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Abstract

The efficient supply of spare parts is of prime concern for OEMs. Next to the traditional spare parts sources in form of final order and remanufacturing, the option to buy back broken products prevents the OEM from fulfilling his spare parts availability obligation in the end-of-life phase and increases his ability to remanufacture. This contribution seeks to identify optimal buy-back strategies for different settings regarding information availability and buy-back flexibility. A numerical study analyzes circumstances under which buy-back is especially beneficial for the OEM.

Keywords: Inventory Management, Spare Parts Management, Reverse Logistics, Buy-back

1 Introduction

In recent years, original equipment manufacturers (OEMs) of durable goods identified the after-sales market as one of their key business segments. For instance, Cohen et al. (2006) provide results of a 1999 AMR Research report stating that by being active in the aftermarket businesses could generate about 45% of their gross profits. Furthermore, by efficiently handling the supply of spare parts, a competitive advantage can be established if the OEM provides a superior service to his customers, e.g. by guaranteeing the availability of spare parts during a comparably long service period. Thus, the length of the service period becomes an important strategic parameter for management. This period is subdivided into two distinct phases, namely the normal phase and the final phase. During the normal phase the primary product is manufactured and sold to the customers. The final phase starts when serial production ceases and it lasts as long as spare parts availability is guaranteed. Therefore, it is often considerably longer than the production period. In the automotive sector, for instance, the final phase usually lasts for 10-15 years. However, several OEMs provide a significantly longer availability for their spare parts as the example of a 30 years service period for Mercedes-Benz cars indicates.

In a recent paper, Kim and Park (2008) propose a model that allows to determine the optimal length of the final phase. They argue that the marketing department seeks to stimulate demand by offering a long period with guaranteed spare parts availability as this signals a high quality of the product (see, e.g., Spence, 1977; Gal-Or, 1989). Obviously, if the final phase would be determined without such considerations by only accounting for the operational costs and revenues of service, it would often be chosen considerably shorter. Our research basically focuses on situations in which both perspectives (marketing and operations) yield large differences in the length of the final period and we propose an efficient method for spare parts management under those circumstances.

From the OEM's perspective, inventory management for spare parts differs considerably from inventory management applied to manufacturing processes, mainly because demand for spare parts is less predictable and highly dynamic on a comparably low level (see, e.g., Kennedy et al., 2002; Huiskonen, 2001). In addition, options for resupply become increasingly rare during the final phase. While in the normal phase production facilities of the primary product can be used, this efficient sourcing option is often no longer recommendable in the final phase due to high fixed costs incurred for a relatively small output. Thus, a frequently adopted strategy is to place a final order at the time when regular production comes to an end. However, this is connected with high stock levels resulting in large holding cost and a high obsolescence risk as all demands occurring in the final phase need to be estimated beforehand. Extra production represents an additional option in the final phase which in contrast to regular production is typically performed in small lots. Nonetheless, this option is under certain circumstances prohibitively expensive or technically infeasible (see Hesselbach et al., 2002, for a comprehensive overview on available options).

Obviously, there is a one-to-one correspondence between a spare part and the broken component. This creates the opportunity to recover the broken part for later use as a spare part. Part recovery, hence, can complement other sources of spare parts supply. An overview on different recovery processing options is provided by Thierry et al. (1995) including repair, refurbishing, and remanufacturing. Although all of these options can be applied in principle to satisfy an existing spare part demand, this contribution focuses solely on remanufacturing processes. Remanufactured parts are considered to be as-good-as-new and OEMs frequently offer the same warranty as for new parts. Compared to extra production, remanufacturing is relatively cheap, but since not all broken parts might be remanufacturable it should be accompanied by other options to avoid shortages (see, e.g., Inderfurth and Mukherjee, 2008; Inderfurth and Kleber, 2009).

In case of not being able to fulfill occurring spare part demands and in order to avoid a penalty or a goodwill loss, further options the OEM can offer to his customers range from swapping to buy-back. Swapping refers to a replacement of the dysfunctional product with a new generation product free of charge for the customer (as has been reported by Pourakbar et al., 2008). This option is favorable for high tech products experiencing a considerable price deterioration between successive product generations but it is less beneficial for durables. Buy-back of products is typically performed in practice in form of trade-in campaigns. These campaigns, though, foremost intend to increase the sales of new products and thus both functional and broken products are accepted. Although there are many examples from industry (see, e.g., Ray et al., 2005), an acquisition of recoverable parts for satisfying spare part demands is (at best) seen as a side effect and is hence not explicitly stated as motivation for such a campaign.

In our study, however, we emphasize the use of more focused trade-in campaigns which explicitly aim to control the OEM's supply of recoverable parts. In doing so, we abstract from the above mentioned sales promoting effects for other products and isolate the sole effect of buying back broken products on spare parts management.

In particular, we are interested in those conditions under which buying back broken products for obtaining spare parts profitably complements the traditional sourcing options final order and remanufacturing. This could for instance be accomplished by using the already existing service network which provides the OEM with a direct access to his customers demanding spare parts.

An active integration of buying back used products into a generic product recovery system has been examined by Minner and Kiesmüller (2002) in a deterministic setting with a stationary price-response function. There, the effects of the acquisition decision on current and future demands are neglected. In our case, however, buying back would on the one hand decrease current and future demands for spare parts since there will be no future spare part demand generated from a bought back product. On the other hand, customers with a dysfunctional product might accept a comparably low compensation yielding a cheap supply of recoverable parts for the OEM. Therefore, the trade-off between cannibalizing current and future demands to release oneself from the obligation to provide spare parts and creating an additional source of supply for satisfying the remaining demand represents the main focus of this work.

The profitability of the buy-back option depends on constraints on price and quantity decisions but also on the availability of required information. First, the OEM might be able to approach different customers in a specific way. In the marketing literature, a number of market-segmentation approaches are discussed (see, e.g., Kotler and Keller, 2008; Wedel and Kamakura, 2000). Especially, it is argued that one can segment the market by observable and unobservable characteristics. Observable criteria for segmenting customers are mostly geographic or demographic data. Here, one might additionally segment on type of relationship, for instance B2B (car rental enterprise) or B2C (private customer). Criteria that are unobservable typically contain psychographic or behavioral characteristics.

Furthermore, the OEM might be restricted in his flexibility to price-discriminate between customers because of legislation like the Robinson-Patman act in the US. We refer to Bernstein et al. (2006) for a more comprehensive motivation for simple pricing schemes. Finally, the OEM might have no control over the buy-back quantity once he offers a price. This might be the case because he communicates a buy-back campaign in the mass-media. Additionally, a quantity restriction of buy-backs for the decentralized repair shops might not be realizable as the demand at each facility is unknown or uncertain in advance.

The remainder is organized as follows. In Section 2 we introduce a basic mathematical model on how to incorporate buy-backs in the decision-making process and state its main assumptions. Afterwards, Section 3 analyzes a base case scenario and elaborates possible benefits from segmenting the OEM's customers into distinct groups. The fourth section elaborates the critical assumptions made in the basic model and shows how to adapt it to be able to deal with additional constraints and limited information availability as described above. Furthermore, the base case parameters set in Section 3 are critically reviewed in a sensitivity analysis. Finally, Section 5 summarizes the main conclusions and gives some directions for future research.

2 A basic model with buy-back

We consider a single product for which the OEM guarantees the availability of spare parts during the final phase of service. The planning horizon of length T starts at the end of regular production, i.e. at the time when no further products are manufactured to be sold. Thus, at this point in time the number of products with the customers (which we will refer to as the install base) no longer increases. For the sake of simplicity, the considered product includes only one vital component that can fail and needs to be replaced by a spare part to restore its functionality. Otherwise, the product's value would reduce considerably. Failures occur deterministically with rate λ, i.e. each period a fraction of the install base requires spare parts to replace the broken components. This is accomplished by the existing service network operated by the OEM which is also used to return broken components to a remanufacturing facility.

