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# An Advanced Heuristic for Multiple-Option Spare Parts Procurement after End-of-Production

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## Abstract

After-sales service is a major profit generator for more and more OEMs in industries with durable products. Successful engagement in after-sales service improves customer loyalty and allows for competitive differentiation through superior service like an extended service period after end of production during which customers are guaranteed to be provided with service parts. In order to fulfill the service guarantee in these cases, an effective and efficient spare parts management has to be implemented, which is challenging due to the high uncertainty concerning spare parts demand over such a long time horizon. The traditional way of spare parts acquisition for the service phase is to set up a huge final lot at the end of regular production of the parent product which is sufficient to fulfill demand up to the end of the service time. This strategy results in extremely high inventory levels over a long period and generates major holding costs and a high level of obsolescence risk. With increasing service time more flexible options for spare parts procurement after end of production gain more and more importance. In our paper we focus on the two most relevant ones, namely extra production and remanufacturing. Managing all three options leads to a complicated stochastic dynamic decision problem. For that problem type, however, a quite simple combined decision rule with order-up-to levels for extra production and remanufacturing turns out to be very effective. We propose a heuristic procedure for parameter determination which accounts for the main stochastic and dynamic interactions between the different order-up-to levels, but still consists of quite simple calculations so that it can be applied to problem instances of arbitrary size. In a numerical study we show that this heuristic performs extremely well under a wide range of conditions so that it can be strongly recommended as a decision support tool for the multi-option spare parts procurement problem.

**Keywords:** Spare Parts, Inventory Management, Reverse Logistics, Final Order

# 1 Introduction

The after-sales service is an important and at the same time often still underestimated source for generating revenue and profit for manufacturing companies. In a recent benchmark study covering more than 120 companies from various industries including aerospace and defense, automotive, and consumer goods, Deloitte Research (2007) revealed that business units related to service provided an on average 75% higher profitability compared with the overall business profitability. Although revenues of these units amount to only a quarter of total revenues they yield almost 50% of total profit. Comparable numbers have been provided by Kim et al. (2007). However, since many companies still neglect the importance of their after-sales market, market shares of original equipment manufacturers (OEMs) in the service business show a considerable growth potential. But there are further reasons, why companies should more deeply engage in after-sales service. Cohen et al. (2006a) stress an improved customer loyalty and a resulting competitive differentiation through superior services. Such services include short repair times or an extended service period in which customers are guaranteed to be provided with spare parts. Additionally, it should be noted that the after-sales markets can be expected to show better resistance to economic downturn situations than markets for primary products.

Inventory management has long been recognized to play a key role in providing adequate after-sales service (Cohen and Lee, 1990). However, the peculiarities involved when dealing with spare parts supply chains considerably differ from those applied to the manufacturing supply chain for various reasons (for a detailed discussion see e.g. Kennedy et al., 2002, and Cohen et al., 2006b). We restrict our discussion to two main issues. Firstly, specific demands for spare parts are usually unpredictable but must be filled almost instantly. Even on an aggregate level as observed by OEMs, demands for spare parts include both dynamics and considerable variability (see Kennedy et al., 2002; Hesselbach et al., 2002). Secondly, a relatively long service period compared to commonly decreasing product life cycles steadily increase the number of products which are no longer produced but for which still spare parts must be provided. Teunter and Klein Haneveld (2002) for instance give evidence from the electronics industry, where service periods between 4 and 30 years are observed for products that normally are produced for less than two years. As a consequence, especially in the end-of-life period, which encompasses the period between end-of-production (EOP) of the primary product and its end-of-service (EOS), the

loss of scale economies makes provisioning of spare parts increasingly unattractive. Not surprisingly, Deloitte Research (2007) identified supplier reliability and long lead times for purchased components as the two major challenges for operations planning.

The traditional way of spare parts acquisition for the final phase of the service period is to place a huge final production order or last-time buy at EOP of the parent product which is sufficient to fulfill spare parts demand up to the end of the service time. However, this strategy results in extremely high inventory levels being held over a long period and it further generates major holding costs and a high level of obsolescence risk. Cattani and Souza (2003) for instance report that write-offs at Hewlett Packard due to scrapped inventory reduce profits each year by up to 1% of revenue. Recknagel (2007) estimates the obsolescence risk accepted for final order decisions in the white goods industry to be around 25%.

