The present model formalizes benefits of such a trust "investment" - the principal's decision to pay a bonus could be seen as an investment -, but the optimal trust level smaller than one suggests that definite fairness or transparency of PMS cannot be an objective for firms. Therefore, disagreements of employees with their performance appraisals (Campbell/Lee, 1988) are likely to persist.³²

In similar vein, empirical research suggests trust between employee and supervisor (Reinke, 2003; Ittner et al., 2003) or trust in the agency in general (Condrey, 1992; Hedge, 2000) as a crucial influence on acceptance of performance appraisal systems. Although the model presentation and interpretation so far focuses on subjective performance measures, similar concerns can be expected even if verifiable accounting measures are used for performance appraisals. As Hopwood (1972, p. 157) notes, accounting PMS can often be improved but not as much to achieve the "ideal system", which would correctly mirror all dimensions of employee performance. As a consequence, even accounting information should be used with discretion in performance evaluation.³³ Here, numbers or outcomes may not be disputed but the cause of numbers (Campbell et al., 1998, p. 133). Taking this insight into our model, both principal and agent agree on observed agent effort or performance and if it was a verifiable accounting number a contract could be written on it. But if agent motivation depends on his willingness to accept the cause his trust in the performance evaluation process would be important. Low accounting noise and a just and transparent evaluation will certainly foster trust in that respect.

As another interpretation of the optimal trust level, consider rapidly changing environments which are likely to be represented by a shorter "life" of trust, i.e. investments in trust would be needed more often. While there is evidence that accounting for trust can help maintain or rebuild trust (Busco et al., 2006; Johansson/Baldvinsdottir, 2003), it remains open to what extent that should be done. Since the optimal level of trust, $\frac{\tau^F}{\tau^F+d}$, is increasing in τ^F but d is only weakly increasing in τ^F , an additional prediction emerging from the analysis is that in rapidly changing environments, possibly manifested by repeated restructuring and/or managerial turnover, the optimal level of trust is lower than in stable environments. (Note that investments in trust are viable in both scenarios.) In fact, there is some empirical evidence (Coyle-Shapiro, 2000; Robinson, 1996; Robinson/Rousseau, 1994) that globalization and organizational restructuring

³²Although this may sound trivial it is actually not. Ittner/Larcker (2003) report that not setting the right performance targets is a common mistake in nonfinancial performance measurement. For example, managers tried to achieve 100% customer satisfaction because they believed it would pay off.

³³See also Murphy/Oyer (2003).

have lead to more frequent breach of implicit contracts or the so-called psychological contract.³⁴ However, the analysis in this paper also suggests that trust measurement in employer-employee relations *could* come to equal results in differently stable environments because optimal trust as well as critical trust are not monotone in the parameter τ^F . Interpreting that parameter as the degree of the agent's bounded rationality, again equal trust measures can go hand in hand with different degrees of bounded rationality.

Adding more structure to the model would allow to further clarify the dependence of optimal and critical trust on other parameters and to derive more testable hypotheses. Such extended model specifications could then be tested in the laboratory. An interesting feature of the model carrying over to them is the range of possible trust levels because experimental trust games usually allow only for the alternative to fully trust or not trust at all.

7 Summary and conclusion

The results of our paper relate to contract design and implicit incentives both from a theoretical perspective as well as from a practical perspective. First of all we introduce an evolution of trust in repeated implicit contract relationships. The distributional assumption how trust evolves with (non)fulfillment decisions by principals allows for adaptive learning on the side of the agent and covers quite different ex ante evaluations about the trustworthiness of principals. Agents are assumed to be characterized by bounded rationality, and unforeseeable contingencies prevented separating equilibria. These two assumptions give rise to rely on trust as part of contracts in the first place. As long as agents are able to recall the entire sequence of play, non-alternating strategies were found to be optimal, i.e. principals build up trust in early periods by fulfillment of contracts (if that is ever optimal) and then harvest the benefits from high trust by not fulfilling the contract in later periods. If discounting is high, alternating strategies may become optimal. We further show that higher levels of ex ante trust increase the value of the agency and can at least partially offset detrimental effects of a high discount rate. As trust works as a moderator in this paper's model, we also contribute to the sparse literature in that respect.

In an extension of the model where agents could not recall the entire history of play, we show that the less periods the agent is able to recall the higher the pressure for the principal to fulfill the implicit contract. Furthermore, we demonstrate optimality of alternating strategies. That is, principals switch back and forth between fulfillment and nonfulfillment of the implicit part of contracts. Here a subset of trigger strategies that is usually exogenous to implicit contract models can be derived endogenously, thus offering some justification for the conventional approach. These strategies imply an average level of trust in agency that is generally smaller than one,

³⁴According to Rousseau (1990, p. 390), a psychological contract are individual beliefs regarding reciprocal obligations.

i.e. is less than maximum possible. As such we formalize the idea of optimal trust suggested by Wicks et al., 1999. An interesting and important interpretation of the analysis relates to performance measurement systems and performance appraisals. Although transparency, fairness and understandability are important to make performance measurement systems work - that is, employees trust the process or system - attaining or maintaining definite transparency and understandability is not in every case viable as it may become too costly. At the other end, we identify a critical trust level that employers optimally do not go below. Therefore, a sufficiently high level of employee trust in the system and the appraisals suffice. We substantiated our claim that these issues need to be taken into account even if verifiable accounting numbers are available.

