



**The Equity-Efficiency Relationship:
the Double Criterion versus Pareto-
Efficiency**

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Abstract

The paper is devoted to the analysis of the equity-efficiency relationship from the standpoint of the proposed double criterion of efficiency as opposed to the criterion of Pareto-efficiency. The equity-efficiency relationship with an extremum, obtained in the paper exceeds the boundaries of the traditional approach to this problem. Empirical verification of the proposed model shows that it corresponds to the facts much better than the traditional equity-efficiency trade-off model.

Keywords:

equity-efficiency trade-off, Pareto-efficiency, double criterion of efficiency.

JEL Classification

D31; D63

Introduction

Recently, the question of the relationship between equity and efficiency has been widely discussed in a growing number of investigations. The traditional approach to this question is that society has to choose between equity and efficiency. This idea was classically formulated by Okun in his well-known book "Equality and Efficiency: The Big Trade-off". He wrote: "The pursuit of efficiency necessarily creates inequalities. And hence society faces a trade-off between equality and efficiency" and "...the conflict between equality and economic efficiency is inescapable" (Okun 1975).

His famous "leaky bucket experiment" has found further reflection in economic literature. The basic idea is that a sum of money obtained by poorer individuals in the tax-transfer process appears to be less than a sum of money taken from richer ones. This results in reduction in total income and less efficient use of resources. For instance, as notes Burtless (1986), summarizing the results of a series of the negative income tax experiments held in the 1970s in the US, in order to increase family incomes by \$1 the government had to spend almost \$2. This leakage was mostly accounted for by the fact that higher transfers induced reductions in labour supply.

Another example of a leaky bucket is unemployment insurance. Katz and Meyer (1990) showed that each additional week of benefit availability increased unemployment spells by 0.16 to 0.20 weeks.

The other side of the problem is inefficiencies related to the tax system. For example, Feldstein (1999) estimates that the efficiency loss from current income taxes is more than 30

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percent and 50 percent if social security taxes are included. According to Ballard, Shoven and Whalley (1985), losses connected with the US tax system were approximately 13% to 24% for every dollar raised. The importance of tax structure optimisation has induced a lot of investigations on this problem. For instance, questions of optimal taxation with respect to economic efficiency are discussed by Auerbach and Hines (2001).

Thus, there is no doubt that leaky buckets and inefficiencies are characteristic of the tax-transfer system. However, some economists take a different approach to the relationship between equity and efficiency. For instance, Baumol (1988) shows that the equity-efficiency trade-off in economic decision-making does not have to be as great as generally believed. Blank (2002), analysing extensive empirical data, focused her attention on policy situations in which equity and efficiency are not inevitably in conflict with each other. According to Jackson (2000), the relationship between taxes as a share of GDP and growth and productivity is weak and insignificant, while the relationship to inequality measures is strong and significant, which is not consistent with the idea that higher public consumption financed through the tax-transfer system comes at the cost of lost efficiency. The basic that there is a complex equity-efficiency relationship with an extremum was set forth in Arkhiereiev (1999).

Much attention in the studies of equity and efficiency has been paid to the relationship between inequality and economic growth. Traditional theory says that inequality and economic growth are positively correlated. For example, Kaldor (1957) and Pasinetti (1962) developed this point. Such an approach has found a lot of opponents. Among them, one could mention, for instance, Alesina and Rodrik (1994). In general, as Aghion, Caroli and Garcia-Penalosa (1999) note, the view that inequality is growth-enhancing has been challenged by a number of empirical studies, often based on cross-country regressions of GDP growth on income inequality. They all find a negative correlation between the average rate of growth and a number of measures of inequality.

Thus, there is no universal approach to the problem of the relationship between equity and efficiency, and opposite points of view can be found in the literature. But the typical character of the studies on this problem is that major attention is paid to redistribution processes affecting inequality (although taxes and transfers alter both the post-tax and pre-tax distribution of earnings (Stiglitz 1987)). Our major goal is to discuss first of all the influence of initial distribution on economic efficiency.

A. Theoretical models

Theoretical substantiation of the inverse equity-efficiency relationship model is usually focussed on pointing out efficiency losses arising in the process of product redistribution. Such an approach is confined to a single factor affecting the analyzed relationship, namely redistribution of income coming into being in the process of production. However, such an approach implicitly presupposes that the character of product distribution in the process of production does not affect its efficiency. Since this assumption is not obvious, it is necessary to consider the case of the influence of distribution relations on production efficiency.

In order to find whether each of the assumptions corresponds to the facts, in this section of the paper we construct the models based on deliberate differentiation of the cases of dependence and independence of production outcome on the character of distribution.

I. The Case of Independence of Production Outcome on Distribution Relations

Consider first the case of independence of production outcome of the character of distribution. In order to simplify the model suppose that there are only two entities (two groups)

taking part in the joint process of production of a single good. They execute different functional roles and distribute the created product of some given volume

$$Q = const, \quad (1)$$

between them such that

$$Q = Q_1 + Q_2, \quad (2)$$

where Q_1 and Q_2 are the quantities of the product obtained by the first and the second entity respectively.

Let μ denote the share of the product obtained by the first entity. Then the relationship between Q_1 , Q_2 and μ are given by the following equations:

$$Q_1 = \mu \cdot Q; \quad (3)$$

$$Q_2 = (1 - \mu) \cdot Q. \quad (4)$$

This case is illustrated in Figure 1 where the volume of production Q is plotted against the share in output of the first entity. In the case of a multi-product model, it is necessary to consider the utility of a product instead of its quantity. Other regularities remain unchanged. The dependence of the volumes of the product Q_{A1} and Q_{A2} obtained by the first and the second entity, respectively, on μ are shown for the volume of production Q_A which is constant and independent of distribution. According to (2), the following equation is true of any point m : $Q_{A1}^m + Q_{A2}^m = Q_A$. In Figure 1, the same dependences are shown for the volume of production Q_B which is greater than Q_A . One can see that the quantities of the product obtained both by the first and the second entity increase for a larger total volume of production and invariable position of point m .

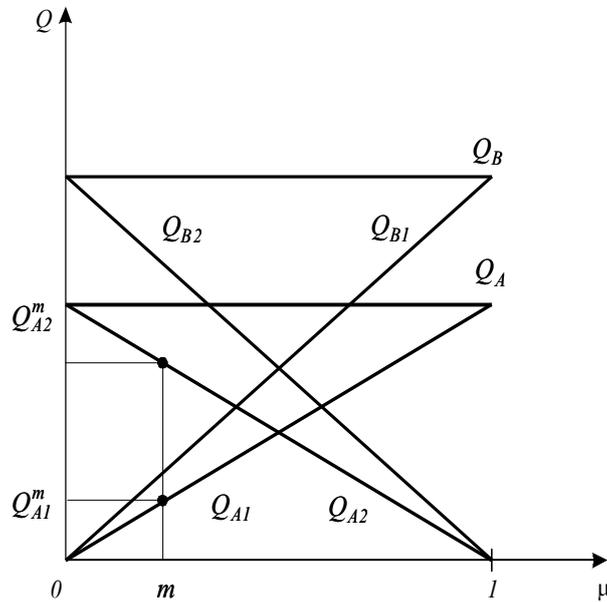


Fig. 1: The Case of Independence of the Volume of Production on the Character of Product Distribution.