More flexibility can be provided by adding options such as extra production and remanufacturing of used products (for an overview on further options see Hesselbach et al., 2002). Extra production can take place in specialized internal facilities in small or medium-size lots or in form of occasional procurement from outside suppliers. It therefore incurs high flexibility since this option can react on additional information about demand obtained during the service period. Because of the loss of scale economies, variable production cost can easily exceed regular production cost by 100% and more, and often considerable lead times apply. When remanufacturing facilities are available, product recovery can be used as a further option for spare parts acquisition. Remanufacturing of used components or parts normally causes only moderate variable cost. However, that cost must be adapted to include a price discount for selling a refurbished part instead of a new one. Furthermore, this option is restricted by a limited availability of recoverable products, yielding further uncertainty in timing and quantity of returns.

For example, Volkswagen AG, a large Germany based car manufacturer, has been aware of both opportunities as well as challenges of the after-sales business and is known to provide superior service to the customer. Volkswagen guarantees the availability of all critical spare parts at reasonable prices for at least 15 years after producing the last car of a certain model. This period is considerably longer than the production period of about 6 years, yielding a large and growing number of stock-keeping units (SKUs), rising from 160,000 in 1995 up to 350,000 in 2005. Volkswagen must supply spare parts

for more than 50 million cars with the customers stemming from around 200 current and previous models. All procurement processes are coordinated at the Genuine Parts Centre (named OTC), a central warehouse for spare parts located near Kassel (Germany). In 2006, the OTC with shelf space of more than 850,000 square meters delivered almost 18 million parts to their customers of 4 billion € in value (Volkswagen AG, 2006). From our company experience, we estimate that about 20% of the stock stems from final order like provisioning. Volkswagen has a long history of product recovery of more than 60 years. Industrial remanufacturing takes place under the same standards applied to serial production und recovered parts carry the same warranty as new parts. Customers profit from price discounts of around 50% when buying genuine exchange parts. In 2008 about 2.5% of all SKUs are additionally sourced from remanufacturing used cars' components or broken parts. Despite this relatively small number this amounts to about 15% of the spare parts turnover (Volkswagen AG, 2008). Providing spare parts in an efficient way poses a large challenge to the manufacturer, requiring decisions on which parts to offer as spares, and about disposition, procurement, and inventory control of a huge number of SKUs.

Service divisions like the OTC at Volkswagen seek for ways to reduce inventory investments by efficiently coordinating all relevant procurement options, i.e. the final order, extra production, and remanufacturing. This is a challenging task in face of the large number of SKUs, time variability and uncertainty involved. In this paper, we analyze acquisition policies aiming to answer the following questions:

1. How effective are simple decision rules which can easily be implemented in practice?
2. How should these rules be applied in terms of setting the policy parameters?

These questions are not only of relevance for automotive companies. Similar problems are present in other industries with durable products, but short innovation cycle. See, e.g., Spengler and Schröter (2003) and van Kooten and Tan (2009) for two cases concerning producers of complex industrial products.

The remainder of the paper is structured as follows: Our contribution to literature is outlined in Section 2. In Section 3, we introduce the modeling assumptions and formulate a dynamic model for spare parts management in the final phase. The high computational effort required for finding optimal solutions necessitates the development of heuristics for

dealing with practical problem sizes. Therefore, a heuristic solution approach is provided in Section 4. The performance of this approach is investigated in Section 5. In Section 6, we present managerial insights for inventory control of spare parts in the final phase and conclude with directions for further research.

## 2 Literature Review

Quantitative approaches being relevant in our context can be distinguished in approaches dealing with decision making during the end-of-life period focusing on the determination of the final order and those that deal with inventory control in product recovery systems.

Fortuin (1980) presents a first mathematical approach aiming to determine the final order size. For given service levels, and assuming an exponentially declining stochastic demand over a given planning horizon, final orders are calculated for a problem environment where no further procurement options are available. Teunter and Fortuin (1999) extend this approach by generalizing the demand pattern and further integrating product recovery using a simple push policy where all returns are recovered upon arrival. This policy structure is optimal when remanufacturing cost can be neglected and holding returned items in stock is at least as costly as keeping new spare parts. Further on, near-optimal policy parameters are determined. Teunter and Fortuin (1998) present an application of this approach to a case study from the electronics industry and provide detailed procedures to forecast future demand distributions based upon historic demand data, price, and the relevant life cycle phase. For a similar push policy system with repairable spare parts facing random yield and lead time for repair, van Kooten and Tan (2009) develop a numerical procedure for final order determination by using a Markov chain approach. Also in Pourakbar et al. (2009) a combination of final order and push repair policy is addressed which is enriched by a switching option to an alternative policy (named product swapping) in form of offering a new product instead of repair. An approach for determining the optimal time to switch is presented. Teunter and Klein Haneveld (2002) consider a system where a final order decision is combined with subsequent extra production orders during the service period. They propose a procurement policy with order-up-to levels for extra production and present a method for calculating these policy parameters.