This paper suggests a model for the formal analysis of a trust relationship between two parties. An agent trusts a principal with a certain probability. Observability of agent effort puts the principal's trust in the agent aside; it is therefore a one-sided trust relationship. Extending it by including non-observability of agent effort such that the principal needs to trust the agent to expend the desired effort level (in the absence of verifiable performance measures); a two-sided trust relationship would result. Another possible extension could be to include control as a matter of choice by the principal. It seems worthwhile and promising to add more structure to the model in these respects. Also, these model extensions then lead to empirical predictions beyond the ones we propose, and experimental research as well as field research could provide in-depth tests of these.

Appendix

Proof of Proposition 1

(a) Consider period t with induced trust γ_{t-1} . If it is optimal to dishonor the implicit contract in period t with discount rate r , given γ_{t-1} , then it must be also optimal to dishonor it in period t with $r' > r$, given γ_{t-1} , as value functions (or equivalently: the left hand side of the non-renegeing constraint (12)) are decreasing in r . Hence, starting with the same initial level of trust, γ_0 , the induced level of trust in period t with discount rate r , γ_{t-1}^r , must be at least as high as with $r' > r$, i.e. $\gamma_{t-1}^r \geq \gamma_{t-1}^{r'}$ for all t ; for $\gamma_{t-1}^r < \gamma_{t-1}^{r'}$ to be true for some period t there must have been a period $\tau < t$ with $\gamma_{\tau-1}^r = \gamma_{\tau-1}^{r'}$ with nonfulfillment for r but fulfillment for r' : a contradiction. Hence, the number of periods in which the implicit contract is honored is weakly decreasing in r . Let θ^* the optimal strategy with r and θ^{**} the optimal strategy with $r' > r$. As $\mathcal{S}(\theta)$ is decreasing in r , $\mathcal{S}(\theta^{**}, r') < \mathcal{S}(\theta^{**}, r)$, and as $\mathcal{S}(\theta^{**}, r) < \mathcal{S}(\theta^*, r)$ it follows that $\mathcal{S}(\theta^{**}, r') < \mathcal{S}(\theta^*, r)$ such that the principal's equilibrium payoff decreases in r . (b) This part follows from the positive affine transformation of (13) that results from a higher level of ex ante trust. ■

Proof of Proposition 3

(a) The surplus function (13) for strategy $\theta^* = (\theta_1 = 1, \theta_2 = 1, \dots, \theta_{T-1} = 1, \theta_T = 0)$ obtains as

$$\begin{aligned} \mathcal{S}(\theta^*) &= S_1^H(\gamma_0) + \sum_{t=2}^{T-1} S_i^H(\gamma_{t-1} = 1)\delta^{t-1} + S_T^D(\gamma_{T-1} = 1)\delta^{T-1} \\ &= \frac{\gamma_0}{2(2-\gamma_0)} + \frac{1}{2} \sum_{t=2}^{T-1} \delta^{t-1} + \frac{3}{2}\delta^{T-1}, \end{aligned}$$

where γ_0 denotes the prior level of trust. In the final period of the agency, strategic principals will not pay the bonus, i.e. $\theta_T = 0$, because $S_T^D(\theta^{T-1}) \geq S_T^H(\theta^{T-1})$ for any history of play. Since alternating strategies are ruled out by (16), any strategy that has $\theta_1 = 0$, leads to a surplus of $\mathcal{S}(\theta_1 = 0, \theta_2 = 0, \dots, \theta_T = 0) = \frac{\gamma_0(4-\gamma_0)}{2(2-\gamma_0)^2} < \frac{3}{2}$, which is clearly dominated by $\mathcal{S}(\theta^*)$ if r is sufficiently low (δ sufficiently close to 1). A strategy $\theta^\tau = (\theta_1 = 1, \theta_2 = 1, \dots, \theta_\tau = 1, \theta_{\tau+1} = 0, \dots, \theta_T = 0)$, $2 \leq \tau \leq T-1$, leads to a surplus $\mathcal{S}(\theta^\tau) = \frac{\gamma_0}{2(2-\gamma_0)} + \frac{1}{2} \sum_{t=2}^{\tau} \delta^{t-1} + \frac{3}{2}\delta^\tau$. Hence, $\mathcal{S}(\theta^*) - \mathcal{S}(\theta^\tau) = \frac{1}{2} \sum_{t=\tau+1}^{T-1} \delta^{t-1} - \frac{3}{2}(\delta^\tau - \delta^{T-1})$ is positive for all $\tau \leq T-1$ if r is sufficiently low.