In this contribution, we focus on the spare parts supply system depicted in Figure 1. The notation used is summarized in Table 1. Demand for spare parts is satisfied from a central stocking point. Let B_t^S denote the OEM's spare parts stock at the end of period t. The OEM can replenish this inventory using two different options. At the beginning of the planning period, he places a final order FO at unit cost c_f. Afterwards,

Table 1: Notation used

Parameters	
n	Number of customer segments
T	Planning horizon
c_r	Unit cost of remanufacturing
c_f	Final order unit cost
h^S	Spare parts holding costs per unit and period
h^R	Returned products holding costs per unit and period
p_i	Reservation price in customer segment i
p_s	Revenue per spare part sold
q	Remanufacturing yield rate
λ	Components failure rate
r	Interest rate
$\bar{y}_i$	Initial product stock in customer segment i
$\bar{B}_0^R$	Initial stock of broken products
$\nu_{i,t}$	Percentage of products leaving the OEM's access of segment i in period t
Decision and state variables	
B_t^S	Spare parts inventory at the beginning of period t
B_t^R	Recoverables inventory at the beginning of period t
FO	Size of the final order
R_t	Number of remanufactured parts in period t
D_t	Number of broken products disposed of in period t
E_t	Fulfilled spare part demand in period t
$x_{i,j,t}$	Number of broken products bought back from segment i at price p_j in period t
$y_{i,t}$	Number of customers in segment i in period t
$\Theta_{i,t}$	Binary pricing variable for customer segment i in period t

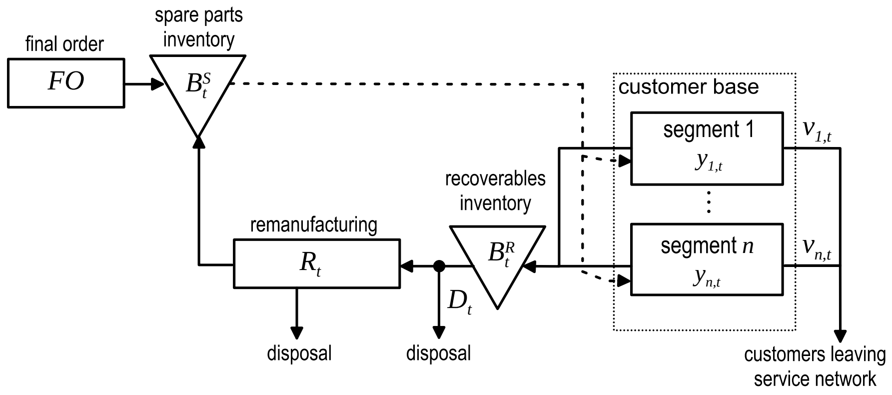

Figure 1: Spare parts supply system

regular production ceases and remanufacturing broken components from the stock of recoverables B_t^R becomes the only sourcing option. The parameters h^R and h^S represent the unit holding cost for broken parts and spare parts per period, respectively. In each period t, the OEM must decide on the amount of broken components that he would like to remanufacture R_t at unit cost c_r and on the quantity of broken components to be disposed of D_t. As it is commonly presumed for practical applications, we suppose that revenues for recovering material and costs of extracting materials are about the same size which means that the disposal costs are negligible. Due to an imperfect remanufacturing process only q% of the remanufactured products fulfill the designated quality standards to be sold as spare parts. All costs and revenues are discounted by the interest rate r.

Both replenishment options exhibit considerable disadvantages. The final order bears the burden of holding spare parts over a long period of time and the option of remanufacturing broken parts cannot provide all spare parts demanded due to the imperfect remanufacturing process. An appropriate way to overcome these deficiencies would be to include the buy-back of broken products as another option. If the OEM decides to buy back, he loses a revenue of p_s per spare part that would be sold otherwise but he also increases the recoverables stock since an additional broken component (included in the product bought back) is returned to the OEM. The compensation paid to the customer to persuade her to sell her broken product depends on her valuation of the product. For this, we assume that all customers value their product differently, but this valuation does not change over time. Different buy-back prices, thus, yield different quantities and decisions upon both must be made simultaneously. In contrast to other approaches (see, e.g., Minner and Kiesmüller, 2002) where a given functional relationship does not change over time, in our long range approach buy-back decisions impact the composition of the install base and therefore, change conditions relevant for later decisions.

For the OEM, individual information upon each customer's valuation for a broken product might hardly be obtainable. He therefore segments his customers into groups $i = 1, ..., n$ in which all customers value their product similarily. The number of functioning products in each customer segment i at the end of period t is denoted by $y_{i,t}$. It is assumed that the initial size of each segment $\bar{y}_i$ is known in advance, and independent of any of the OEM's decisions a fraction $\nu_{i,t}$ of all products in a customer segment i leaves the service network as they are, for example, salvaged at a breakers yard. Let p_i denote the reservation price of all customers in segment i representing their valuation of a defective product. Without loss of generality, the customer segments are arranged such that the inequality $p_1 < ... < p_n$ is satisfied. It is easy to see that only these prices are relevant for the buy-back decision. If the OEM would propose a price to a segment that lies between two adjacent reservation prices, he could easily reduce this price to the lower of the two reservation prices while still being able to acquire the same quantity.

In an idealized setting (denoted by M1) the OEM can decide for each segment separately on which quantities he wishes to buy back for which price. This requires, that the OEM can assign each customer to her segment, i.e. that individual information is available on all customers. Buy-back quantities are denoted by $x_{i,j,t}$ representing the number of broken products bought back from customer segment i at price p_j in period t. Consequently, the amount of broken products that is bought back reduces the number of spare parts sold in period t which will be denoted by E_t. Additionally, the OEM needs to determine the size of the final order FO and in each period t he decides on the number of remanufactured R_t and disposed of parts D_t. Problem M1 can be formulated as follows:

$$
\begin{aligned}
\max \Pi_1 &= -c_f \cdot FO + \sum_{t=1}^{T} (1+r)^{-t} \left[p_s \cdot E_t - c_r \cdot R_t - h^R \cdot B_t^R - h^S \cdot B_t^S - \sum_{i=1}^{n} \sum_{j=1}^{n} x_{i,j,t} \cdot p_j \right] && (1)\\
\text{s.t.} & && \\
E_t &= \lambda \sum_{i=1}^{n} y_{i,t-1} - \sum_{i=1}^{n} \sum_{j=1}^{n} x_{i,j,t} \quad t = 1, ..., T && (2)\\
B_t^S &= B_{t-1}^S - E_t + q \cdot R_t \quad t = 1, ..., T && (3)\\
B_0^S &= FO && (4)\\
B_t^R &= B_{t-1}^R - R_t - D_t + \lambda \cdot \sum_{i=1}^{n} y_{i,t-1} \quad t = 1, ..., T && (5)\\
B_0^R &= \bar{B}_0^R && (6)\\
y_{i,t} &= y_{i,t-1} \cdot (1 - \nu_{i,t}) - \sum_{j=1}^{n} x_{i,j,t} \quad i = 1, ..., n \quad t = 1, ..., T && (7)\\
y_{i,0} &= \bar{y}_i \quad i = 1, ..., n && (8)\\
\sum_{j=1}^{n} x_{i,j,t} &\leq \lambda \cdot y_{i,t-1} \quad i = 1, ..., n \quad t = 1, ..., T && (9)\\
x_{i,j,t} &= 0 \quad i, j = 1, ..., n \quad j < i && (10)
\end{aligned}
$$

$$
B_t^S, B_t^R, E_t, R_t, x_{i,j,t}, y_{i,t} \geq 0 \quad i, j = 1, ..., n \quad t = 1, ..., T \qquad (11)
$$