Extending the number of procurement options to more than only two considerably increases the complexity of the required approach. Spengler and Schröter (2003) evaluate various strategies to meet spare parts demand for electronics equipment using a complex System Dynamics approach. Inderfurth and Mukherjee (2008) present a basic formulation of the multiple-option spare parts procurement problem as a stochastic dynamic decision problem. However, both contributions do not investigate the optimal policy structure and do not provide methods for finding optimal parameters for a given policy. We extend the approach put forth in Inderfurth and Mukherjee (2008) and propose a modeling framework incorporating the final order decision, extra production with lead-times, and remanufacturing of used parts.

Lessons learned from inventory control in product-recovery systems can be applied in this context. For a given final order, we are dealing with a hybrid manufacturing/remanufacturing system under stochastic demands and returns as it is known from literature on stochastic inventory control in reverse logistics (for an overview see van der Laan et al., 2004). Inderfurth (1997) analyzes optimal policy structures of such a system and concludes that simple order-up-to policies only are present in the case of negligible or equal lead times. In addition, finding optimal policy parameters is not a simple task even in the special case. An efficient approach for determining close-to-optimal policy parameters has been presented by Kiesmüller and Scherer (2003). They show that an approximation of the value function considerably simplifies the search for the parameters. However, no similar approximations are available in case of lead-time differences for production and remanufacturing. In a related approach, Kiesmüller and Minner (2003) propose the use of simple order-up-to policies also for this case and develop simple procedures based upon a myopic newsvendor approach to determine policy parameters in the static infinite-horizon framework.

The main contribution of this paper is to provide insights into the usability of simple order-up-to policies in combination with the final order decision for end-of-life spare parts acquisition. An efficient heuristic is developed explicitly taking into account demand and return dynamics and a finite horizon setting. Furthermore, in contrast to the previous approaches, we are able to assess the performance loss due to the suboptimal policy structure.

### 3 Model for multiple-option spare parts procurement

In this section, based upon the formulation proposed in Inderfurth and Mukherjee (2008) a multi-period stochastic model is presented. The model can be used to determine both the optimal final order level and subsequent remanufacturing and extra production decisions. Although this optimization problem is hard to solve for realistic problem sizes, exploring it provides us with important insights that can be exploited to elaborate an effective heuristic. Additionally, we will use it for generating benchmarks in order to assess the performance of our heuristic approach.

#### 3.1 Assumptions

We consider a periodic review system with a finite planning horizon of length  $T$ . The first planning period begins at EOP, and the last one ends at EOS. The length of a period can reflect a time span between a month and a year, depending on the specific practical background. In each period  $t$ , a random number of spare parts  $D_t$  is demanded and a random number of used products  $R_t$  returns becoming available for remanufacturing in the following period. The probability distributions of demands and returns can vary over time according to the dynamic nature of the demand and return process during the service period of a part. It is assumed that spare parts demand and product returns are not correlated on the aggregate level of an OEM's perspective. We also do not consider correlation of the stochastic variables between successive periods as we do not have any respective evidence from practice.

In the first period the size of the final order, denoted by  $y$ , is fixed. Subsequently, in each period the stocks of serviceables and recoverables are observed and decisions are made on the remanufacturing quantity  $r_t$  and on the number of units  $p_t$  procured by extra production. While remanufacturing can usually be carried out in any period without major setup time, it might take considerable lead time to run extra production or initiate outside procurement. Thus, extra production is assumed to increase the stock level only after a lead time of  $l$  periods. Two inventory state variables are relevant for describing the information necessary to make optimal decisions in each period: the net serviceables stock  $I_t^S$  and the stock of used products  $I_t^R$ .

The cost parameters are assumed to be time-invariant. They include sourcing unit cost

for final lot production  $c_F$ , extra production  $c_P$ , and remanufacturing  $c_R$ . Fixed costs are not considered. The final order is part of the last regular production run so that there are no extra setup costs. Remanufacturing usually is a quite flexible sourcing option which is connected with only minor setup activities. Extra production often is rather characterized by a major lead time for reactivating respective manufacturing facilities or outside supply sources than by a considerable setup cost. The stocking of serviceable spare parts is charged by a holding cost  $h$  per unit and period. We consider two different types of cost when not being able to satisfy a demanded item during the period when it is required. A backorder cost  $v$  per unit and period applies for all periods in which the delivery of a spare part can be postponed (i.e. for  $t < T$ ). A per unit penalty for unsatisfied demand  $p$  is charged if at the end of the service period a required part cannot be delivered so that the customer has to be compensated in other ways. In practice we usually find a cost relationship where remanufacturing is more costly than regular large-scale production, but cheaper than extra procurement. Given a penalty  $p$  that is higher than unit procurement cost of all options results in situations where all three options are regularly utilized for spare parts procurement. This is just the environment of interest which we will model here. Thus the following cost inequality is assumed to hold:  $c_F \leq c_R \leq c_P < p$ .