The consumption possibilities line $Q_A Q_A$, corresponding to the volume of production Q_A , is constructed in the coordinates Q_1 and Q_2 in Figure 2. It is positioned at an angle of 45° to the coordinate axes and shows the relationship between the quantities of the product consumed by both entities under conditions of invariability of its total volume. The values of Q_{A1}^M and Q_{A2}^M are defined by the value μ which is reflected in Figure 2 by point M at the intersection of the consumption possibilities line $Q_A Q_A$ and the distribution line Od drawn at an angle of α to the axis OQ_1 from the origin of coordinates, and

$$tg\alpha = \frac{Q_{A2}^M}{Q_{A1}^m} = \frac{1-\mu}{\mu} \quad (5)$$

indicates the proportion of distribution of the produced goods irrespective of their absolute value. A larger production level $Q_B > Q_A$ corresponds to larger consumption possibilities, and the corresponding consumption possibilities line shifts to position Q_BQ_B .

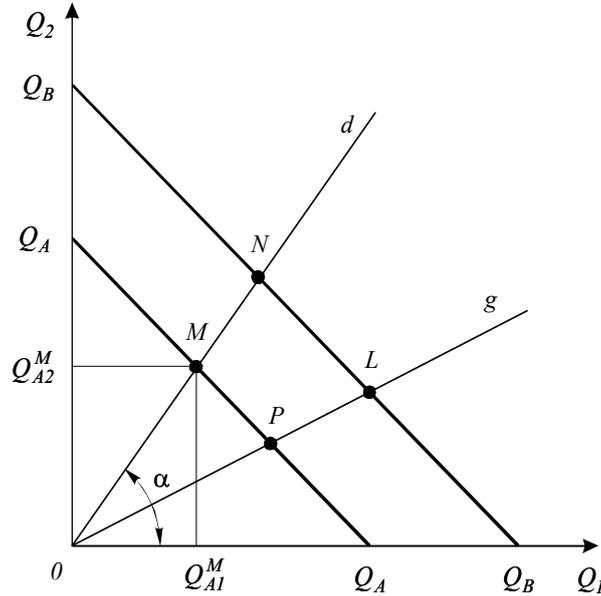


Fig. 2: The Relationship between the Values of the Product Obtained by Entities if the Volume of Production is Independent on the Character of Distribution.

The value of μ being unchanged, the values Q_{B1}^N and Q_{B2}^N of the product obtained by the entities are defined by point N at the intersection of the consumption possibilities line Q_BQ_B with the same distribution line Od . An increase in the share of the first entity, which manifests itself in an increase in μ , leads to a decrease in the angle α , i.e. to a turn of the distribution line, for example, to position Og . The values of the product Q_{A1}^P , Q_{A2}^P and Q_{B1}^L , Q_{B2}^L , obtained by the entities, are determined in this case by points P and L at the intersection of the consumption possibilities lines Q_AQ_A and Q_BQ_B with a new distribution line Og .

Changes in $tg\alpha$, reflecting the character of product distribution between entities, do not result in changes in the total amount of the product (Q_A or Q_B respectively), the latter being a good indicator of production efficiency. Therefore, the equity-efficiency relationship at this stage of product reproduction may be represented as a vertical line. If losses arising in the process of product redistribution are taken into account, it is transformed into the inverse relationship characteristic of the traditional model.

II. The Case of Dependence of Production Outcome on Distribution Relations

Now we shall move on to consider the case of the influence of distribution relations on the volume of production in the single-product model. As in the previous case, where the volume of production was unchanged, consider first the dependence of the volume of production and the amount of the product obtained by the first and the second entity on the share of the product obtained by the first entity (μ). Suppose the first entity performs direct production activities, while the other performs entrepreneurial functions concerning the organization of production including

the combination of productive factors. The character of the function $Q(\mu)$ may be determined on the basis of the following well-known principles.

If the share of the first entity in the product produced is small enough and increases gradually, one may expect an increase in its value due to overcoming the “avoidance effect”, i.e. incomplete use of potential productivity caused by low wages. This principle is reflected by the rising part of the graph $Q(\mu)$ in Figure 3. (For more information about wage models and efficiency, see Yellen 1984).

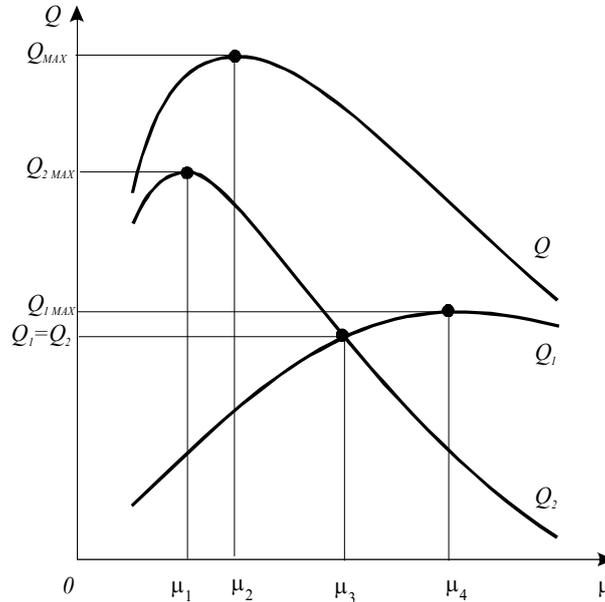


Fig. 3: The Case of Dependence of the Volume of Production on the Character of Product Distribution.

A further increase in μ results in a decrease in Q as a consequence of growing disincentives to efficient entrepreneurship due to the reduction in entrepreneurial income (as was in detail analysed in supply-side economics and practically implemented in Reaganomics). It should be noted that this argument is also used in classical economics for justifying the necessity of granting sufficient freedom to market forces to bring about a reduction in the wage level. The influence of this factor is reflected by the decreasing part of the graph $Q(\mu)$.

The result of the simultaneous influence of the mentioned factors is shown in Figure 3. The information about the concave curve $Q(\mu)$ allows us to find the dependence of the volume of the product obtained by each of the producers on the share of the first entity in the total volume of production. These dependences are shown in Figure 3 by means of the curve Q_1 for the first entity and the curve Q_2 for the second one. The volume of the product produced Q is of maximal value if the distribution rate μ equals μ_2 . The value of μ at which Q_1 reaches its maximum, given the function $Q(\mu)$, can be found by setting the derivative of the function (3) with respect to μ equal to zero.