The OEM maximizes his discounted profit Π_1 which includes expenses for producing the final order as well as each period's net cash flow consisting of the revenue of selling E_t spare parts minus the cost incurred for remanufacturing, stock-keeping, and buy-back. Constraints (2)-(11) are interpreted as follows. The number of spare parts sold to the customers E_t is determined in (2) by the number of products that break down in t reduced by the amount of broken products the OEM buys back. Constraints (3) and (5) are inventory balance equations for the spare parts and recoverables inventory with initial levels set in (4) and (6). The initial spare parts stock equals the size of the final order. The stock of spare parts at the end of period t B_t^S is determined by the stock at the end of the previous period B_{t-1}^S reduced by the fulfilled spare parts demand E_t plus the yield from the remanufacturing process $q \cdot R_t$. Starting from an initial value $\bar{B}_0^R$, the stock of recoverables is reduced in each period by the number of remanufactured R_t and disposed of parts D_t and increases by the number of broken products that return to the OEM.

Table 2: Base case parameter values

n	T	$\bar{y}_1$	ν_1	λ	p_s	c_f	c_r	q	r	h^S	h^R	p_1
1	80	400	1.5%	10%	10	3	1.5	50%	2.5%	0.2	0.1	20

The development of the number of products in each customer segment is given in balance equation (7) while (8) represents the initial size of each segment. The segment size reduces by the exogenous drain of leaving customers $y_{i,t-1} \cdot (1 - \nu_{i,t})$ and the total number of bought-back products from that segment. Constraint (9) ensures that the number of bought-back products from customer segment i must not exceed the number of broken products in the respective period. By establishing logical constraint (10) it is guaranteed that no buy-back occurs for a lower price than the segment's reservation price. For instance, the OEM cannot acquire any broken product from segment 2 for the price p_1 since this would not be sufficient. The non-negativity restrictions (11) assure validity of decisions.

In the idealized setting it can be easily seen that it is not optimal to buy back products for a different price than the segment's specific reservation price. Thus, an optimal solution of M1 will always show $x_{i,j,t} = 0$ for $i \neq j$.

3 The value of buy-back under idealistic conditions

3.1 Base case parameters

In this section, an example is used to illustrate the potential benefit of buying back broken products and to elaborate the gains of a more detailed customer segmentation. The respective parameter values of the base case scenario are summarized in Table 2.

We start our analysis with a single customer segment ($n = 1$) for which all spare part demands must be satisfied for the next 80 periods. A period is hereby defined to be a quarter of a year which means the OEM faces a 20 year planning horizon. The OEM estimates the initial number of products in the install base to be $\bar{y}_1$=400 out of which a fraction of $\nu_1 = 1.5\%$ are leaving the service network each period. The main component fails at a rate $\lambda = 10\%$, i.e. each product has to be repaired on average once in two and a half years. Each spare part demand satisfied yields a revenue of

Table 3: Optimal final order FO, discounted profit Π, relative profit surplus Δ and first period in which buy-back takes place z in the benchmark solution and M1

Benchmark		Optimal buy-back in M1			
FO_{BM}	Π_{BM}	FO_1	z_1	Π_1	Δ_1
935	2390	658	46	3127	+30.8%

p_s=10. The OEM estimates that each broken product can be bought back for a price of $p_1 = 20$ being twice the revenue from selling a spare part. Spare parts are procured by placing a final order at unit cost c_f=3 yielding an initial profit margin of 70%. All products returning to the OEM will be remanufactured at unit cost $c_r = 1.5$. It is assumed that remanufacturing is successful in $q = 50\%$ of the cases, i.e. only one of two broken parts can be used further. Thus, there is no direct cost advantage for neither parts procured in the final order nor for parts succesfully remanufactured. The discount rate is set to $r = 2.5\%$ per quarter or about 10% per year. Out of pocket holding cost are $h^S = 0.2$ and $h^R = 0.1$ per unit and period for spare parts and recoverable parts, respectively. Taking both discounting and holding cost into account, it would be economically beneficial to satisfy demand from parts procured in the final order for at most 20 periods (5 years) and then to switch to remanufacturing. Hence, the base case parameters depict the situation motivated in the introduction, i.e. the operations manager is confronted with a much longer final phase than he would choose individually.

3.2 The value of buy-back without segmentation

By inserting the parameter values into model M1 outlined in Section 2, the optimal solution was obtained by using the optimization software CPLEX 11 and compared with a benchmark solution that does not allow for buying back broken products. This has been accomplished by forcing all buy-back quantities $x_{i,j,t}$ to zero. The main results are shown in Table 3.

The benchmark solution shows a structure where (as has been examined in a related approach by Kleber and Inderfurth, 2007) there are two phases to be distinguished. In a first phase (periods 1 to 29) the demand for spare parts is satisfied from the final order of size $FO_{BM} = 935$. All broken parts that return are held in the recoverables inventory and none is disposed of. In the second phase (periods 30 to 80) the strategic stock of returned products built up in the first phase is used to serve the entire demand by remanufacturing broken parts from the recoverables inventory. Thus, the size of the final order equals that part of total demand over the planning horizon which cannot be satisfied by remanufacturing. The benchmark solution to the base case scenario yields a total profit of 2390.

When including the buy-back option, the final order reduces to 658 implying a substantial reduction in holding cost. Although considerably shorter (the first phase ends in period 19), both of the above phases are found as well in the optimal solution. In an adjacent third phase (starting in period $z_1 = 46$), the OEM buys back as many products as are needed to satisfy demand. Interestingly, no stock is build up in the recoverables inventory during that phase since all returns are instantly remanufactured. Hence, each period's buy-back quantity is set to just compensate the yield loss. The discounted profit of the base case scenario increases by about 31% to 3127 when buy-backs are included. For a detailed description of the policy structure see the Appendix.

3.3 The value of customer segmentation

This subsection broadens the above analysis by allowing the OEM to segment the install base w.r.t. differences in the customers' valuation of the product. This analysis might provide managers with valuable insights on how much effort they should invest in segmenting the install base.

In order to keep the results consistent, the only difference between customer segments is the buy-back price. All other parameters remain the same as in the base case, e.g. the fraction of customers leaving the service network ν_i is kept at 1.5 % for all segments i. For determining the segment specific buy-back prices it is assumed that the willingness to accept a buy-back, i.e. the reservation price, is uniformly distributed among the 400 customers within an interval between 0 and 20. Given n segments, $400/n$ customers with the lowest reservation price are assigned to the first segment, the next $400/n$ customers to segment 2, and so on. Each buy-back price, thus, indicates the value for which all customers of a respective segment would sell their broken

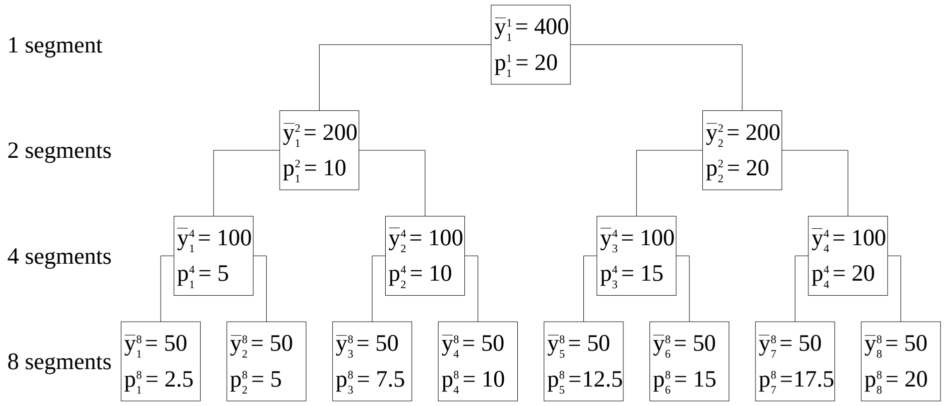

Figure 2: Initial segment sizes $\bar{y}_i^n$ and corresponding buy-back prices p_i^n for different numbers of segments n