(b) If r is sufficiently large, only the first period payoff of the surplus function matters. The first period payoff is maximized by nonfulfillment, $\theta_1 = 0$. ■

Proof of Lemma 1

Assume that r is sufficiently low such that the time value of surpluses can be ignored. Consider a representative sequence $\theta^R(\tau^F, 1)$ consisting of τ^F fulfillments and one nonfulfillment. The

representative³⁵ surplus from the second or higher repetition of the representative sequence $\theta^R(\tau^F, 1)$ is independent of the period in which the nonfulfillment is placed within $\theta^R(\tau^F, 1)$ as at each fulfillment the agent recalls $\tau^F - 1$ nonfulfillments and one fulfillment and at the period of nonfulfillment the agent recalls τ^F fulfillments (full trust). It is equal to $\mathcal{S} = \tau^F S^H\left(\frac{\tau^F - 1}{\tau^F}\right) + S^D(1)$.

Now assume a second nonfulfillment being optimal, $d = 2$. We consider two different strategies in placing the second nonfulfillment. In strategy *A* it is placed immediately after the first nonfulfillment and in strategy *B* the two nonfulfillments are not placed in a row. The surpluses related to strategies *A* and *B* are given by

$$\begin{aligned}\mathcal{S}^A &= S^D(1) + S^D\left(\frac{\tau^F - 1}{\tau^F}\right) + (\tau^F - 1) S^H\left(\frac{\tau^F - 2}{\tau^F}\right) + S^H\left(\frac{\tau^F - 1}{\tau^F}\right) \\ \mathcal{S}^B &= 2S^D\left(\frac{\tau^F - 1}{\tau^F}\right) + (\tau^F - 2) S^H\left(\frac{\tau^F - 2}{\tau^F}\right) + 2S^H\left(\frac{\tau^F - 1}{\tau^F}\right)\end{aligned}$$

The difference of surpluses is $\Delta = \mathcal{S}^A - \mathcal{S}^B = S^D(1) - S^D\left(\frac{\tau^F - 1}{\tau^F}\right) - \left(S^H\left(\frac{\tau^F - 1}{\tau^F}\right) - S^H\left(\frac{\tau^F - 2}{\tau^F}\right)\right)$. Notice that $1 - \frac{\tau^F - 1}{\tau^F} = \frac{\tau^F - 1}{\tau^F} - \frac{\tau^F - 2}{\tau^F} = \frac{1}{\tau^F}$. As both S^D and S^H are increasing convex functions of $\gamma_{(\cdot)}$, and as for the marginal surpluses it holds $S^{D'} > S^{H'}$ for all $\gamma_{(\cdot)}$, Δ is strictly positive. Hence, if a second nonfulfillment is optimal it must be placed immediately after the first nonfulfillment. The same argumentation applies if $d > 2$ nonfulfillments are optimal. Hence, if a repetition of representative strategies $\theta^R(\tau^F, d)$ is consistent with equilibrium behavior, d nonfulfillments must be placed in a row such that there is at most one change from nonfulfillment to fulfillment within a representative sequence. ■

Proof of Proposition 4

Assume $r = 0$ for the whole proof.

a) We first prove that independent of the initial trust γ_0 at the beginning of the first period it is always optimal to establish full trust right from the beginning of the relationship. Assume $T > 2\tau^F$. We first show that strategy $\theta(\tau^F, \mathbf{0}) = (\theta_1 = \theta_2 = \dots = \theta_{\tau^F} = 1, 0, 0, \dots, 0)$, i.e. fulfill τ^F periods and then never again, dominates all strategies $\theta(\tau^F - \mathbf{i}, \mathbf{0}) = (\theta_1 = \theta_2 = \dots = \theta_{\tau^F - i} = 1, 0, 0, \dots, 0)$, $i = 1, \dots, \tau^F - 1$ if $\gamma_0 < 1$. Strategy $\theta(\tau^F, \mathbf{0})$ yields the surplus³⁶

$$\mathcal{S}(\theta(\tau^F, \mathbf{0})) = \sum_{t=1}^{\tau^F} S^H(\theta^{t-1}) + \sum_{t=\tau^F+1}^{2\tau^F} S^D(\theta^{t-1}), \quad (18)$$

³⁵After the representative sequence has been played once, the surplus is the same for every future repetition due the agent's limited recall.

³⁶Assuming $T > 2\tau^F$ under strategy $\theta(\tau^F, \mathbf{0})$ or $\theta(\tau^F - i, \mathbf{0})$, respectively, trust is completely destroyed at the end of period $2\tau^F$ or $(2\tau^F - i)$, respectively, such that all future payoffs are zero.