Holding costs for returned used products are neglected since the capital tied up in returns is near zero in many cases. Also physical holding costs play a very minor role, because stock-keeping of used products often does not require a special treatment and results in negligible additional costs. Disposal costs for used products and spare parts left at the end of the planning period are not taken into account since in the case of durable products, the value of the material tied up in these parts quite often outweighs the costs of disassembling and cleaning. In principle, the consideration of disposal as well as recoverables holding cost is possible within our framework, but it would not contribute much in many practical cases. The objective is to minimize the total expected cost over the entire planning horizon. We treat all cost parameters as real values and thus neglect discounting.

### 3.2 Model

Given the above assumptions, we consider the following model (let  $(x)^+$  denote  $\max\{x; 0\}$  and  $p_t = 0 \forall t < 1$ ):

$$\min TEC = E \left\{ c_F \cdot y + \sum_{t=1}^{T-l} c_P \cdot p_t + \sum_{t=1}^{T-1} \left[ c_R \cdot r_t + h \cdot (I_{t+1}^S)^+ + v \cdot (-I_{t+1}^S)^+ \right] + c_R \cdot r_T + h \cdot (I_{T+1}^S)^+ + p \cdot (-I_{T+1}^S)^+ \right\} \quad (1)$$

$$I_{t+1}^S = \begin{cases} y + p_{1-l} + r_1 - D_1 & \text{for } t = 1 \\ I_t^S + p_{t-l} + r_t - D_t & \text{for } t = 2, \dots, T \end{cases} \quad (2)$$

$$I_1^R = 0 \quad \text{and} \quad I_{t+1}^R = I_t^R - r_t + R_t \quad \text{for } t = 1, \dots, T \quad (3)$$

$$y \geq 0, p_t \geq 0, \quad \text{and} \quad 0 \leq r_t \leq I_t^R \quad \text{for } t = 1, \dots, T \quad (4)$$

The objective function in (1) describes the expected value (regarding the stochastic demand and return flow) of the total cost  $TEC$  over the entire planning horizon  $T$ . It includes sourcing cost for final order, extra production, as well as remanufacturing. Further, holding cost for serviceable parts, backorder cost when not immediately satisfying demand and a penalty for unmet demand at the end of the planning horizon are considered. Constraints (2) and (3) represent inventory balance equations, where for ease of presentation initial stocks are set to zero. Restrictions (4) assure non-negativity of all decisions and pose a limit for the remanufacturing quantity to not exceed the number of available used products in each period.

In order to solve this dynamic stochastic optimization problem the corresponding functional equations have to be formulated. For a production lead time  $l > 0$  these read as follows:

**Period  $T$**

$$f_T(I_T^R, I_T^S, p_{T-l}) = \min_{r_T} \left\{ c_R \cdot r_T + \frac{E}{D_T} \left\{ h \cdot (I_T^S + p_{T-l} + r_T - D_T)^+ + p \cdot (D_T - I_T^S - p_{T-l} - r_T)^+ \right\} \right\} \quad (5)$$

**Periods  $t = T - l + 1, \dots, T - 1$**

$$f_t(I_t^R, I_t^S, p_{t-l}, \dots, p_{T-l}) = \min_{r_t} \left\{ c_R \cdot r_t + \frac{E}{D_t, R_t} \left\{ h \cdot (I_t^S + p_{t-l} + r_t - D_t)^+ + v \cdot (D_t - I_t^S - p_{t-l} - r_t)^+ + f_{t+1}(I_t^R - r_t + R_t, I_t^S + p_{t-l} + r_t - D_t, p_{t-l+1}, \dots, p_{T-l}) \right\} \right\} \quad (6)$$

**Period**  $t = l + 1, \dots, T - l$

$$f_t(I_t^R, I_t^S, p_{t-l}, \dots, p_{t-1}) = \min_{r_t, p_t} \left\{ c_P \cdot p_t + c_R \cdot r_t + \underset{D_t, R_t}{E} \left\{ h \cdot (I_t^S + p_{t-l} + r_t - D_t)^+ + v \cdot (D_t - I_t^S - p_{t-l} - r_t)^+ \right. \right. \\ \left. \left. + f_{t+1}(I_t^R - r_t + R_t, I_t^S + p_{t-l} + r_t - D_t, p_{t-l+1}, \dots, p_t) \right\} \right\} \quad (7)$$