Table 4: Influence of the number of segments n on the final order and discounted profit

n	FO_1	z_1	Π_1	Δ_1
1	658	46	3127	+30.8%
2	621	41	3383	+41.5%
4	592	38	3514	+47.0%
8	582	36	3578	+49.7%
16	576	35	3610	+51.0%
32	573	35	3626	+51.7%
64	573	35	3634	+52.1%

products. In the first segment it would be $p_1^n = 20/n$, in the second one $p_2^n = 2 \cdot 20/n$, and so on. The segmentation of customers is sketched in Figure 2.

Table 4 depicts the results of the experiments which can be interpreted as follows. As the solution to M1 can react more flexible, the profitability of the buy-back option increases as more different segments have been identified. That is because a more precise fragmentation of the install base allows the OEM to approach each customer's actual reservation price. If there is only a rough segmentation of the install base, the OEM offers some customers too high prices since they would have also sold their broken product for a much lower price. However, it can be seen that the additional benefit of a more detailed segmentation does decrease as more different segments are established.

4 Robustness with respect to critical assumptions and parameters

This section deals with more realistic conditions than those required for the basic model. First, we delineate potential problems when approaching customer segments individually, and assess secondly the impact of such deficiencies on the profitability of the buy-back option.

4.1 Critical assumptions

While analyzing the model context, a subset of problems can be established that arises due to possibly existing exogenous constraints, such as communication, information, and pricing constraints. Communication constraints will be analyzed both from an external as well as an internal point of view. The internal view refers to an internal communication within the OEM's service network. Thus, the OEM is able to approach each customer individually to offer her a buy-back and has therefore the flexibility to decide on the quantity he buys back in each planning period. The external view, on the other hand, corresponds to a setting in which the OEM communicates the buy-back offer to all customers simultaneously via a mass-media marketing campaign. As the OEM cannot withdraw his offer, he has to accept all broken products the customers wish to sell. Whether the communication focuses on his service network or his customers, thus, determines the OEM's buy-back *quantity flexibility*.

The OEM can face furthermore *information constraints*, if he cannot assign a customer to her corresponding segment and does hence not know from which segment he bought back a broken product. This will typically be the case when the segmentation is based on unobservable criteria, such as psychological or behavioral characteristics (see Kotler and Keller (2008), Chapter 8).

Finally, the OEM can face limited *pricing flexibility*. Pricing constraints describe the OEM's restriction to address each segment independently, i.e. his pricing flexibility is restricted. Therefore, the OEM might be limited to set only one price per period. In this case, he is not able to buy back products from different segments for different prices in a given period. Bernstein et al. (2006) discuss reasons why the OEM might

Table 5: Three dimensions of flexibility and information availability

<table>
<tr><td colspan="2" rowspan="2"></td><th colspan="4">quantity flexibility</th></tr>
<tr><td colspan="2">yes</td><td colspan="2">no</td></tr>
<tr><td></td><th>individual information</th><td>available</td><td>not avail.</td><td>available</td><td>not avail.</td></tr>
<tr><th rowspan="2">pricing flexibility</th><td>yes</td><td>full pricing and quantity flexibility, full information availability (M1)</td><td colspan="3"></td></tr>
<tr><td>no</td><td>limited pricing and full quantity flexibility, full information availability (M2)</td><td>limited pricing and full quantity flexibility, limited information availability (M3)</td><td colspan="2">limited pricing and quantity flexibility, limited information availability (M4)</td></tr>
</table>

be restricted in his pricing format, e.g. the Robinson-Patman act.

These three dimensions, namely pricing and quantity flexibility as well as individual information availability result in eight subclasses of problems (see Table 5). However, it can be shown that several subclasses are redundant (shaded cells). First, the OEM cannot exploit any pricing and quantity flexibility if he cannot assign his customers to the respective segments (as every customer will apparently claim that she has a high reservation price). Second, we argue that the OEM is only able to communicate one buy-back price per period directly to all his customers if the external view of communication prevails. Hence, if the pricing flexibility acts as an additional constraint only one case needs to be analyzed regardless the information availability. Because of the limited buy-back quantity flexibility, the OEM is required to buy back all products the customers intend to sell. As the OEM is, by assumption, able to estimate the total size of each customer segment, he has no advantage of assigning the customers to the segments since he cannot utilize this information satisfactorily. Yet, there is the possibility to advertise ‘up-to’ prices. This case, however, could be treated in the same way as the internal view.

4.2 The economic impact of critical assumptions

In order to assess the economic impact of the assumptions made the model M1 needs to be adapted. Subsequently, we describe the required changes.

The second setting M2 is characterized by less flexibility than M1 due to its restricted pricing flexibility. This is supported by the fact that only a single buy-back price can be set in each period. However, in this setting it is still possible to assign each customer to her segment and to choose which quantity to buy from which customer segment. The OEM's pricing decision is described by an additional binary decision variable $\Theta_{i,t}$ that determines which buy-back price the OEM sets in period t. It is 1 if the buy-back price p_i is offered and 0 else. The following constraints must be added to M1:

$$\sum_{i=1}^{n} \Theta_{i,t} \leq 1 \quad t = 1, ..., T \tag{12}$$

$$x_{i,j,t} \leq M \cdot \Theta_{j,t} \quad i, j = 1, ..., n \quad t = 1, ..., T \tag{13}$$

$$\Theta_{i,t} \in \{0, 1\} \quad i = 1, ..., n \quad t = 1, ..., T \tag{14}$$

Restriction (12) ensures that at most one buy-back price is chosen in a period (or none if there is no buy-back). Constraint (13) (with M being a sufficiently large number) is required to ensure that products from segment i can only be bought back for a sufficient price which may also be higher than the corresponding segment's price. Constraint (14) ensures that Θ can only be 0 or 1. Obviously, due to the additional restrictions imposed the profit of M2 (denoted by Π_2) cannot exceed the profit of M1.

In the third setting M3, the OEM can again only set a single price in each period and he can also choose upon the quantity to take back. However, the absence of available information regarding each customer's assignment results in the problem that it cannot be easily determined how many items were bought back from which customer segment. Hence, further assumptions are required in order to keep track of the number of customers in each segment. However, it is easy to conclude that the profit of this setting must lie between the profits of the less restricted setting M2 and the even more restricted setting M4. A more detailed analysis of this setting will be left for future research.

Setting M4 provides us with the least flexible environment that still allows for customer segmentation. Due to its limited pricing flexibility only a single price can be selected per period, but since this price is externally communicated, all customers for which the offered price exceeds their reservation price return their dysfunctional product to the OEM. Constraint (13) must hence be replaced by (15).

$$\lambda \cdot y_{i,t-1} - x_{i,j,t} \leq M \cdot (1 - \Theta_{j,t}) \quad i, j = 1, ..., n \quad i < j \quad t = 1, ..., T \tag{15}$$

Constraint (15) captures the fact that for a given buy-back price p_j (i.e. $\Theta_{j,t} = 1$) all customers from segments $i = 1, ..., j - 1$ are going to sell their broken products.