Thus, we obtain the following condition for maximization of Q_1 :

$$\frac{dQ}{d\mu} \cdot \mu + Q = 0 \quad (6)$$

which requires the elasticity of the curve $Q(\mu)$ being equal to one:

$$E_{Q(\mu)} = \frac{dQ}{d\mu} \cdot \frac{\mu}{Q} = -1. \quad (7)$$

In Figure 3, the condition for a maximal value of Q_1 holds if $\mu = \mu_4$.

Analogically, equating the derivative of function (4) with respect to μ to zero, we obtain the following condition for the maximization of Q_2 :

$$\frac{dQ}{d\mu} \cdot (1 - \mu) - Q = 0 \quad (8)$$

to which the following elasticity of the curve $Q(\mu)$ corresponds:

$$E_{Q(\mu)} = \frac{dQ}{d\mu} \cdot \frac{\mu}{Q} = \frac{\mu}{1 - \mu}. \quad (9)$$

In Figure 3, the maximum of Q_2 obtains if $\mu = \mu_1$.

The analysis of the obtained relationships shows that both agents can increase the values of the product obtained by them within the range of variable μ to the left of μ_1 . If $\mu = \mu_1$, maximization of the product obtained by the second agent (entrepreneur) takes place, and it is evident that the Pareto-efficient states of the economy, i.e. states in which the first entity's consumption cannot be increased without a decrease in the other entity's consumption, are obtained within the range of variable μ from μ_1 to μ_4 inclusive. Thus, as theory tells us, maximization of entrepreneurial income, achieved by the second entity if $\mu = \mu_1$, results in attainment of a Pareto-efficient state of the economy. However, as is seen from Figure 3, such maximization in general case does not lead to the achievement of maximal production efficiency which is determined by the value of total output produced, given costs unchanged, and takes place if $\mu = \mu_2$. Maximization of the product obtained by the first entity (providing direct production activities), which is the case if $\mu = \mu_4$, does not allow us either to attain maximal efficiency of production though it results in a Pareto-efficient state of the economy.

We can come to the same conclusion analyzing the consumption possibilities curve. It can be constructed in the coordinates Q_1 and Q_2 on the basis of the information contained in Figure 3 by analogy with the case of independence of production results of the character of distribution depicted in Figure 2. Taking into account the influence of distribution relations, the shape of the consumption possibilities curve will be essentially different from the latter case. If in Figure 2 the consumption possibilities line coincides with one of the equal product lines, the concave consumption possibilities curve, obtained in Figure 4, cuts a set of such lines. Each of the distinctive points R , S , T , U on this curve, corresponding to the values of μ equal to μ_1 , μ_2 , μ_3 , μ_4 in Figure 4, is characterized by its own value of the product and the shares obtained by each of the entities. The value Q_2 is maximized at point R ($\mu = \mu_1$), therefore, the tangent to the consumption possibilities line drawn through it is a horizontal line. The greatest volume of production Q_{max} corresponds to point S ($\mu = \mu_2$), since the tangent at this point is the line with the volume of production greater in comparison to all other equal product lines having common points with the consumption possibilities line. Point T ($\mu = \mu_3$) is at the intersection of this line with the equal distribution line starting from the origin of coordinates at an angle of 45° . Finally, point U ($\mu = \mu_4$) corresponds to maximization of the value Q_1 , therefore, the tangent to the consumption possibilities line is vertical at this point. The part of the consumption possibilities line between points R and U is a set of Pareto-efficient distributions, where an increase in the quantity consumed by one of the individuals is accompanied with a decrease in that consumed by the other one.

III. Distribution Relations and Social Welfare Functions

Points R , S , T , U bear direct relation to different social welfare functions. The tangents to points R and U in Figure 4 are indifference lines showing equal social welfare levels on the

assumption that welfare of one of the entities of production is completely disregarded. The tangent to point S is the indifference line for the Benthamian social welfare function reflecting maximization of aggregate consumption irrespective of its distribution. The line starting from the origin of coordinates at an angle of 45° corresponds to the social welfare maximization line in accordance with the theory of supporters of full equality.

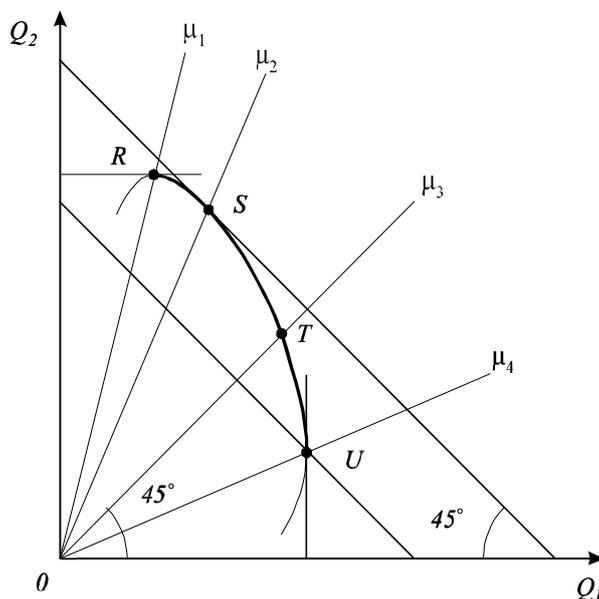


Fig. 4: The Relationship between the Values of the Product Obtained by Entities if the Volume of Production Depends on the Character of Distribution.

Indifference lines for other social utility levels can be represented according to this theory as an aggregate of pairs of lines diverging upwards and downwards from the equal distribution line at equal angles marked as β in Figure 5.

In the same figure, one can see that some other social welfare functions have maximums at the same distinctive points. The right-angled indifference curve VTW , drawn through point T , corresponds to the Rawlsian social welfare function which determines the welfare level by the amount of the product consumed by the least well-off members of society. The right-angled indifference curve XYZ , drawn through point R , pertains to the so-called Nietzschean social welfare function reflecting maximization of welfare of the most well-off members of society.

However, in reality, society does not follow any of the mentioned criteria, and peculiarities of distribution take shape depending on the bargaining power of entities of production. Theoretically, it is possible to reach a compromise within the section RU of the consumption possibilities line lying between the points at which maximization of the respective entity's consumption takes place. Such a solution could be possible in case of a bilateral monopoly which could be considered within the limits of this model but is rarely found in reality. The different roles of the entities involved in the process of production should be taken into account in practice. Since the second entity performs the main organizational functions, its bargaining power may be expected to be much greater, and a compromise will be reached within the part of the consumption possibilities line acceptable for it. The second entity as the agent performing the functions of organization of production is most likely guided by the production efficiency criterion as an integrated social welfare criterion, where production efficiency manifests itself in maximization of the product produced. We know that such maximization is reached at point S , thus a compromise acceptable for the organizer of production will be chosen within the section RS .