whereas strategy's $\boldsymbol{\theta}(\tau^F - \mathbf{i}, \mathbf{0})$ surplus obtains as

$$\mathcal{S}(\boldsymbol{\theta}(\tau^F - \mathbf{i}, \mathbf{0})) = \sum_{t=1}^{\tau^F - i} S^H(\theta_i^{t-1}) + \sum_{t=\tau^F - i}^{2\tau^F - i} S^D(\theta_i^{t-1}), \quad (19)$$

where the subscript i indicates the number of fewer fulfillments under $\boldsymbol{\theta}(\tau^F - \mathbf{i}, \mathbf{0})$ compared with $\boldsymbol{\theta}(\tau^F, \mathbf{0})$. Note that (18) contains i strictly positive elements more than (19) due to i additional fulfillments and limited recall. The profit difference $\Delta = \mathcal{S}(\boldsymbol{\theta}(\tau^F, \mathbf{0})) - \mathcal{S}(\boldsymbol{\theta}(\tau^F - \mathbf{1}, \mathbf{0}))$ using (18) and (19) amounts to

$$\Delta = S^H(\theta^{\tau^F - 1}) + \sum_{t=\tau^F + 1}^{2\tau^F} [S^D(\theta^{t-1}) - S^D(\theta_i^{t-2})].$$

Because of $\frac{dS^H}{dE(\gamma)} > 0$ and $\frac{dS^D}{dE(\gamma)} > 0$ for any history of play, the fulfillment in period τ^F under $\mathcal{S}(\boldsymbol{\theta}(\tau^F, \mathbf{0}))$ provides for

$$S^D(\theta^{t-1}) - S^D(\theta_i^{t-1}) > 0, \quad t = \tau^F + 1, \dots, 2\tau^F. \quad (20)$$

This proves $\mathcal{S}(\boldsymbol{\theta}(\tau^F, \mathbf{0})) > \mathcal{S}(\boldsymbol{\theta}(\tau^F - \mathbf{1}, \mathbf{0}))$. By iteration, it can be shown that $\mathcal{S}(\boldsymbol{\theta}(\tau^F - \mathbf{i}, \mathbf{0})) > \mathcal{S}(\boldsymbol{\theta}(\tau^F - \mathbf{i} - \mathbf{1}, \mathbf{0}))$, $i = 2, \dots, \tau^F - 1$. Thus, $\mathcal{S}(\boldsymbol{\theta}(\tau^F, \mathbf{0})) > \mathcal{S}(\boldsymbol{\theta}(\tau^F - \mathbf{i}, \mathbf{0}))$ for all $i = 1, \dots, \tau^F - 1$.

Next, we show that strategy $\boldsymbol{\theta}(\tau^F, \theta_{\tau^F + 1}, \dots, \theta_T)$, where the θ_t 's, $t = \tau^F + 1, \dots, T$, are optimally chosen, dominates all other possible strategies. Assume that strategy $\boldsymbol{\theta}(\tau^F - \mathbf{i}, \mathbf{0})$ is changed by replacing a nonfulfillment decision in period $t = \tilde{t} = \tau^F - i + 1, \dots, (2\tau^F - i)$ with a fulfillment decision. Because $\frac{d^2 S^H}{d[E(\gamma)]^2} > 0$ and $\frac{d^2 S^D}{d[E(\gamma)]^2} > 0$ – which only holds in case of limited recall so that the sample size or the denominator in the expected level of trust remains constant at τ^F – *marginal* gains (losses) from fulfillment (nonfulfillment) are *increasing* in previous fulfillments (nonfulfillments). Hence, if a nonfulfillment is replaced by a fulfillment it has to be in period $\tilde{t} = \tau^F - i + 1$. Optimality of that replacement follows from the steps of the proof above implying $\mathcal{S}(\boldsymbol{\theta}(\tau^F - \mathbf{i} + \mathbf{1}, \mathbf{0})) > \mathcal{S}(\boldsymbol{\theta}(\tau^F - \mathbf{i}, \mathbf{0}))$. Again, by iteration the optimality of additional replacements in periods $\tilde{t} = \tau^F - i + 2, \dots, \tau^F$ can be shown leading (again) to $\boldsymbol{\theta}(\tau^F, \mathbf{0}) \succ \boldsymbol{\theta}(\tau^F - \mathbf{i}, \mathbf{0})$ for all $i = 1, \dots, \tau^F - 1$. Obviously, $\boldsymbol{\theta}(\tau^F, \theta_{\tau^F + 1}, \dots, \theta_T) \succeq \boldsymbol{\theta}(\tau^F, \mathbf{0})$ proving optimality of $\boldsymbol{\theta}(\tau^F, \theta_{\tau^F + 1}, \dots, \theta_T)$.

b) We next prove that for all τ^F , $d > 0$ is a necessary condition for optimality. Assume a sequence of τ^F fulfillment decisions has been played leading to full trust. Now compare the following sequences

$$\boldsymbol{\theta}_{d=1} = (\theta_t = 0, 1, 1, \dots, \theta_{t+\tau^F+1} = 1) \quad (21)$$

$$\boldsymbol{\theta}_{d=0} = (\theta_t = 1, 1, 1, \dots, \theta_{t+\tau^F+1} = 1), \quad (22)$$

starting in period t . Note that both sequences consist of $(\tau^F + 1)$ – elements to ensure that full trust is (re)established at the end of the sequences. The crucial step in the proof is that the