By solving the respective optimization problems M2 and M4 for n=2 segments the economic impact of the assumptions regarding pricing and quantity flexibility as well as information availability can be evaluated. In Table 6, the total discounted profits and the final order sizes are presented for each of the different settings. Interestingly, while showing in general the same solution structure with three phases as M1, the third (buy-back) phase of both settings M2 and M4 is characterized by switching price decisions. While in most periods the low price p_1 is set and broken products from the first customer segment are bought back only, sporadically the price p_2 is set. In those 'campaign' periods a stock of broken products is build up, i.e. more broken products are bought back than are actually needed to satisfy the current period's demand. For a detailed description of the policy structure see the Appendix.

The comparably small gap between M1, M2, and M4 can be explained by the similarity of the optimal solution structures. Firstly, it can be observed that the different assumptions do not influence the size of the final order substantially. Secondly, a variation in the model assumptions results in changes in the optimal solution structure that occur quite late in the planning horizon. As all cash flows are discounted, a deviation in one of the later periods does therefore only have a limited effect on the total discounted profit. Although M3 is not explicitly treated, interpreting the optimal ob-

Table 6: Total discounted profit, relative deviation from M1 and corresponding final order sizes.

	Benchmark	M1	M2	M4
Total discounted profit Π	2390	3383	3358	3343
Relative deviation from Benchmark Δ	–	+41.6%	+40.5%	+39.9%
Final order size FO	935	621	622	626

jective values of M2 as a lower and of M4 as an upper bound of the optimal objective value analogous results are to be expected for the not explicitly modeled setting M3. Thus, a main insight from this example is that the OEM can significantly enhance his performance by including buy-backs into his decision-making process even with only limited pricing and quantity flexibility and information availability.

Since we only dealt with a single example so far, the following subsection conducts a sensitivity analysis to provide insights into the robustness of our findings.

4.3 Sensitivity to changing parameters

Taking the base case from Section 3 with two segments as starting point, a sensitivity analysis is performed that focuses on the question under which parameter combinations the buy-back option appears to be especially valuable. To achieve this, all relevant parameters are modified to a considerably higher and lower value while keeping all other parameters constant. Since we did not find a substantial difference for the settings M2 and M4, we restrict our discussion to a comparison of M1 and the benchmark solution without buy-back. The corresponding results for M2 and M4 can be found in the Appendix. Table 7 presents those parameters that seem to have a substantial impact on the profitability of the buy-back option, i.e. the remanufacturing yield rate q, the interest rate r, the final lot unit cost c_f, the length of the planning horizon T as well as both holding cost parameters h^R and h^S.

These findings can be explained as follows. In the benchmark setting, the spare part demand can only be satisfied by two options, either by spare parts from the final order or by remanufacturing. As serving customers close to the end of the planning horizon becomes more and more expensive, the benchmark solution worsens as the

Table 7: Optimal final order FO, discounted profit Π, first buy-back period z and relative profit change Δ in the benchmark solution and M1 for parameters with significant impact.

		Benchmark		Optimal buy-back in M1			
		FO_{BM}	Π_{BM}	FO_1	z_1	Π_1	Δ_1
base case		**935**	**2390**	**621**	**42**	**3383**	**+41.6%**
	40%	1122	836	689	36	2415	+188.7%
q	**50%**	**935**	**2390**	**621**	**42**	**3383**	**+41.6%**
	60%	748	3821	541	47	4396	+15.1%
	1.25%	935	4142	758	55	4513	+8.9%
r	**2.5%**	**935**	**2390**	**621**	**42**	**3383**	**+41.6%**
	5%	935	567	462	29	2287	+303%
	1.5	935	3793	724	48	4369	+15.2%
c_f	**3**	**935**	**2390**	**621**	**42**	**3383**	**+41.6%**
	4.5	935	986	553	36	2510	+154.4%
	60	795	3156	628	42	3454	+9.4%
T	**80**	**935**	**2390**	**621**	**42**	**3383**	**+41.6%**
	100	1039	1644	610	41	3371	+105%
	0.15	935	2868	652	45	3604	+25.7%
h^S	**0.2**	**935**	**2390**	**621**	**42**	**3383**	**+41.6%**
	0.25	935	1912	587	39	3185	+66.6%
	0.05	935	3210	674	47	3816	+18.9%
h^R	**0.1**	**935**	**2390**	**621**	**42**	**3383**	**+41.6%**
	0.15	935	1789	576	38	3108	+73.7%

final order becomes larger compared to setting M1. For instance, this is the case if the remanufacturing yield rate q is low and if the interest rate r, the final order unit cost c_f or one of both holding cost parameters become larger. A larger h^R, for instance, means that the remanufacturing operations could have started earlier which reduces the number of spare parts procured in the final order. However, due to its limited flexibility the benchmark solution cannot react appropriately and is therefore less profitable than setting M1. Regarding the length of the planning horizon, it can be said that a longer planning horizon reduces the total profits of the OEM if he does not account for the buy-back option. In turn, incorporating the buy-back option into the spare parts fulfillment strategy allows the OEM to offer even longer service periods while keeping the costs for this additional service at an adequate level.

Table 8 presents those parameters that change the advantageousness of the buy-back option only slightly, i.e. the outflow rates ν_1 and ν_2, the buy-back prices p_1 and p_2, and the initial segment sizes $\bar{y}_1$ and $\bar{y}_2$. It can be seen that a decreasing outflow from one of the customer segments improves the relative performance of M1 slightly. This is because the less flexible benchmark solution needs to increase the final order while M1 can react by buying back more broken products. The influence of buy-back prices appears to be relatively small as well. The larger one of these prices is, the smaller the possible gain becomes. The buy-back price effect shows its impact also when the initial assignment of customers to segments is changed while keeping the total number of customers constant at 400. If, for instance, the initial install base in segment one comprises 300 customers while it contains only 100 in the second segment, the average buy-back price will decrease as p_1 and p_2 remain at 10 and 20, respectively. Interestingly, the deviation Δ_1 remains constant if the number of customers in each segment is multiplied by the same factor.

Finally, other parameters that do not influence the outcome significantly need to be mentioned as well. Among these parameters, the failure rate λ can be found. The numerical investigation has revealed that a change in the failure rate does not have a large impact on the profitability of the buy-back option as all decisions are increased or decreased approximately proportionally. This means that for $\lambda = 5\%$ the size of the final order and all subsequent decisions decrease to about half of their

Table 8: Optimal final order FO, discounted profit Π, first buy-back period z and relative profit change Δ in the benchmark solution and M1 for parameters with relatively small impact.

		Benchmark		M1			
		FO_{BM}	Π_{BM}	FO_1	z_1	Π_1	Δ_1
base case		**935**	**2390**	**621**	**42**	**3383**	**+41.6%**
	1%	1020	2222	643	41	3467	+56.1%
ν_1	**1.5%**	**935**	**2390**	**621**	**42**	**3383**	**+41.6%**
	2%	868	2492	598	42	3305	+32.6%
	1%	1020	2222	645	41	3385	+52.4%
ν_2	**1.5%**	**935**	**2390**	**621**	**42**	**3383**	**+41.6%**
	2%	868	2492	590	42	3368	+35.1%
	5	935	2390	595	39	3534	+47.9%
p_1	**10**	**935**	**2390**	**621**	**42**	**3383**	**+41.6%**
	15	935	2390	638	44	3249	+36%
	15	935	2390	605	41	3456	+44.6%
p_2	**20**	**935**	**2390**	**621**	**42**	**3383**	**+41.6%**
	25	935	2390	628	42	3317	+38.8%
	(300/100)	935	2390	594	40	3495	+46.2%
$(\bar{y}_1/\bar{y}_2)$	**(200/200)**	**935**	**2390**	**621**	**42**	**3383**	**+41.6%**
	(100/300)	935	2390	635	41	3256	+36.2%

initial base case values. Furthermore, the cost of remanufacturing one broken product c_r has no substantial influence. This can be explained by the fact that all broken products have to be remanufactured if they are not disposed of beforehand. Thus, no important influence on the buy-back decision can be derived from this parameter. The corresponding results can be found in the Appendix.