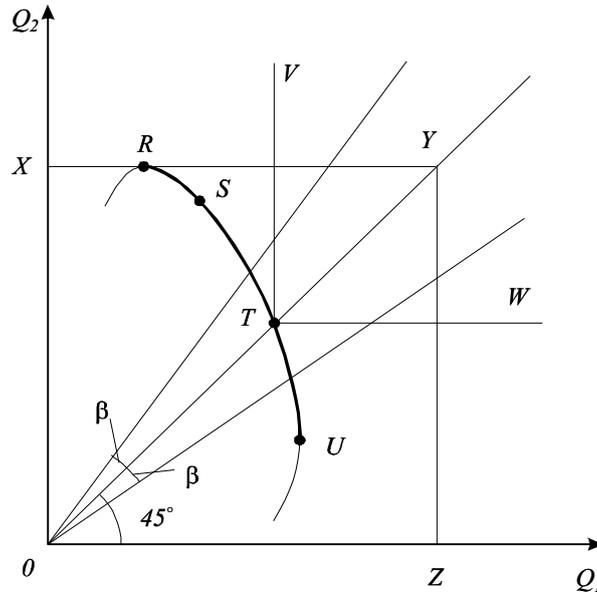


Fig. 5: Estimation of the Product Distribution Efficiency in accordance with Social Welfare Functions.

Consider the optimality of this choice from the standpoint of the whole society. The section RS is Pareto-optimal as it is part of the curve RU . However, it is universally recognized that the criterion of Pareto efficiency is individualistic, i.e. connected with welfare of each individual and is not connected with inequality, i.e. with the character of income distribution (Stiglitz J. E., 1988). At the same time, a social optimality criterion with obvious efficiency and inequality indicators should be used for the evaluation of consumption allocations. In this respect, two criteria may be proposed since it is impossible to give preference to any of them.

The first criterion may be the one which is natural if the situation is analyzed from the viewpoint of the whole society: given equal inequality rates, from two variants of distribution the one providing a greater total value of the product produced should be chosen. In other words, distribution is efficient from the standpoint of the first criterion if any other distribution, given the same inequality rate, cannot provide a higher total product to the society. It is plausible to use a ratio between the income of the least well-off entity and the income of the most well-off one as an indicator of inequality, i.e. to consider

$$J = \begin{cases} \frac{Q_2}{Q_1} & \text{if } Q_1 \geq Q_2, \\ \frac{Q_1}{Q_2} & \text{if } Q_1 < Q_2. \end{cases}$$

The section of the analyzed consumption possibilities line $RSTU$ satisfying this criterion can be found by means of constructing distribution lines diverging to equal angles γ from the full equality line and comparing the values of the product produced Q at intersections with the consumption possibilities line.

In Figure 6, such distribution lines Oe and Of cross the consumption possibilities line at points E and F through which the lines QEQ_E and QFQ_F are drawn showing corresponding values Q_E and Q_F of the product produced, Q_E being greater than Q_F . In the example under consideration, the points satisfying the first criterion are above point T . Thus, on the one hand, they cannot include part of the Pareto-efficient points, and on the other hand, they may include points which are not Pareto-optimal.

The second criterion is also natural from the social point of view: from two distributions of the same total product the one providing more equality should be chosen. In other words, distribution is efficient from the viewpoint of the second criterion if an output of the same value cannot be achieved at a greater equality rate. In Figure 7, the line $Q_H Q_H$ has two points of intersection with the consumption possibilities line (H_1 and H_2). In order to compare these points from the standpoint of the equality rate achieved, compare the values of angles φ and ψ between distribution lines OH_1 and OH_2 , and the equal distribution line OT . The second angle is less than the first one. Therefore, distribution at point H_2 is closer to equality.

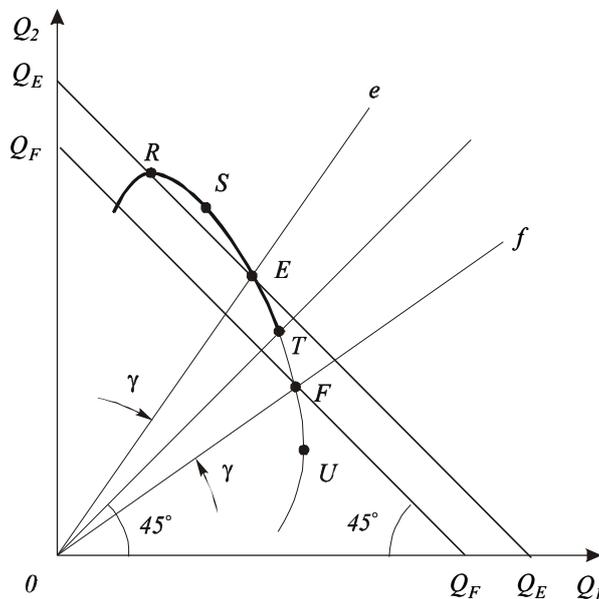


Fig. 6: The first Criterion of Efficiency from the Standpoint of the Whole Society.

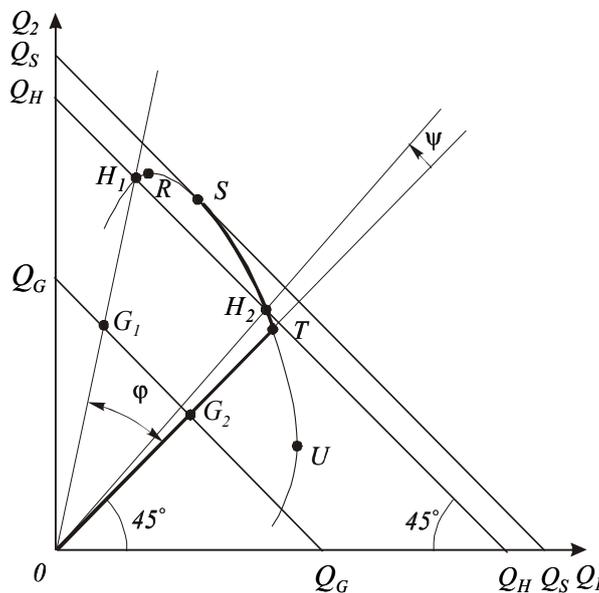


Fig. 7: The Second Criterion of Efficiency from the Standpoint of the Whole Society.

One can easily see that part of points satisfying the second criterion is within the section ST (these points are always closer to the equal distribution line OT). The other part of such points is within the segment OT of the equal distribution line since it provides the most even distribution in

From the standpoint of society, it is expedient to reduce the inequality rate within this section to enhance efficiency. However, this goal can be attained only if society has mechanisms at its disposal compelling the separate entities to act in the social instead of in the individual interest. Naturally, society is ready to solve this problem to varying degrees in different countries.

IV. The Equity-Efficiency Relationship Model

The proposed model allows us to construct a graph of the equity-efficiency relationship reflecting the double criterion of efficiency. Let efficiency E be estimated using an indicator which is natural in case of a fixed amount of resources, namely the value of the product produced Q . Estimation of equity will be based on the inequality rate J .