trust level given $\theta_{d=1}$ remains *constant* after decision in period t , $\theta_t = 0$, because when moving on from period $(t+i)$ to $(t+i+1)$, $i \in \{1, 2, \dots, \tau^F\}$, the agent's limited recall capability "deletes" the fulfillment in period $(t-\tau^F)$ from memory while "storing" the fulfillment decision from period $(t+i)$. As such the single nonfulfillment in period t reduces the trust level from full trust, $E(\gamma) = 1$, under $\theta_{d=1}$ to $E(\gamma) = \frac{\tau^F-1}{\tau^F}$. Surpluses associated with (21) and (22) then obtain as

$$\begin{aligned}\mathcal{S}(\theta_{d=1}) &= \frac{3}{2} + \tau^F \cdot S^H\left(\frac{\tau^F-1}{\tau^F}\right) \\ &= \frac{3}{2} + \tau^F \cdot \frac{\tau^F-1}{2(\tau^F+1)} \\ \mathcal{S}(\theta_{d=0}) &= (\tau^F+1) \cdot \frac{1}{2}.\end{aligned}$$

Simple algebra shows that $\mathcal{S}(\theta_{d=1}) > \mathcal{S}(\theta_{d=0})$ holds for any τ^F .

c) After the first representative sequence has been played, induced trust is less than one. Applying the same arguments as in a) at the optimum the representative sequence must be repeated as long as possible given there are at least τ^F -periods remaining to "harvest" trust after it has been raised to its maximum. The periods after the last repetition of $\theta^R(\tau^F, d)$ are subject to separate optimization.

As the results in a), b), and c) are derived for $r = 0$ they also hold for a sufficiently low discount rate. ■

Proof of Proposition 5

The idea of the proof is to transform the optimal strategy into a profit annuity. [Clearly, the optimal strategy will have the highest profit annuity.] Optimal strategies are characterized by their representative sequence. Effects of the ex ante distribution of trust are eliminated after initial play of τ^F fulfillments (which have already been proven optimal). Subsequent repetitions of the representative sequence will then be played with recurring levels of trust solely determined by τ^F and d . The first decision and its associated profit which is not influenced by ex ante trust is the first nonfulfillment after τ^F fulfillments. Therefore we rearrange the representative sequence such that d nonfulfillments are followed by τ^F fulfillments. With τ^F and d given, the profit annuity based on the decision sequence $\theta(d, \tau^F)$ obtains as

$$a = \frac{(1+r)^{(\tau^F+d)} \cdot r}{(1+r)^{(\tau^F+d)} - 1} \cdot \pi_0(r, d, \tau^F), \quad (23)$$

where r is the interest rate and $\pi_0 = \left(\sum_{t=1}^d \frac{S^D(\gamma_t)}{(1+r)^t} + \sum_{t=d+1}^{\tau^F+d} \frac{S^H(\gamma_t)}{(1+r)^t} \right)$ denotes the present value resulting from playing the sequence once. The derivative of (23) with respect to d amounts to

$$\frac{\partial}{\partial d} a = r \cdot (1+r)^{(\tau^F+d)} \cdot \left[\frac{\frac{\partial \pi_0}{\partial d}}{(1+r)^{(\tau^F+d)}} - \frac{\pi_0 \cdot \ln(1+r)}{[(1+r)^{(\tau^F+d)} - 1]^2} \right]. \quad (24)$$

From (24) it follows

$$\frac{\partial}{\partial d} a \geq 0 \Leftrightarrow \left[\frac{\frac{\partial \pi_0}{\partial d}}{(1+r)^{(\tau^F+d)}} - \frac{\pi_0 \cdot \ln(1+r)}{[(1+r)^{(\tau^F+d)} - 1]^2} \right] \geq 0,$$

which after rearranging obtains as

$$\frac{\partial}{\partial d} a \geq 0 \Leftrightarrow \frac{\frac{\partial \pi_0}{\partial d}}{\pi_0} \geq \left| \frac{\partial AF}{\partial d} \right|, \quad (25)$$

where $AF = \frac{(1+r)^{(\tau^F+d)} \cdot r}{(1+r)^{(\tau^F+d)} - 1}$ denotes the annuity factor. (25) says that an additional nonfulfillment inserted into the representative sequence is efficient if the normalized marginal present value increases faster in d than the annuity factor decreases in d . When is condition (25) likely to hold? If τ^F increases, the right-hand side of (25) decreases. At the same time the left-hand side increases because the marginal *loss* from nonfulfillment *decreases* if τ^F increases: By definition, $d \leq (\tau^F - 1)$, and with τ^F increasing, the number of *possible* subsequent nonfulfillments increases. At the same time, establishing full trust takes more periods. From proposition 4 we know that establishing full trust is optimal. With τ^F increasing the marginal loss from nonfulfillment decreases, whereas the marginal gain from an early resumption of fulfillment decreases as

$$\frac{dS_{t+1}^H(\theta^{t-1}, \theta_t)}{d\theta_t} = \frac{\alpha + \beta + t}{\left[2(\alpha + \beta + t) - \left(\alpha + \sum_{i=1}^{t-1} \theta_i + \theta_t \right) \right]^2} > 0 \quad (26)$$

and

$$\frac{dS_{t+1}^D(\theta^{t-1}, \theta_t)}{d\theta_t} = \frac{4(\alpha + \beta + t)^2}{\left[2(\alpha + \beta + t) - \left(\alpha + \sum_{i=1}^{t-1} \theta_i + \theta_t \right) \right]^3} > 0 \quad (27)$$

show; (26) and (27) increase in $t = \tau^F$ and θ_t which - for expositional brevity - is here assumed to be continuous. Therefore, d weakly increases in τ^F . ■