Regarding the other model settings (M2 and M4), the examined numerical examples reveal that the profit loss from restricted information availability and/or quantity and pricing flexibility only reacts slightly when one of the parameter values is altered. The largest loss in total profit that has been observed was 2.7% between settings M4 and M1 in a situation with a large remanufacturing yield rate q=60%. However, tendencies can be identified. The relative deviation between the profits of M4 and M1 seems to increase for a small failure rate λ, for a high per unit final lot cost c_f, and for comparably large holding costs h^R and h^S, respectively. These tendencies could also be observed when comparing M2 and M1 but on a less prominent scale. For details we refer again to the Appendix.

5 Conclusions

Due to a high profitability, after-sales management has received an ever increasing attention in the recent past. This study was particularly motivated by the automotive industry which continues to give long lasting mobility warranties for their cars. These warranties are obviously an attractive instrument for the marketing and sales department while they impose a challenge for the management of spare parts. This study highlights that buying back broken products is, under certain circumstances which can be found in practice, an attractive instrument to manage the end-of-life service period, especially in situations in which options to resupply are limited to placing a final order and later remanufacturing broken parts.

For different settings regarding the availability of information required for buy-back as well as limited pricing and quantity flexibility we propose simple MILP models that are able to find optimal strategies. After evaluating these strategies in a numerical study we find, that buying back defective products is a beneficial substitute for building up a large inventory of spare parts at the beginning of the planning horizon by procuring parts in a final order. It seems that the availability of detailed information and limitations of pricing and quantity flexibility do not affect the profitability of a product recovery system with buy-back option substantially. A main reason for this result can be found in the structural similarities of the optimal policies that could be observed by numerically examining a representative base case. Interestingly though, the buy-back is performed in form of campaigns in situations where the pricing flexibility is limited, i.e. a regular low price buy-back interval is interrupted by single periods in which a high price is offered to the customers.

It was shown which parameters especially influence the profitability of the buy-back option. Here it seems that those parameters which determine the profit impact of the final order size (like unit production cost and holding cost) seem to be of highest importance, while the influence of buy-back related parameters like prices show only limited impact. This is because our benchmark solution without buy-back only shows small flexibility to react on parameter changes while in the buy-back case, a trade-off is struck between the final order size and later buying back more or less products. In case of a high remanufacturing yield rate, the system can be handled like a repair system (see, e.g., Sherbrooke, 2004) where the buy-back option is less favorable. If the cost of the employed capital is high, it becomes more and more attractive to reduce the final order while instead compensating the customers for not fulfilling the spare parts availability guarantee.

This study is to our knowledge the first attempt to investigate the value of a buy-back option in a closed loop supply chain for spare parts. There are certainly some limitations to this study which can be overcome by further research. We limit our analysis to a MILP formulation which is numerically solved. Even though the numerical study is restricted to parameters that do not change over time (like e.g. the failure rate or customer valuation of their product), time dependent parameters can be addressed as well. General structural properties of optimal solutions could be obtained by using optimal control methods, as have been successfully applied in product recovery systems (see Kiesmüller et al., 2004; Kleber, 2006). Complementing our deterministic approach, a stochastic simulation could be used to evaluate more realistic models involving uncertainty. Here, due to the high flexibility, buying back broken products becomes an even more attractive option. Finally, another extension would include the case where multiple parts are included in a product and thus, the buy-back would yield inflows of several remanufacturable parts.

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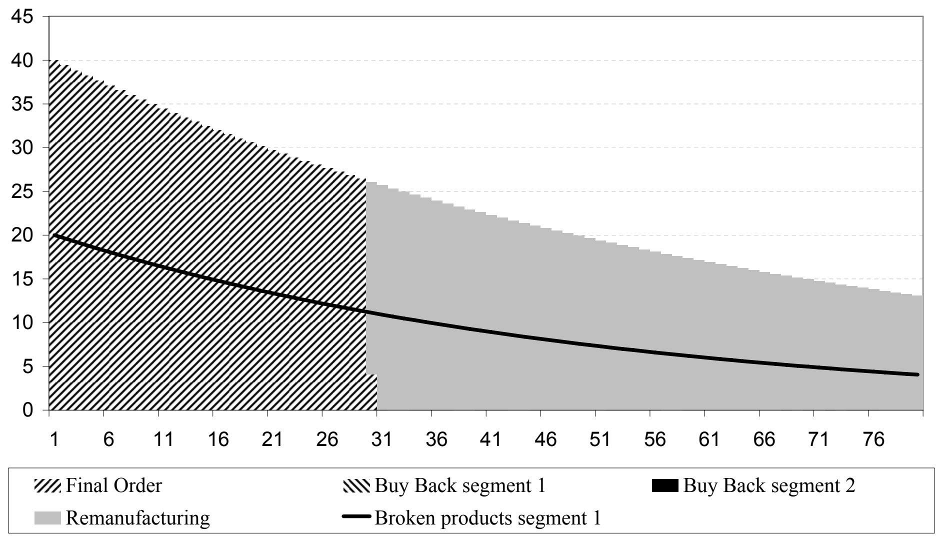

Figure 3: Fulfillment of spare parts demand the benchmark solution

A Appendix

A.1 Detailed discussion of policy structure

In the following, we discuss structural properties of optimal solutions. Since the main effects are already present when two customer segments are distinguished, we restrict to that case.

Benchmark without buy back. Figure 3 depicts the optimal solution structure. Here, total demand for spare parts (the height of each bar) is presented over the entire planning horizon. Thus, it can be seen that the total demand for spare parts equals to 40 units in the first period. As customers leave the service network, the demand for spare parts declines over the planning horizon, reaching 31.8% of its first period's value in the last period. Since the total demand for spare parts consists of the demand of two different customer segments, the black line in Figure 3 indicates the first segment's demand for spare parts, and the distance between the height of each bar and the black line depicts the second segment's demand. Additionally, the color-coded bars in this figure present the respective source from which the demand for spare parts has been satisfied. Two phases can be distinguished without buy-back option. In the first phase (periods 1 to 29) the demand for spare parts is satisfied completely from the final lot. All broken products that return to the OEM in this phase are immediately

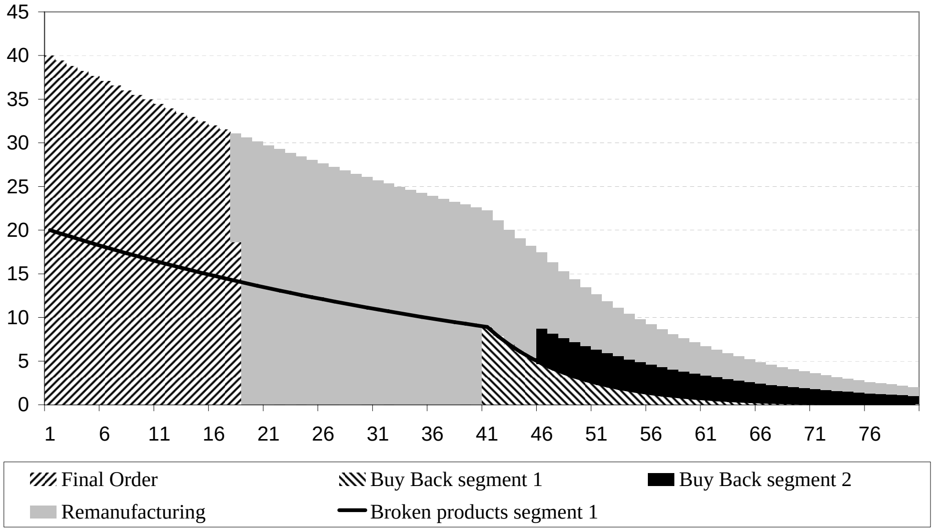

Figure 4: Fulfillment of spare parts demand in case M1

disassembled and the parts obtained by this procedure are held in the recoverables inventory and none is disposed of. In the second phase (periods 30 to 80) the strategic stock of returned products built up in the first phase is used to serve the entire demand by remanufacturing the recoverables inventory.