The equity-efficiency relationship obtained in this way is shown in Figure 9. Section TU is not taken into account since reaching a compromise within it contradicts the leading organizational role of the second entity. True, this section reflects peculiarities of stagnant socialism when remuneration of persons who played the leading role in the organization of production was often less than that obtained by subordinate executors. The malignancy of such an arrangement is graphically demonstrated in Figure 9 by the branch TU where efficiency of production is less in comparison with the branch TR for the same values of inequality in distribution. It is clear that the character of distribution, reflected by the branch TU , is both inefficient and unfair.

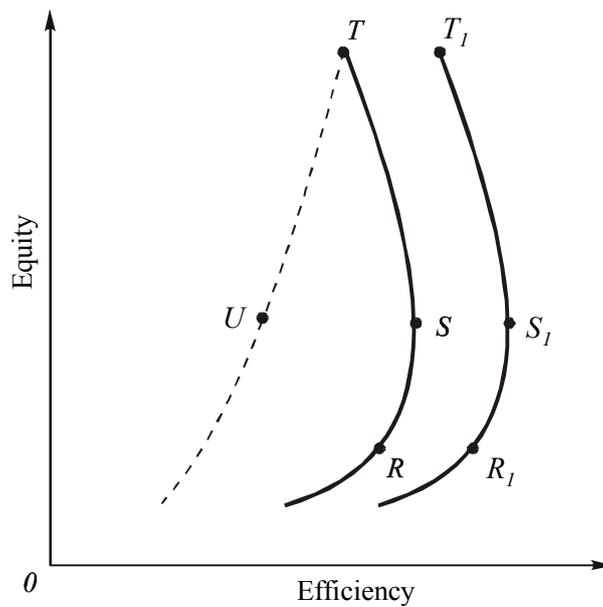


Fig. 9: The Equity-Efficiency Relationship in the Case of Dependence of the Volume of Production on the Character of Product Distribution.

The other Pareto-efficient points are represented by the section TR of the graph. It should be noted that only the part between points S and T is consistent with the traditionally recognized conception of an inverse equity-efficiency relationship, i. e. a trade-off between equity and efficiency. Distributional considerations cannot, however, be a-priori confined to this trade-off since the allocations between S and T do not include all Pareto-optimal distributions but only those satisfying the double criterion of social optimality, while a perfectly competitive market economy, guaranteeing the fulfillment of the Pareto criterion, does not assure the satisfaction of the double criterion. The complete equity-efficiency relationship curve is represented in Figure 9 by the branch TR and includes the section SR characterized by the direct equity-efficiency relationship.

guided not only by the character of primary distribution but also by rational expectations of the value of disposable income generated as a result of redistribution processes.

The double criterion of efficiency has its own sphere of application and possesses apparent advantages in comparison with the Pareto-criterion. The latter cannot be applied to any redistribution processes in principle, since they are accompanied by costs decreasing efficiency. On the contrary, the double criterion can be applied to the analysis of any processes of both primary distribution and redistribution.

Application of the double criterion to the processes of primary distribution allows to single out a section of absolute efficiency in accordance with the double criterion (the section *ST* in Figure 10) which is more narrow in comparison with Pareto-efficiency and includes part of Pareto-efficient points. Application of the double criterion to redistribution processes allows to single out a section of relative efficiency in accordance with the double criterion (the section *GJ* in Figure 10) which has no relation to Pareto-efficient points but is more efficient than part of actually achievable points of Pareto-efficiency according to the double criterion.

Thus, the double criterion is applicable to the analysis of redistribution processes and allows to single out sections which in accordance with it are more efficient than part of Pareto-efficient points achievable in reality as a result of primary distribution.

B. Empirical Verification of the Model

Verification of the proposed equity-efficiency relationship model with an extremum was carried out using macroeconomic indicators of efficiency of production and inequality. It appeared expedient to use the Gini index as a universal indicator characterizing the level of inequality.

For the purpose of reliability of the model verification results, a period of time close to the duration of a business cycle was chosen. The recent decade with the centre at the edge of the millennium may be considered such a period.

The data on the values of the Gini index in various countries has been comprehensively collected by the United Nations University WIDER (World Institute for Development Economics Research) out of various international and national primary sources. It also appeared true for the period of time selected by us for carrying out the analysis. During the period of investigation, these data were available in “UNU-WIDER World Income Inequality Database, Version 2.0a, June 2005” (WIID). However, it appeared to contain data up to 2003 year, and we considered it possible to restrict our analysis to rather a long 8-year period.

There are Gini indices based on different income/consumption/expenditure concepts in WIID. These concepts are incomparable both theoretically and practically, since the data on Gini indices, corresponding to them, are greatly different for the same countries and the same years. For these reasons, it was impossible to use data, calculated on the basis of different concepts, for one and the same country and one and the same year.

For model verification, the most appropriate indicators are calculated on the basis of income. However, Gini indices, based on different income concepts, vary by the number of countries for which they are presented. In this respect, we used an income concept for which there are comparable data for most countries. The analysis revealed “*Income, Disposable*” to be such an income concept for which there is information by 58 countries for the period under investigation. At the same time, the database contains information, for example, for “*Income, Gross*” by 28 countries and for “*Income, Factor*” – just by 1 country.

However, in some cases, data based on the disposal income concept (this also holds true for the other concepts), appeared to cover not all population and/or not all territory of a country. We considered it not appropriate to use such incomplete data because of their incomparability with Gini

indices calculated on the basis of data with full coverage of territory and population. Thus, we dropped 4 countries from the analysis for which only incomplete data were available.

We also used data calculated on the basis of “*Monetary Income, Disposable*” together with data based on disposable income in case of absence of the former in view of the insignificant difference (for our purposes) between these concepts. The use of the data for disposable monetary income allowed us to involve 8 countries more in the analysis.

In the obtained sample, for some of the countries there were data based on disposable income originating from more than one primary sources giving different information about the values of Gini indices. In this respect, for further comparability of data they were selected with the aim of achieving a maximally possible homogeneity of primary sources.

Since data on EU-15 are most fully presented by European Commission, in all cases it was this source that was used if it was available. Another source, containing information on the indicator under consideration by a considerable number of countries, was “Transmonee”, and preference was given to this base in all cases of the absence of data from European Commission. Luxembourg Income Study is a well-known inequality database, and it was this base that was used as a primary source if there were not available data from the two mentioned above sources. Finally, information on Latin America is widely presented in Deininger&Squire database which was used as a primary source of information by corresponding countries. In the cases of the absence of information from the mentioned databases, data were used from national statistical sources.

Since the Gini index, based on disposable income, was chosen as the basic inequality indicator, it was expedient to use a corresponding efficiency indicator. As such an overall indicator, we chose GDP per capita. We also decided to proceed from the principle of maximum comparability of data when selecting data for this indicator. In this connection, there was used the World Bank World Development Indicators database containing a large set of comparable data on the values of GDP and the number of population in various countries including the countries previously selected for the analysis.