References

- [1] Adler, P. S., 2001. Market, Hierarchy, and Trust: The Knowledge Economy and the Future of Capitalism. *Organization Science* 12, 215-234
- [2] Aumann, R.J., Maschler, M.B., Repeated Games with Incomplete Information. Cambridge: MIT Press.
- [3] Bachmann, R., 2001. Trust, Power and Control in Trans-Organizational Relations. *Organization Studies* 22, 337-365
- [4] Baker, G., 1990. Pay-For-Performance for Middle Managers: Causes and Consequences. *Journal of Applied Corporate Finance* 3, 50-61
- [5] Baker, G., Gibbons, R., Murphy, K.J., 1994. Subjective Performance Measures in Optimal Incentive Contracting. *Quarterly Journal of Economics* 109(4), 1125-1156
- [6] Basu, S., Waymire G.B., 2006. Recordkeeping and Human Evolution. *Accounting Horizons* 20(3), 201-229
- [7] Basu, S., Dickhaut, J., Hecht, G., Towry, K., Waymire, G., 2007. Recordkeeping Alters Economics History by Promoting Reciprocity, Working Paper.
- [8] Binmore, K., McCarthy, J., Ponti, G., Samuelson, L., 2002. A Backward Induction Experiment. *Journal of Economic Theory* 104(1), 48-88
- [9] Bone, J.D., Hey, J.D., Suckling, J.R., 2003. Do People Plan Ahead? *Applied Economics Letters* 10(5), 277-280
- [10] Boot, A.W.A., Greenbaum, S.A., Thakor, A.V., 1993. Reputation and Discretion in Financial Contracting. *The American Economic Review* 83(5), 1165-1183
- [11] Bull, C., 1987. The Existence of Self-Enforcing Implicit Contracts. *The Quarterly Journal of Economics* 102(1), 147-160
- [12] Busco, C., Riccaboni, A., Scapens, R.W., 2006. Trust for Accounting and Accounting for Trust. *Management Accounting Research* 17(1), 11-41
- [13] Butler, Jr., J.K., 1983. Reciprocity of trust between professionals and their secretaries. *Psychological Reports* 53, 411-416
- [14] Camerer, C., 1995. Individual Decision Making. In: Kagel, J., Roth, A.E. (Eds.). *Handbook of Experimental Economics*. Princeton: Princeton University Press.

- [15] Camerer, C., Johnson, E.J., 2004. Thinking about Attention in Games: Backward and Forward Induction. In: Brocas, I., Carrillo, J.D. (Eds.). *The Psychology of Economic Decisions, Volume 2: Reasons and Choices*. Oxford: Oxford University Press.
- [16] Camerer, C., Weigelt, K., 1988. Experimental Tests of a Sequential Equilibrium Reputation Model. *Econometrica* 56(1), 1-36
- [17] Campbell, D., Lee, C., 1988. Self Appraisal in Performance Evaluation: Development versus Evaluation. *Academy of Management Review* 13, 302-314
- [18] Campbell, D.J., Campbell, K.M., Chia, H.-B., Merit Pay, Performance Appraisal, and Individual Motivation: An Analysis and Alternative. *Humane Resource Management* 37(2), 131-146
- [19] Casadesus-Masanell, R., 2004. Trust in Agency, *Journal of Economics & Management Strategy* 13(3), 375-404
- [20] Christensen, P.O., Feltham, G.A., *Economics of Accounting, Vol II: Performance Evaluation*. New York: Springer.
- [21] Coleman, J.S., 1990. *Foundations of Social Theory*. Cambridge: Belknap Press.
- [22] Condrey, S.E., Brudney, J.L., Performance-Based Managerial Pay in the Federal Government: Does Agency Matter? *Journal of Public Administration Research and Theory* 2, 157-174
- [23] Conlisk, J., 1996. Why Bounded Rationality? *Journal of Economic Literature* 34, 669-700
- [24] Coyle-Shapiro, J., Kessler, I., 2000. Consequences of the psychological contract for the employment relationship: A large scale survey. *Journal of Management Studies* 37(7), 903-930
- [25] Dasgupta, P., 1988. Trust as Commodity. In: Gambetta, D. (Ed.). *Trust: Making and Breaking of Cooperative Relations*. Oxford: Blackwell, 49-72
- [26] DeGroot, M.H., 1970. *Optimal Statistical Decisions*. New York: McGraw-Hill.
- [27] Dirks, K.D., Ferrin, D.L., 2001. The Role of Trust in Organizational Design. *Organization Science* 12(4), 450-467
- [28] Englmaier, F., Segal, C., 2006. A Note on Dynamic Patterns of Implicit Contracts with Imperfect Public Monitoring, Working Paper Harvard Business School.
- [29] Folger, R., Konovsky, M., 1989. Effects of procedural and distributive justice on reactions to pay raise decisions. *Academy of Management Journal* 32, 115-130