M1. In the following, we analyze how this structure changes when the buy-back option is included. Figure 4 depicts the development of spare part demand in setting M1 in the case of two customer segments. Although considerably shorter, both phases of the benchmark solution can be found in the optimal solution of setting M1 as well. While the first phase consists of 17 periods (from period 1 to 17) in which the entire spare part demand is satisfied by acquisitions made in the final lot, the second phase covers 23 periods (from period 18 to 40). In this phase, all demands are fulfilled by remanufacturing broken products that have been brought to the recoverables inventory in the first phase. In contrast to the benchmark solution, the recoverables inventory is not depleted at the end of the second phase. In a third phase (starting in period 41), the OEM starts buying back from the first segment. These products are remanufactured instantly and are used to satisfy the current demand if the designated quality standards are met. The remaining demand which cannot be satisfied by remanufacturing the bought-back products from the first segment is satisfied by remanufacturing broken parts left in stock from the second phase. This strategy is followed until the recoverables

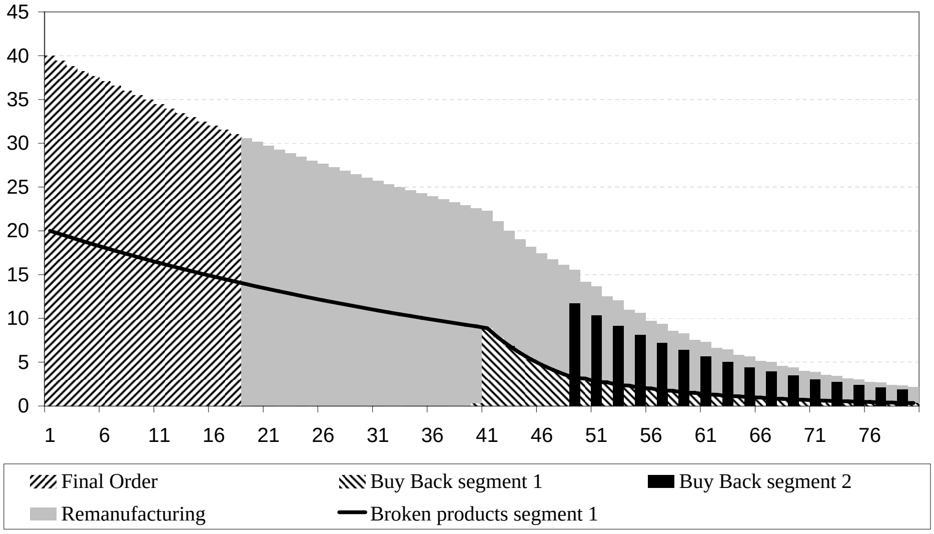

Figure 5: Fulfillment of spare parts demand in case M2

stock is depleted. Then, the OEM buys back from both segments. Interestingly, no further stock will be build up in the recoverables inventory. Hence, the total buy back quantity is set to just compensate the yield loss. As the OEM has perfect knowledge of its customers and can offer each customer an individual buy back price, he will only procure broken products from the first segment for the buy-back price p_1.

M2. Figure 5 depicts the structure of the optimal solution if the presumption of full pricing flexibility is lifted, still showing all three phases already explained for the setting M1. However, quantity and pricing decisions change in the third phase due to the limited pricing flexibility. In contrast to setting M1, the third phase of setting M2 is characterized by switching price decisions. While in most periods the low price p_1 is set and broken products from the first customer segment are bought back only, sporadically the price p_2 is set. In those 'campaign' periods a stock of broken products is build up, i.e. more broken products are bought back than are actually needed to satisfy the current period's demand. This strategy is driven by the fact that the OEM wants to set the price p_2 as low as possible. Yet, the entire demand of the second segment cannot be satisfied by only using bought-back products from the first segment. Thus, without stock-keeping the OEM would be forced to set p_2 in every period, which cannot be optimal.

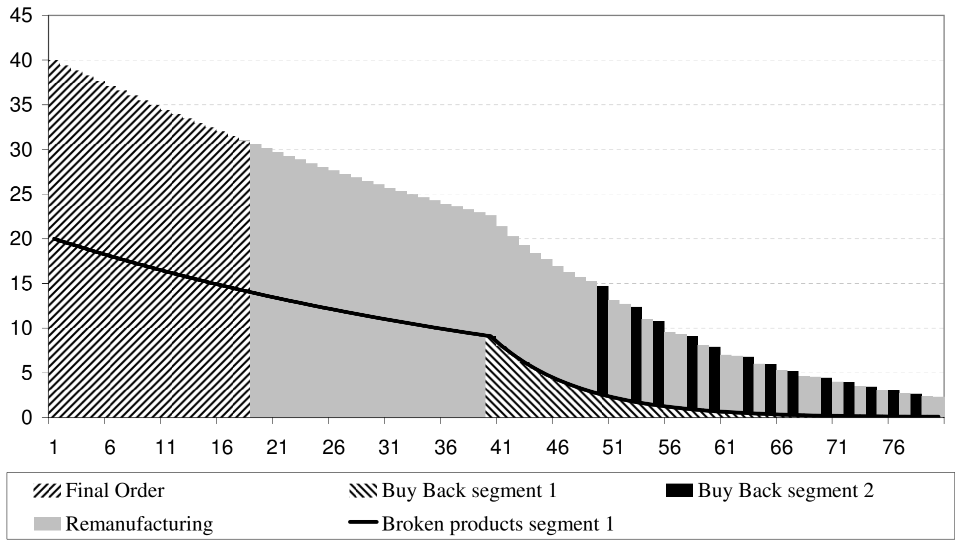

Figure 6: Fulfillment of spare parts demand in case M4

M4 The structure of the optimal solution is illustrated in Figure 6. As there is no quantity flexibility in this setting, the OEM has to buy back all broken products from both segments if the price p_2 is set, i.e. he pays all customers in segment one a higher price than their reservation price. The third phase exhibits the same switching pattern as in setting M2, except the fact that fewer periods can be observed during which the higher price p_2 is set. This can be explained by the missing quantity flexibility. If the price p_2 is set, the OEM has to buy back all products from the first and the second customer segment. Thus, a higher temporary stock is build up in the recoverables inventory, which lasts longer to fulfill future spare part demands than in M2.