The ultimate sample included 50 countries for which there was available information for both indicators (Gini index and GDP per capita). Among these countries there appeared European Union countries (with the exception of Cyprus and Malta), OECD countries (with the exception of Iceland, Japan, New Zealand and Turkey), 18 Central Europe, Eastern Europe and Central Asia countries, 10 Latin America countries, as well as China. Thus, the obtained sample is representative insofar as it includes transition and developing countries together with developed ones.

The preliminary analysis of the selected yearly statistical data on the value of GDP per capita and Gini coefficients, carried out by separate countries of various regions, has shown that the relationship between these indicators may be much more accurately described with a second-degree polynomial rather than with a linear function, which is proved by corresponding coefficients of determination. At the same time, application of polynomials of higher degrees rather slightly affects coefficients of determination; therefore, their use is inexpedient.

The second-degree equation of the relationship between the value of GDP per capita, designated as Y , and the value of the Gini coefficient, denoted as g , may be written for a separate country k as follows:

$$\hat{Y}_k = a_{2k} g_k^2 + a_{1k} g_k + a_{0k}. \quad (10)$$

The values of coefficients a_{0k} , a_{1k} and a_{2k} in this equation may be found by means of minimizing the function

$$F(a_{2k}, a_{1k}, a_{0k}) = \sum_{i=1}^{n_k} \left(\sum_{p=0}^2 a_{pk} g_{ki}^p - Y_{ki} \right)^2, \quad (11)$$

where i is the index number of the year in the sample;

n_k is the general number of years in the sample for which there are statistical data for country k .

Solving the normal system of equations of the least-squares method using the Gaussian method, one can find the values of the coefficients in equation (10) for a country k and further find the value of the Gini index g_{0k} which corresponds to the maximal value of GDP per capita $Y_{k \max}$:

$$g_{0k} = -\frac{a_{1k}}{2a_{2k}}. \quad (12)$$

However, the number of statistical data in samples by each separate country is insufficient to develop a reliable model of quadratic relationship between the indicators under investigation. In this connection, we developed a special model for calculating an optimal average level of inequality, determined by an optimal Gini index g_0 , for every single country included in the sample. In this model, some changes in the standard method of determining parameters of the quadratic equation using the least square method were needed.

In order to make these changes, the value of coefficient a_{1k} in the initial equation (10) of the relationship between the value of GDP per capita \hat{Y}_k and the value of the Gini index g_k for country k was determined on the basis of equation (12) by means of substitution of a particular optimal for a separate country k Gini index g_{0k} to an optimal Gini index g_0 which is considered as a single parameter for all countries.

$$a_{1k} = -2g_0 a_{2k}. \quad (13)$$

Taking into account expression (13), the regression equation for country k will take the following shape:

$$\hat{Y}_k = -a_{2k} g_k^2 + 2g_0 a_{2k} g_k + a_{0k}. \quad (14)$$

The expressions for determining values of coefficients in equation (14) are found by means of minimizing the function:

$$F(a_{2k}, a_{0k}, g_0) = \sum_{i=1}^{n_k} (-a_{2k} g_{ki}^2 + 2g_0 a_{2k} g_{ki} + a_{0k} - Y_{ki})^2 \quad (15)$$

where coefficients a_{2k} and a_{0k} are unknown quantities and the value of the optimal Gini index g_0 at this stage of calculations is considered as an adjustable parameter. The equations for determining coefficients a_{2k} and a_{0k} for each country can be found by means of the least squares method by equating to zero the expressions for derivatives of the right part of equation (15) with respect to these coefficients:

$$\left(\sum_{i=1}^{n_k} g_{ki}^4 - 4g_0 \sum_{i=1}^{n_k} g_{ki}^3 + 4g_0^2 \sum_{i=1}^{n_k} g_{ki}^2 \right) a_{2k} + (2g_0 \sum_{i=1}^{n_k} g_{ki} - \sum_{i=1}^{n_k} g_{ki}^2) a_{0k} = 2g_0 \sum_{i=1}^{n_k} g_{ki} Y_{ki} - \sum_{i=1}^{n_k} g_{ki}^2 Y_{ki}; \quad (16)$$

$$(2g_0 \sum_{i=1}^{n_k} g_{ki} - \sum_{i=1}^{n_k} g_{ki}^2) a_{2k} + n_k a_{0k} = \sum_{i=1}^{n_k} Y_{ki}. \quad (17)$$

The solution of the system of equations (16) and (17) allows us to write equations for coefficients a_{2k} and a_{0k} in the following shape:

$$a_{2k} = - \frac{(\sum g_{ki}^2 Y_{ki} - 2g_0 \sum g_{ki} Y_{ki})n_k - (\sum g_{ki}^2 - 2g_0 \sum g_{ki}) \sum Y_{ki}}{(\sum g_{ki}^4 - 4g_0 \sum g_{ki}^3 + 4g_0^2 \sum g_{ki}^2)n_k - (\sum g_{ki}^2 - 2g_0 \sum g_{ki})^2}; \quad (18)$$

$$a_{0k} = \frac{(\sum g_{ki}^4 - 4g_0 \sum g_{ki}^3 + 4g_0^2 \sum g_{ki}^2) \sum Y_{ki} - (\sum g_{ki}^2 - 2g_0 \sum g_{ki})(\sum g_{ki}^2 Y_{ki} - 2g_0 \sum g_{ki} Y_{ki})}{(\sum g_{ki}^4 - 4g_0 \sum g_{ki}^3 + 4g_0^2 \sum g_{ki}^2)n_k - (\sum g_{ki}^2 - 2g_0 \sum g_{ki})^2}. \quad (19)$$

The value of the optimal Gini coefficient g_0 was obtained on the basis of statistical data for all ($m = 50$) countries of the sample by means of minimizing average quadratic deviations, i.e. solving equation

$$u = \frac{1}{m} \sum_{k=1}^m \left[\frac{1}{n_k} \sum_{i=1}^{n_k} (\hat{Y}_{ki} - Y_{ki})^2 \right] \rightarrow \min \quad (20)$$

on the basis of organizing an iteration cycle in the range of the values of the Gini index from 0 to 100. The parameters of the quadratic equation for each country were determined in accordance with expressions (18) and (19).

As a result of solution of the formulated task, there was found the value of the optimal Gini coefficient $g_0 = 34$ corresponding to maximal efficiency for the set of the selected group of countries. In its turn, it allowed us to find the values of the potential efficiency of economies of each country out of the sample within the proposed model. To that end, equations of pair regression (14) were used in relation to each separate country. The values of potential GDP, corresponding to optimal inequality level $g_0 = 34$ within the proposed model, were calculated on the basis of these equations.

