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Risk-Neutral Monopolists are Variance-Averse.

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Abstract

If the production of a risk-neutral monopolist is influenced by a random variable, then the expected profit is decreasing in the variance of the production process.

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1 Introduction

This paper evaluates the expected profit function of a risk-neutral monopolist who carries out a risky production process. If production is influenced by a random variable, then output and sales price are both random variables. As they are negatively correlated, the monopolist's expected revenue is not just the product of expected output and expected price. Rather, the expected revenue is smaller. The difference between expected revenue on the one hand, and the product of expected price and expected output on the other, increases with the variance of the random variable which influences the production process. Therefore, the expected profit of the monopolist is decreasing in the variance. In other words: A risk-neutral monopolist derives negative marginal utility from variance; he is variance-averse.

2 The model

Consider a monopolist who faces inverse demand $p = a - bQ$, with $a, b > 0$, where p denotes the price and Q the quantity of the good he offers. The monopolist carries out a stochastic production process,[1] described by

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[1]Firms may face other sources of risk besides stochastic production, in particular uncertainty in regulatory policy, see Isik (2005).

$Q(e,\eta) = e + \eta$, where e denotes the amount of the only input factor, and η is a random variable with zero expected value, $\mathrm{E}[\eta] = 0$, and non-negative variance $\mathrm{Var}[\eta] = \sigma^2 > 0$. Hence, $\mathrm{E}[Q] = e$ and $\mathrm{Var}[Q] = \sigma^2$.
Assume that a unit of the input factor costs c, with $0 < c < a$.[2] The monopolist is risk-neutral, in other words: He seeks to maximize his expected monetary profit $\mathrm{E}[\pi(Q)]$; his utility derived from monetary income X is described by the utility function $U(X) = X$ as well as any positive affine transformation of U.
Closer inspection of the expected revenue $\mathrm{E}[p(Q)Q]$ reveals that price $p(Q) = a - bQ(e,\eta)$ and output $Q(e,\eta)$ are both random variables which are negatively correlated. Hence, the expected revenue is not just the product of expected quantity and expected price.

Lemma: *The expected revenue of the monopolist is given by* $E[p(Q)Q] = ae - be^2 - b\sigma^2$.

Proof: $\mathrm{E}[p(Q)Q] = \mathrm{E}[aQ - bQ^2] = a\mathrm{E}[Q] - b\mathrm{E}[Q^2]$. Applying the definition of variance, $\sigma^2 = \mathrm{E}[Q^2] - (\mathrm{E}[Q])^2$, which is equivalent to $\mathrm{E}[Q^2] = (\mathrm{E}[Q])^2 + \sigma^2$, expected revenue can be rewritten as $\mathrm{E}[p(Q)Q] = a\mathrm{E}[Q] - b(\mathrm{E}[Q])^2 - b\sigma^2$. Using $\mathrm{E}[Q] = e$ leads to $\mathrm{E}[p(Q)Q] = ae - be^2 - b\sigma^2$. $\square$

The correlation coefficient between $p(Q)$ and $Q(e)$ is -1, and the covariance is $-b\sigma^2$. The Lemma is helpful to demonstrate the main results of this note:

Proposition: *i) If a risk-neutral monopolist engages in a stochastic production process and faces constant marginal factor cost, then his marginal utility of output variance is negative. ii) The monopolist chooses the same expected output level as in the case with risk-free production.*

Proof: First, it has to be shown that $\partial\mathrm{E}[\pi(Q)]/\partial\sigma^2 < 0$. Choosing his input e, the monopolist maximizes $\mathrm{E}[\pi(Q)] = \mathrm{E}[p(Q(e))Q(e) - ce]$. Using the Lemma, the right hand side can be rewritten as $(a-c)e - be^2 - b\sigma^2$ which implies $\partial\mathrm{E}[\pi(Q)]/\partial\sigma^2 = -b < 0$.
Turning to the second part of the proposition, a monopolist with deterministic production would maximize his profit by choosing an output level $Q^M = \arg\max[(a-c)Q - bQ^2] = (a-c)/2b$, hence $e^M = (a - c/2b)$. The first-order condition for the risk-neutral monopolist with stochastic production is $(a-c) - 2be = 0$ who, thus, would also choose the input level e^M.$\square$

3 Conclusion

If a risk-neutral firm enjoys market power, its firm demand (price-sales schedule) is decreasing. If, moreover, production is stochastic, then the

[2] The factor cost $c(e)$ is no random variable, but the marginal cost is a random variable.

firm derives marginal disutility from the variance of the random variable that influences production.[3] This is a surprising result, as a risk-neutral decision-maker usually would not care for a mean-preserving spread. The reason for this effect is the negative correlation between price and output under imperfect competition.[4] The marginal disutility of variance is $-b$, i.e., the slope of the demand curve in the case of a linear demand function.

In a mean-variance-diagram, a risk-neutral decision-maker has horizontal indifference curves when facing a choice between risky assets. A risk-neutral monopolist who faces a choice between different risky production processes would have upwards sloped iso-profit curves in such a diagram: When accepting a higher variance, he would require higher expected revenues (e.g., a higher demand) in order to maintain a specific profit level. His iso-profit curves, thus, appear as if he was risk-averse,[5] because a higher variance negatively affects the monopolist's expected profit.

The optimal input decision of the monopolist is unaffected as long as the variance of the random variable which influences production is independent of the input level.[6] These conjectures regarding the iso-profit curves and the output choice may prove helpful when trying to test the model empirically. One interpretation of the stochastic production model could be a principal-agent-model of the moral hazard type. Under moral hazard, the risk-neutral principal is unable to verify the agent's effort. One way to include this into a model would be a production function with two arguments, agent's effort and a random variable.[7] This approach would directly lead to the problem considered here: If, from the principal's point of view, production is stochastic, and he enjoys price-setting power on the output market, then sales price and output are negatively correlated random variables, and the expected revenue had to be adjusted accordingly.[8] If the variance in production is only due to the fact that production has to be carried out by an agent, this

[3] Holden and Subrahmenyam (2003) start with the assumption that monopolistic traders are risk-averse and analyze a dynamic model. Yin (2008) studies the behavior of risk-averse monopolists under a two-part tariff.

[4] The results obtained for the case of a monopolistic firm could also be applied to oligopolists producing under risk.

[5] Risk-aversion in the CAPM is extensively analyzed by Schneeweiss (2003).

[6] Kaniovski (2003) derives that risk-averse monopolists who decide with regard to aspiration levels would choose lower output levels, and higher prices than their risk-neutral counterparts.

[7] See, e.g., the initial discussion in Lazear and Rosen (1981, 843) on piece-rates as an incentive contract for single agents. The authors assume that the employer operates in a competitive product market and, thus, receives a constant price for each output unit. Gibbons (1987, 418) implicitly assumes the output unit price to be one. Thus, in these model no (negative) correlation exists between stochastic output and market price.

[8] With the typical assumption of quadratic effort cost on the agent's side, the optimal solution would be $e = a/2(1 + b)$; leaving this modification aside, all results stated in the Lemma and the Proposition would still hold.

could be seen as a new, and up to now unexplored, source of agency cost.[9] Another application of the model would be the derivation of an optimal specific investment into a monopoly position with risky production. Neglecting the variance effect would lead to an overestimation of the expected revenue (and profit) from such an investment.

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[9]See Jensen and Meckling (1976) the conventional sources of agency cost.