A.2 Results of the sensitivity analysis

Table 9: Impact of selected parameters.

altered parameter		Benchmark		M4				M2				M1			
		FO_{BM}	Π_{BM}	FO_4	z_4	Π_4	Δ_4	FO_2	z_2	Π_2	Δ_2	FO_1	z_1	Π_1	Δ_1
base case		**935**	**2390**	**626**	**41**	**3343**	**+39.9%**	**622**	**41**	**3358**	**+40.5%**	**621**	**42**	**3383**	**+41.6%**
q	40%	1122	836	689	35	2352	+181.2%	685	35	2366	+182.9%	689	36	2415	+188.7%
	50%	**935**	**2390**	**626**	**41**	**3343**	**+39.9%**	**622**	**41**	**3358**	**+40.5%**	**621**	**42**	**3383**	**+41.6%**
	60%	748	3821	545	48	4372	+14.4%	541	47	4383	+14.7%	541	47	4396	+15.1%
r	1.25%	935	4142	781	55	4493	+8.5%	777	55	4496	+8.5%	758	55	4513	+8.9%
	2.5%	**935**	**2390**	**626**	**41**	**3343**	**+39.9%**	**622**	**41**	**3358**	**+40.5%**	**621**	**42**	**3383**	**+41.6%**
	5%	935	567	471	29	2249	+296.3%	461	28	2263	+298.7%	462	29	2287	+303%
c_f	1.5	935	3793	700	48	4332	+14.2%	694	48	4341	+14.5%	724	48	4369	+15.2%
	3	**935**	**2390**	**626**	**41**	**3343**	**+39.9%**	**622**	**41**	**3358**	**+40.5%**	**621**	**42**	**3383**	**+41.6%**
	4.5	935	986	559	35	2457	+149.1%	552	35	2478	+151.2%	553	36	2510	+154.4%
T	60	795	3156	644	42	3436	+8.9%	639	42	3437	+8.9%	628	42	3454	+9.4%
	80	**935**	**2390**	**626**	**41**	**3343**	**+39.9%**	**622**	**41**	**3358**	**+40.5%**	**621**	**42**	**3383**	**+41.6%**
	100	1039	1644	626	41	3327	+102.3%	620	41	3346	+103.5%	610	41	3371	+105%
(h^R/h^S)	(0.1/0.15)	935	2868	657	44	3569	+24.5%	654	44	3581	+24.9%	652	45	3604	+25.7%
	(0.1/0.2)	**935**	**2390**	**626**	**41**	**3343**	**+39.9%**	**622**	**41**	**3358**	**+40.5%**	**621**	**42**	**3383**	**+41.6%**
	(0.1/0.25)	935	1912	604	39	3138	+64.2%	590	38	3157	+65.1%	587	39	3185	+66.6%
	(0.05/0.2)	935	3210	677	46	3789	+18%	677	46	3798	+18.3%	674	47	3816	+18.9%
	(0.1/0.2)	**935**	**2390**	**626**	**41**	**3343**	**+39.9%**	**622**	**41**	**3358**	**+40.5%**	**621**	**42**	**3383**	**+41.6%**
	(0.15/0.2)	935	1789	582	37	3056	+70.8%	586	38	3076	+71.9%	576	38	3108	+73.7%
	(0.05/0.15)	935	3688	725	51	4061	+10.1%	718	50	4067	+10.3%	715	51	4082	+10.7%
	(0.1/0.2)	**935**	**2390**	**626**	**41**	**3343**	**+39.9%**	**622**	**41**	**3358**	**+40.5%**	**621**	**42**	**3383**	**+41.6%**
	(0.15/0.25)	935	1091	555	35	2785	+155.2%	549	35	2809	+157.5%	539	35	2846	+160.8%

Table 10: Impact of selected parameters (contd.)

altered parameter		Benchmark		M4				M2				M1			
		FO_{BM}	Π_{BM}	FO_4	z_4	Π_4	Δ_4	FO_2	z_2	Π_2	Δ_2	FO_1	z_1	Π_1	Δ_1
(ν_1/ν_2)	(1.5%/1%)	1020	2222	664	41	3335	+50.1%	649	40	3356	+51.1%	645	41	3385	+52.4%
	(1.5%/1.5%)	**935**	**2390**	**626**	**41**	**3343**	**+39.9%**	**622**	**41**	**3358**	**+40.5%**	**621**	**42**	**3383**	**+41.6%**
	(1.5%/2%)	868	2492	593	41	3335	+33.8%	589	41	3346	+34.3%	590	42	3368	+35.1%
	(1%/1.5%)	1020	2222	651	41	3422	+54%	643	41	3438	+54.7%	643	41	3467	+56.1%
	(1.5%/1.5%)	**935**	**2390**	**626**	**41**	**3343**	**+39.9%**	**622**	**41**	**3358**	**+40.5%**	**621**	**42**	**3383**	**+41.6%**
	(2%/1.5%)	868	2492	604	41	3269	+31.2%	601	41	3283	+31.8%	598	42	3305	+32.6%
	(1%/1%)	1105	2051	671	40	3412	+66.3%	680	41	3434	+67.4%	668	41	3468	+69.1%
	(1.5%/1.5%)	**935**	**2390**	**626**	**41**	**3343**	**+39.9%**	**622**	**41**	**3358**	**+40.5%**	**621**	**42**	**3383**	**+41.6%**
	(2%/2%)	801	2592	571	41	3259	+25.8%	568	41	3269	+26.1%	567	42	3289	+26.9%
(p_1/p_2)	(10/15)	935	2390	614	41	3423	+43.2%	603	40	3437	+43.8%	605	41	3456	+44.6%
	(10/20)	**935**	**2390**	**626**	**41**	**3343**	**+39.9%**	**622**	**41**	**3358**	**+40.5%**	**621**	**42**	**3383**	**+41.6%**
	(10/25)	935	2390	640	41	3276	+37.1%	635	41	3291	+37.7%	628	42	3317	+38.8%
	(5/20)	935	2390	597	37	3488	+46%	600	38	3505	+46.7%	595	39	3534	+47.9%
	(10/20)	**935**	**2390**	**626**	**41**	**3343**	**+39.9%**	**622**	**41**	**3358**	**+40.5%**	**621**	**42**	**3383**	**+41.6%**
	(15/20)	935	2390	646	44	3220	+34.7%	644	44	3232	+35.2%	638	44	3249	+36%
	(5/15)	935	2390	582	37	3567	+49.3%	578	37	3584	+50%	576	38	3612	+51.1%
	(10/20)	**935**	**2390**	**626**	**41**	**3343**	**+39.9%**	**622**	**41**	**3358**	**+40.5%**	**621**	**42**	**3383**	**+41.6%**
	(15/25)	935	2390	657	44	3151	+31.9%	661	45	3164	+32.4%	652	45	3187	+33.4%
$(\bar{y}_1/\bar{y}_2)$	(300/100)	935	2390	605	41	3473	+45.3%	595	40	3484	+45.8%	594	40	3495	+46.2%
	(200/200)	**935**	**2390**	**626**	**41**	**3343**	**+39.9%**	**622**	**41**	**3358**	**+40.5%**	**621**	**42**	**3383**	**+41.6%**
	(100/300)	935	2390	638	39	3217	+34.6%	642	40	3235	+35.4%	635	41	3256	+36.2%
c_r	0.75	935	2808	612	40	3859	+37.4%	620	41	3877	+38%	610	41	3905	+39.1%
	1.5	**935**	**2390**	**626**	**41**	**3343**	**+39.9%**	**622**	**41**	**3358**	**+40.5%**	**621**	**42**	**3383**	**+41.6%**
	2.25	935	1971	629	41	2827	+43.5%	626	41	2840	+44.1%	623	42	2863	+45.3%
λ	5%	468	1195	316	42	1652	+38.3%	318	42	1656	+38.6%	316	43	1679	+40.5%
	10%	**935**	**2390**	**626**	**41**	**3343**	**+39.9%**	**622**	**41**	**3358**	**+40.5%**	**621**	**42**	**3383**	**+41.6%**
	15%	1403	3584	916	40	5087	+41.9%	910	40	5112	+42.6%	911	41	5134	+43.2%