The same calculations were carried out for the classical model of the inverse equity-efficiency relationship. Their implementation allowed us to obtain alternative values of the potential GDP. In our opinion, these estimations of the potential GDP may be used as an additional criterion of revealing the model best corresponding to the facts. To that end, there were found losses sustained by each country depending on the deviation of inequality level from the level corresponding to maximal efficiency for each of the analyzed models by means of comparing the values of potential GDP with actual data for the years under consideration. Based on these data, there were created tables summarizing calculations for both models under consideration. Table I contains the results of the analysis of the relationship between the values of GDP per capita and the Gini index according to the proposed model with an extremum which explains losses as a result of either the choice between equity and efficiency or excessive inequality. Table II summarizes the theoretical estimation of the losses in GDP as a result of the equity-efficiency trade-off only in accordance with the classical model.

Table I.
Actual Efficiency and Potential Efficiency in accordance with the Equity-Efficiency Relationship Model with an Extremum (quite realistic results).

Countries	Actual efficiency (US\$ per capita)	Potential efficiency (US\$ per capita)	Estimated efficiency losses (%)
Countries under the double criterion conditions			
Australia	20 936	23 826	13.8
Austria	25 603	33 781	31.9
Belarus	2 207	2 346	6.3
Belgium	23 920	24 171	1.1
Canada	20 812	25 587	22.9
Czech Republic	5 672	6 181	9.0
Denmark	31 595	38 986	23.4
Finland	25 250	30 302	20.0
France	24 201	25 946	7.2
Germany	25 836	28 523	10.4
Hungary	4 929	7 138	44.8
Ireland	23 583	25 633	8.7
Italy	20 089	21 611	7.6
Latvia	2 906	3 899	34.2
Luxembourg	43 480	46 245	6.4
Macedonia, FYR	1 877	1 923	2.5
Netherlands	24 560	27 109	10.4
Norway	36 356	41 229	13.4
Poland	4 256	4 294	0.9
Romania	1 729	1 883	8.8
Slovak Republic	3 858	4 436	15.0
Slovenia	9 614	10 451	8.7
Spain	15 086	15 406	2.1
Sweden	26 908	29 819	10.8
Switzerland	35 735	36 291	1.6
United Kingdom	24 342	27 249	11.9
Countries beyond the double criterion conditions			
Armenia	601	800	33.1
Bolivia	1 005	1 101	9.5
Bulgaria	1 508	1 571	4.1
Chile	4 705	5 257	11.7

China	1 065	1 386	30.2
Dominican Republic	1 850	2 059	11.3
Ecuador	1 303	3 207	146.1
El Salvador	1 957	2 935	50.0
Estonia	3 836	4 469	16.5
Georgia	697	765	9.7
Greece	11 366	11 903	4.7
Honduras	829	1 179	42.1
Israel	17 531	17 786	1.5
Korea, Rep.	8 595	18 036	109.8
Kyrgyz Republic	299	313	4.6
Lithuania	3 166	3 300	4.2
Mexico	5 079	7 377	45.2
Moldova	372	545	46.5
Panama	3 269	4 420	35.2
Portugal	10 961	11 136	1.6
Russian Federation	2 146	2 798	30.4
Ukraine	734	890	21.2
United States	32 673	45 041	37.9
Venezuela	4 041	6 989	72.9
Average losses			22.3

Table II.

Actual Efficiency and Potential Efficiency in accordance with the Inverse Relationship Model if the Maximal Gini Index = 100 (unrealistically too high results).

Countries	Actual efficiency (US\$ per capita)	Potential efficiency (US\$ per capita)	Estimated efficiency losses (%)
Armenia	601	2 387	297.1
Australia	20 936	76 251	264.2
Austria	25 603	84 965	231.9
Belarus	2 207	2 650	20.1
Belgium	23 920	25 860	8.1
Bolivia	1 005	1 137	13.1
Bulgaria	1 508	1 588	5.3
Canada	20 812	104 272	401.0
Chile	4 705	5 866	24.7
China	1 065	3 133	194.3

Czech Republic	5 672	9 392	65.6
Denmark	31 595	77 337	144.8
Dominican Republic	1 850	2 640	42.7
Ecuador	1 303	4 761	265.4
El Salvador	1 957	2 505	28.0
Estonia	3 836	13 234	245.0
Finland	25 250	63 348	150.9
France	24 201	49 578	104.9
Georgia	697	986	41.4
Germany	25 836	44 891	73.8
Greece	11 366	20 608	81.3
Honduras	829	1 652	99.2
Hungary	4 929	24 115	389.2
Ireland	23 583	68 107	188.8
Israel	17 531	23 584	34.5
Italy	20 089	46 947	133.7
Korea, Rep.	8 595	30 534	255.3
Kyrgyz Republic	299	451	50.8
Latvia	2 906	18 053	521.2
Lithuania	3 166	10 216	222.7
Luxembourg	43 480	68 963	58.6
Macedonia, FYR	1 877	3 296	75.6
Mexico	5 079	11 472	125.9
Moldova	372	1 365	267.2
Netherlands	24 560	44 434	80.9
Norway	36 356	95 008	161.3
Panama	3 269	5 395	65.0
Poland	4 256	19 256	352.4
Portugal	10 961	15 009	36.9
Romania	1 729	1 896	9.7
Russian Federation	2 146	5 520	157.3
Slovak Republic	3 858	9 231	139.3
Slovenia	9 614	15 943	65.8
Spain	15 086	23 886	58.3
Sweden	26 908	48 905	81.8
Switzerland	35 735	46 200	29.3

Ukraine	734	900	22.7
United Kingdom	24 342	65 613	169.5
United States	32 673	156 448	378.8
Venezuela	4 041	14 229	252.1
Average losses			143.8

In our opinion, the results, obtained for the model with an extremum (22.3% losses of GDP on average), seem quite realistic while the influence of the factor of inequality in compliance with the classical model (143.8% losses of GDP on average) appears to be unrealistically too high. However, the level of inequality (Gini index = 100) for which efficiency losses were calculated in accordance with the inverse equity-efficiency relationship model seems to be considerably far from reality. Suppose that the maximal value of the Gini index in WIID = 77.6 (Zambia; 1991) is close to the absolute maximum of inequality which is characterized by the Gini index = 80. Therefore, we go over from purely theoretical value of the Gini index to its actual maximal value. Table 3 shows that such an assumption is more realistic than the above stated but is still considerably inferior to the optimal value of the Gini index calculated in accordance with the proposed equity-efficiency relationship model with an extremum.

Table III.

Actual Efficiency and Potential Efficiency in accordance with the Inverse Relationship Model if the Maximal Gini Index = 80 (unrealistically too high results).