- [30] Forges, F., 1992. Repeated Games of Incomplete Information: Non-Zero-Sum. In: Aumann, R.J., Hart, S. (Eds.). *Handbook of Game Theory with Economic Applications*. North Holland: Amsterdam, 155-177
- [31] Friedman, J.A., 1971. A Non-cooperative Equilibrium for Supergames. *The Review of Economic Studies* 38(1), 1-12
- [32] Fudenberg, D., Maskin, E., 1986. The Folk Theorem in Repeated Games with Discounting or with Incomplete Information. *Econometrica* 54(3), 533-554
- [33] Gibbs, M., Merchant, K. A., Van der Stede, W.A., Vargus, M.E., 2004. Determinants and Effects of Subjectivity in Incentives. *The Accounting Review* 79(2), 409 - 436
- [34] Giddens, A., 1976. *New rules of sociological method*. London: Hutchinson.
- [35] Giddens, A., 1984. *The Constitution of Society*. Cambridge: Polity Press.
- [36] Giddens, A., 1990. *The consequences of modernity*. Stanford: Stanford University Press.
- [37] Green, E.J., Porter, R.H., 1984. Noncooperative Collusion under Imperfect Price Information. *Econometrica* 52(1), 87-100
- [38] Gürtler, O., 2006. *Implicit Contracts: Two Different Approaches*. Working Paper University of Bonn.
- [39] Hedge, J.W., 2000. Exploring the Concept of Acceptability as a Criterion for Evaluating Performance Measures. *Group & Organization Management* 25(1), 22-44
- [40] Herrnstein, R.J., 1991. Experiments on Stable Suboptimality in Individual Behavior. *The American Economic Review* 81(2), 360-364
- [41] Herrnstein, R.J., Prelec, D., 1991. Melioration: A Theory of Distributed Choice. *Journal of Economic Literature* 5(3), 137-156
- [42] Hey, J.D., Knoll, J.A., 2007. How far ahead do people plan? *Economic Letters* 96 8-13
- [43] Hopwood, A.G., 1972. An Empirical Study of the Role of Accounting Data in Performance Evaluation. *Journal of Accounting Research* 10, 156-182
- [44] Hosmer, L.T., 1995. Trust: The Connecting Link between Organizational Theory and Philosophical Ethics. *Academy of Management Review* 20(2), 379-403
- [45] Ittner, C. D., Larcker, D. F., 2003. Coming up Short on Nonfinancial Performance Measurement. *Harvard Business Review* 81(11), 88-95

- [46] Ittner, C. D., Larcker, D. F., Meyer, M.W., 2003. Subjectivity and the Weighting of Performance Measures: Evidence from a Balanced Scorecard. *The Accounting Review* 78(3), 725-758
- [47] James, H.S.J., 2002. The Trust Paradox: A Survey of Economic Enquiries into the Nature of Trust and Trustworthiness. *Journal of Economic Behavior & Organization* 47, 291-307
- [48] Johansson, I.-L., Baldvinsdottir, G., 2003. Accounting for trust: some empirical evidence. *Management Accounting Research* 14, 219-234
- [49] John, K., Nachman, D.C., 1985. Risky Debt, Investment Incentives, and Reputation in a Sequential Equilibrium. *The Journal of Finance* 40(3), 863-878
- [50] Johnson, E.J., Camerer, C., Sen, S., Rymon, T., 2002. Detecting Failures of Backward Induction: Monitoring Information Search in Sequential Bargaining. *Journal of Economic Theory* 104(1), 16-47
- [51] Jones, G.R., George, J.M., 1998. The experience and evolution of trust: Implications for cooperation and teamwork. *The Academy of Management Review* 23(3), 531-546
- [52] Jonker, C.M., Schalken, J.J.P., Theeuwes, J., Treur, J., 2004. Human Experiments in Trust Dynamics, *Lecture Notes in Computer Science* 2995. Berlin, Heidelberg: Springer, 206-220
- [53] Kagel, J.H., Roth, A.E. (Eds.), 1995. *The Handbook of Experimental Economics*. Princeton University Press: Princeton.
- [54] Kreps, D.M., Milgrom, P., Roberts, J., Wilson, R., 1982. Rational Cooperation in the Finitely Repeated Prisoners' Dilemma. *Journal of Economic Theory* 27, 245-252
- [55] Kreps, D.M., 1990. Corporate Culture and Economic Theory. In: Alt, J.E., Shepsle, K.A. (Eds.). *Perspectives on Positive Political Economy*. Cambridge University Press: Cambridge, 90-143
- [56] Kwon, Y.K., 2005. Accounting Conservatism and Managerial Incentives. *Management Science* 51(11), 1626-1632
- [57] Levin, J., 2003. Relational Incentive Contracts. *The American Economic Review* 93(3), 835-857
- [58] Lukas, C., 2005. *Leistungsorientierte Vergütung als Qualifizierungsanreiz*, Wiesbaden: Gabler.
- [59] Luhmann, N., 1979. *Trust and Power*. Chichester: Wiley.
- [60] Luhmann, N., 1989. *Vertrauen. Ein Mechanismus zur Reduktion sozialer Komplexität*, 3rd print, Stuttgart: Enge.

