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Eike B. Kroll • Bodo Vogt

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Otto-von-Guericke-University Magdeburg
Faculty of Economics and Management
P.O. Box 4120
39016 Magdeburg, Germany
http://www.ww.uni-magdeburg.de/

The St. Petersburg Paradox despite risk-seeking preferences: An experimental study

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Abstract

The St. Petersburg is one of the oldest violations of expected utility theory. Thus far, explanations of the paradox aim at small probabilities being perceived as zero and the boundedness of utility. This paper provides experimental results showing that neither risk attitudes nor perception of small probabilities explain the paradox. We find that even in situations where subjects are risk-seeking, the St. Petersburg Paradox exists. This indicates that the paradox lies at the very core of human decision-making processes and cannot be explained by the parameters discussed in previous research so far.

1 Introduction

The St. Petersburg Paradox has attracted various researchers so far and the work provides a puzzle to the very core of economic theories (Cox *et al.*, 2008b). While the St. Petersburg Game in its original version offers an infinite expected value, people are found not to pay more than $25 for participating in the game (Hacking, 1980). Various researchers have provided explanations for the paradox, but with every explanation a new version of the initial game was constructed that brought the puzzle back (Samuelson, 1977).

The first explanation for the observed behaviour was decreasing marginal utility of risk-averse agents (Bernoulli, 1954), however, the game can be constructed correcting for decreasing marginal utility and the paradox remains. Therefore, the focus shifted towards the question of infinity. Limited time was introduced as the factor putting a bound to the utility of the St. Petersburg Game (Brito, 1975) (Cowen & High, 1988). In contrast it was argued, that the utility of the game could in principle be unbounded but the offer is most probably not considered genuine (Shapley, 1977) causing the decision patterns found. The most straightforward solution of the paradox, however, is that utility is bounded since otherwise one can always create lotteries leading to counterintuitive solutions (Aumann, 1977). To avoid infinity the St. Petersburg Game was broken down into a series of finite games, but the paradox still exists (Samuelson, 1960). This fact does not indicate that infinity is the underlying cause of the paradox.

Other work argues that the small probabilities cause the paradox since sufficiently small probabilities are regarded as zero (Brito, 1975) or small chances for large prices create big risks for the agents (Allais, 1952; Weirich, 1984). Another approach on using probabilities as an explanation for the phenomenon, more recent work introduced a new weighting function for Cumulative Prospect Theory solving the problem of infinity (Blavatskyy, 2005; Rieger & Wang, 2006).

In this paper we will report experimental results indicating that risk attitudes cannot explain the St. Petersburg Paradox. In our version of the game we provide a choice situation for waiting time where subjects are found to be risk-seeking (Kroll & Vogt, 2008). We can show that neither risk attitudes nor small probabilities explain the occurrence of the St. Petersburg Paradox. Furthermore, we will argue that bounded utility is not a suitable explanation either.

2 Experiment

The experiment was conducted with 50 students of the University of Magdeburg from different fields of study in at the MaXLab at the Faculty of Economics and Management.

At the beginning of the experiment the participants were paid a show-up fee of 8 Euros. After that the instructions to the experiment were handed out and the participants could make their choices. All subjects had a base waiting time (varying across different treatments) and were offered to participate in a game where this waiting time could be reduced or increased depending on the outcome of the game. This game was designed analogous to the St. Petersburg game. For participation in the game the waiting time was reduced by n minutes and a coin is tossed until tails occurs with a maximum of n tosses. If tails occurs at the i-th toss, the waiting time was increased by 2i minutes. Each participant was offered 9 games with only one choice being realized (Grether & Plott, 1979), with the games differing by the maximum number of tosses (see 1).

Tails occurs the first time at toss no.	Probability	Addtitional waiting time (in minutes)
1	0.5	2
2	0.25	4
3	0.125	8
4	0.0625	16
5	0.03125	32
6	0.015625	64
7	0.0071825	128
8	0.0039065	256
9	0.0019535	512
not at all		+/- 0

Table 1: St. Petersburg Game for waiting time

After the participants made their choices, the experimenter drew a ball from a bingo cage numbered from 1 through 9 determining which choice was selected for realization. If the participant chose not to play the game offered, the base waiting time was realized and started immediately. If the participant chose to play the game, the experimenter tossed the coin as described above and determined the actual waiting time. All participants spent their waiting time in an experimental cabin without communication devices or other kinds of entertainment possibility. To control for reference-dependence of preferences (Köszegi & Rabin, 2007; Farber, 2008), we ran two treatments with different base-waiting times of 10 and 45 minutes.

3 Results

In a previous study on risky decisions over waiting time, we found that subject showed risk-seeking choice patterns (Kroll & Vogt, 2008) as can be seen in 2. That paper elicited risk preferences for waiting time through choices between two lotteries one offering less risk with lower payoffs and higher risk with higher payoffs (Holt & Laury, 2002). Results showed significant risk-seeking behaviour of the subjects.

risk attitude	risk-seeking	not risk-seeking
no. of subjects	27	9

Table 2: Results of Kroll and Vogt (2008)

Knowing the results from that previous study, one can conclude that the subjects in this study would tend to play all of the offered games. The expected value of the offered gambles on waiting times is equal to the base waiting time. Therefore, a risk-seeking individual would choose to participate in all offered gambles. The results of this experiment 3 show, that while individuals do participate in the gambles for small reductions of the base waiting time, they do not for higher possible reductions of the base waiting time.

Reduction of Waiting Time for Participation	1	2	3	4	5	6	7	8	9
Frequency of participation (base waiting time: 10 min)	15	13	13	12	8	2	3	3	4
Frequency of participation (base waiting time: 45 min)	13	12	7	5	2	0	0	0	2

Table 3: Frequencies of participation in the offered games

In the treatment with a base waiting time of 10 minutes, 2 of the 25 participants choose never to play the game, while the rest mostly starts playing the first game, but switches to answering 'no' along the line. None of the participants chooses to play all offered games.

The data from the treatment with a base waiting time of 45 minutes yields similar results. There is a lower number of subjects to play games where high reductions of the base waiting time than observed in the first treatment. However, the difference is not significant on a statistical level.

4 Conclusion

While in earlier studies the St. Petersburg Paradox has been discussed with reference to calibration arguments (Cox *et al.*, 2008a; Cox *et al.*, 2008b), this paper reports decisions in situations where subjects are risk-seeking. The data clearly indicates that the St. Petersburg Paradox still exists although a risk-seeking individual would always be in favour of playing the games offered. Additionally, perceiving small probabilities as non-existent is not a suitable explanation for the paradox since in our game the small probabilities indicate the unfavourable outcome.

In our experiment most of the subjects stop playing before the maximum waiting time reaches 32 minutes. Although one might argue that this is the lower bound on the utility of time, however, a waiting time of 42 minutes cannot be considered as the maximum length of time a person would have. In conclusion, none of factors discussed in the literature with regard to the St. Petersburg Paradox helps to explain the data of our experiment. Therefore, it seems as if the paradox occurs in the very core of human decision-making processes.

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