Countries	Actual efficiency (US\$ per capita)	Potential efficiency (US\$ per capita)	Estimated efficiency losses (%)
Armenia	601	1 806	200.4
Australia	20 936	60 337	188.2
Austria	25 603	69 446	171.2
Belarus	2 207	2 558	15.9
Belgium	23 920	25 350	6.0
Bolivia	1 005	1 068	6.3
Bulgaria	1 508	1 570	4.1
Canada	20 812	80 425	286.4
Chile	4 705	5 352	13.7
China	1 065	2 425	127.8
Czech Republic	5 672	8 418	48.4
Denmark	31 595	65 710	108.0
Dominican Republic	1 850	2 338	26.4
Ecuador	1 303	3 143	141.2
El Salvador	1 957	2 202	12.5
Estonia	3 836	10 225	166.6
Finland	25 250	53 320	111.2

France	24 201	42 395	75.2
Georgia	697	879	26.1
Germany	25 836	39 889	54.4
Greece	11 366	17 811	56.7
Honduras	829	1 297	56.5
Hungary	4 929	18 966	284.8
Ireland	23 583	55 163	133.9
Israel	17 531	21 680	23.7
Italy	20 089	39 237	95.3
Korea, Rep.	8 595	23 887	177.9
Kyrgyz Republic	299	401	34.1
Latvia	2 906	13 565	366.8
Lithuania	3 166	8 042	154.0
Luxembourg	43 480	62 064	42.7
Macedonia, FYR	1 877	2 875	53.2
Mexico	5 079	8 841	74.1
Moldova	372	1 013	172.5
Netherlands	24 560	39 148	59.4
Norway	36 356	78 695	116.5
Panama	3 269	4 402	34.7
Poland	4 256	14 736	246.2
Portugal	10 961	13 731	25.3
Romania	1 729	1 859	7.5
Russian Federation	2 146	4 323	101.5
Slovak Republic	3 858	7 776	101.6
Slovenia	9 614	14 278	48.5
Spain	15 086	21 286	41.1
Sweden	26 908	43 093	60.2
Switzerland	35 735	43 191	20.9
Ukraine	734	848	15.6
United Kingdom	24 342	53 631	120.3
United States	32 673	115 233	252.7
Venezuela	4 041	10 267	154.1
Average losses			98.5

Taking into account the above-stated, the carried out analysis allows us to come to the conclusions and recommendations as follows.

Conclusions and Recommendations

The goal of this paper has been to elaborate on the equity-efficiency trade-off theory and to consider the relationship between equity and economic efficiency while income distribution. With this purpose, we construct the following groups of models: the models of the relationship between production outcome and proportions of distribution, consumption possibilities curves and the equity-efficiency relationship models at the levels of initial and final distribution.

We extended the application of the equity-efficiency trade-off theory on the initial distribution, which broadens the traditional boundaries of the analysis typically concentrated on redistribution processes. Developing an approach from the standpoint of the equity-efficiency relationship with an extremum, we go beyond the limits of both the traditional equity-efficiency trade-off theory (standing up for the inverse equity-efficiency relationship) and the alternative studies showing that, on the contrary, there is a direct equity-efficiency relationship.

We came to such a conclusion owing to the proposed double criterion of social efficiency. While the Pareto criterion is individualistic, the double criterion is, on the contrary, social by nature. Since a market economy does not ensure fulfillment of the double criterion automatically, its application was considered under conditions of state interference in economy. This was based on the supposition that there are rational expectations in the society allowing participants of production processes to be guided by results of final distribution but not by those of primary distribution.

There are serious theoretical grounds to hold that there is a complex dependence (including parts of direct and inverse relationships) of efficiency on equity characteristic of primary product distribution. Redistribution processes essentially obscure this relationship which may reveal itself in the relationship between macroeconomic indicators or not.

Thus, a new conception of the equity-efficiency relationship was proposed. The only possible way of its verification was empirical verification which was carried out in part B of the paper with the use of the values of the Gini index and GDP per capita.

The paper shows that the relationship between the Gini index and GDP per capita can be well described using the equity-efficiency relationship model with an extremum allowing to achieve the greatest efficiency if the Gini coefficient = 34 (the limit of double criterion). However, this value of the optimal Gini index needs further elaboration based on a wider data set.

The only thing that can be claimed quite definitely is that the traditional inverse equity-efficiency relationship model is unrealistic, reflects only redistributive aspect of this relationship and corresponds to the facts only up to a certain limit. Since this model describes empirical data essentially worse than the proposed equity-efficiency relationship model with an extremum, it is expedient to go over to using the equity-efficiency relationship model with an extremum and to carry out further investigations in order to find the value of the Gini index, corresponding to the potential efficiency of social production, more accurately.

The investigations carried out have also shown that *the double criterion* of efficiency does not hold in all countries in the present-day world. In contrast to the Pareto-efficiency, its fulfillment can be ensured only by conscious actions of the state but is not guaranteed by the market. In our opinion, it is the fulfillment of *the double criterion* that should be taken as a principle for ascribing to a country the attribute of a social market economy. The presence of redistribution processes as such is rather a poor criterion according to which all present-day economies may be called “social”, while at the same time, among them there are economies characterized by extremely high levels of inequality, so high even that it contradicts the purposes of achieving economic efficiency. Such

countries can hardly be correctly awarded the attribute of social market economies in spite of the presence of redistribution processes in them.

One should also note the difference in the position of transition economies. If the double criterion holds in rather a large part of Central European countries, in countries of Eastern Europe and Central Asia either this criterion does not hold or the countries are at the dangerous boundary beyond which the double criterion does not hold. In general, transition countries seem to have used the factor of increasing inequality at least up to the boundary behind which it will not make for increasing efficiency. Conscious observance of the second criterion in these countries will allow to prevent an irrational increase in inequality and a decrease in the efficiency of production, will thus bring about a further movement of transition economies towards social market economy.

The obtained optimum is a boundary between countries which really have to choose between equity and efficiency and countries in which reduction in inequality does not at all mean reduction in efficiency, but, on the contrary, will result in its increase. Accordingly, social and economic policy of inequality regulation is to be corrected. What would be needed is not only accuracy of determining the optimal level of the Gini index but also a more precise measurement of the level of inequality in each country. However, presently these levels differ in different sources even for developed economies not to mention developing or transition economies.

Only exact determination of the position of a country in relation to the optimum can show the necessary direction of social policy. This holds in full measure for inequality estimations for Ukraine which are significantly different one from another. The problem is complicated by a variety of different values of the Gini index for different income concepts. A shift from one concept to another sometimes depends not on requirements of economic analysis but on the wish to put statistical data in a favourable light.

In principle, it is advisable to opt for the indicators most widespread in world practice. As it was mentioned above, such an indicator is the Gini index based on disposable income. On the contrary, Derzhkomstat (Statistical Office of Ukraine) calculates this indicator on the basis of expenditure, which makes it impossible to define the value of the Gini index in Ukraine more precisely in comparison with estimations of international organizations. If the latter are correct, the value of the Gini index in Ukraine corresponds to the average value of the Gini index in OECD countries, and it is close to the optimum calculated according to the equity-efficiency relationship model with an extremum. Therefore, the programs of income redistribution are rather optimal and should not be cut down not only from the standpoint of social protection but also with the aim of maintaining efficiency of production.

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