- [61] MacLeod, W.B., Malcomson, J.M., 1988. Reputation and Hierarchy in Dynamic Models of Employment. *Journal of Political Economy* 96(4), 832-854
- [62] MacLeod, B., 2003. Optimal contracting with subjective evaluation *The American Economic Review* 93(1), 216-240
- [63] Mailath, G.J., Samuelson, L., 2006. *Repeated Games and Reputations: Long-Run Relationships*. Oxford: Oxford University Press.
- [64] Mayer, R.C., Davis, J.H., Schoorman, F.D., 1995. An integrative model of organizational trust. *Academy of Management Review* 20, 709-734
- [65] Mayhew, B.W., 2001. Auditor Reputation Building. *Journal of Accounting Research* 39(3), 599-617
- [66] Milkovich, G.T., Newman, J.M., 2002. *Compensation*, 7th ed. Irwin: McGraw-Hill.
- [67] Mitusch, K., 2006. Non-commitment in performance evaluation and the problem of information distortions. *Journal of Economic Behavior & Organization* 60, 507-525
- [68] Mui, L., Mohtashemi, M., Halberstadt, A., 2002. A Computational Model of Trust and Reputation for E-Businesses. *Proceedings of the 35rd Hawaii International Conference on System Sciences*, Vol 7.
- [69] Murphy, K.J., Oyer, P., 2003. Discretion in Executive Incentive Contracts: Theory and Evidence. Working paper.
- [70] Nooteboom, B., 2002. *Trust: Forms, Foundations, Functions, Failures and Figures*. Cheltenham: Edward Elgar.
- [71] Pearce, D.G., Stacchetti, E., 1998. The Interaction of Implicit and Explicit Contracts in Repeated Agency. *Games and Economic Behavior* 23, 75-96
- [72] Pitz, G.F., Sachs, N.J., 1984. Judgment and Decision: Theory and Application. *Annual Review of Psychology* 35, 139-163
- [73] Prendergast, C., 1999. The Provisions of Incentives in Firms. *Journal of Economic Literature* 37, 7-63
- [74] Rapoport, A., 1997. Order of Play in Strategically Equivalent Games in Extensive Form. *International Journal of Game Theory* 26, 113-136
- [75] Reinke, S.J., 2003. Does the Form Really Matter? Leadership, Trust, and Acceptance of the Performance Appraisal Process. *Review of Public Personnel Administration* 23(1), 23-37

- [76] Resnik, P., Zeckhauser, R., 2002. Trust among strangers in internet transactions: Empirical analysis of ebay's reputation system. *The Economics of the Internet and E-Commerce*. In: Baye, M.R. (Ed.). *Advances in Applied Microeconomics*, Vol. 11, 127-157
- [77] Robinson, S.L., Rousseau, D.M., 1994. Violating the Psychological Contract: Not the Exception but the Norm. *Journal of Organizational Behavior* 15(3), 245-259
- [78] Robinson, S.L., 1996. Trust and Breach of the Psychological Contract. *Administrative Science Quarterly* 41(4), 574-599
- [79] Rosen, S., 1992. Contracts and the Market for Executives. In: Wessin, L., Wijkander, H. (Eds.). *Contract Economics*. Oxford: Blackwell, 181-211
- [80] Rousseau, D.M., 1990. New Hire Perceptions of Their Own and Their Employer's Obligations: A Study of Psychological Contracts. *Journal of Organizational Behavior* 11(5), 389-400
- [81] Savage, L., 1954. *The Foundation of Statistics*. New York: Wiley.
- [82] Sydow, J., 1998. Understanding the constitution of inter-organizational trust. In: Lane, C., Bachmann, R. (Eds.). *Trust Within and Between Organizations*. Oxford: Oxford University Press, 31-63
- [83] Simon, H.A., 1955. A Behavioral Model of Rational Choice. *The Quarterly Journal of Economics* 69(1), 99-118
- [84] Wicks, A.C., Berman, S.L., Jones, T.M., 1999. The structure of optimal trust: Moral and strategic implications. *The Academy of Management Review* 24(1), 99-116
- [85] Williamson, O.E., 1993. Calculativeness, Trust, and Economic Organization. *Journal of Law and Economics* 36(1), 453-486.
- [86] Wixted, J.T., 2004. The Psychology and Neuroscience of Forgetting. *Annual Review of Psychology* 55, 235-269
- [87] Zak, P. J., Knack, S., 2001. Trust and Growth. *The Economic Journal* 111, 295-321
- [88] Zucker, L.G., 1986. Production of trust: Institutional sources of economic structure. In: Staw, B., Cummings, L.L. (Eds.). *Research in Organizational Behavior*, Vol. 8, 53-111