**Periods**  $t = 2, \dots, l$

$$f_t(I_t^R, I_t^S, p_1, \dots, p_{t-1}) = \min_{r_t, p_t} \left\{ c_P \cdot p_t + c_R \cdot r_t + \underset{D_t, R_t}{E} \left\{ h \cdot (I_t^S + r_t - D_t)^+ + v \cdot (D_t - I_t^S - r_t)^+ \right. \right. \\ \left. \left. + f_{t+1}(I_t^R - r_t + R_t, I_t^S + r_t - D_t, p_1, \dots, p_t) \right\} \right\} \quad (8)$$

**Period 1**

$$f_1 = \min_{y, p_1} \left\{ c_F \cdot y + c_P \cdot p_1 + \underset{D_1, R_1}{E} \left\{ h \cdot (y - D_1)^+ + v \cdot (D_1 - y)^+ + f_2(R_1, y - D_1, p_1) \right\} \right\} \quad (9)$$

In case of zero lead time ( $l = 0$ ) only the inventory variables  $I_t^R$  and  $I_t^S$  have to be taken into account as state variables of the functional equations. In general, these recursive equations reveal that the optimal policy in each period has to be determined with respect to two plus up to  $l$  state variables, depending on the number of open production orders that exist in any period. This is caused by the fact that the time of arrival of each single open order must be considered when evaluating the impact of extra production on the cost of a period. Consequently, the computational effort required to solve the optimization problem (5)-(9) highly depends on the length of the extra production lead time  $l$ , contributing to the curse of dimensionality we face in stochastic dynamic programming. It therefore is necessary to develop a heuristic solution approach that can be used to solve problems of real-life size for arbitrary length of production lead time. Such a heuristic will be presented in the next section.

## 4 Heuristic for spare parts procurement after EOP

### 4.1 Overview

In order to develop a heuristic which combines applicability and effectiveness, we apply two steps of simplification. First we choose a simple but effective policy for period-by-period decision making which is easy to handle and includes a limited number of policy parameters. Second, we propose a heuristic procedure to determine all policy parameters

such that they are near-optimal. To this end we start with considering what is known regarding the policy structure of our problem.

The dynamic spare parts procurement problem in (1)-(4) combines a single final order decision at the beginning of the first period with a time sequence of remanufacturing and extra production decisions. Given the final order decision, the remaining problem has the basic structure of a multi-period hybrid manufacturing/remanufacturing problem with stochastic demand and returns under proportional costs. From Inderfurth (1997) it is known, that for this problem type the structure of the optimal policy is highly complex except for the case of equal lead times for both options. Under this specific lead time condition the optimal policy has a simple  $(S, M)$ -structure with two order-up-to levels in each period  $t$  ( $S_t$  for extra production and  $M_t$  for remanufacturing).

The heuristic approach that we propose is based on the application of this simple policy also in situations where the production lead time is larger than the lead time for remanufacturing. Thus, for sake of simplicity and easier applicability in case of  $l > 0$ , our solution procedure is relying on a suboptimal policy structure. The same policy simplification is also found in Kiesmüller and Minner (2003) in a related setting.

For using this  $(S, M)$  policy class, appropriate inventory positions for each decision have to be defined. The inventory position being relevant for the extra production decision is

$$IP_t^S = I_t^S + I_t^R + p_{t-1} + \dots + p_{t-l}. \quad (10)$$

It includes both serviceables and recoverables stocks as well as previous production orders that did not yet result in a material inflow (inventory on order). For remanufacturing decisions, the relevant inventory position is

$$IP_t^R = I_t^S + p_{t-l}, \quad (11)$$

i.e. serviceables stock plus that open order which becomes available to satisfy demand during the considered period. According to the  $(S, M)$  policy remanufacturing and extra

production decisions are made in the following way:

$$r_t = \begin{cases} 0 & \text{for } t = 1 \\ \min\{(M_t - IP_t^R)^+, I_t^R\} & \text{otherwise} \end{cases} \quad (12)$$

$$p_t = \begin{cases} (S_1 - y)^+ & \text{for } t = 1 \\ (S_t - IP_t^S)^+ & \text{for } 2 \leq t \leq T - l \\ 0 & \text{otherwise} \end{cases} \quad (13)$$

Even if we restrict the solution procedure on applying a simple  $(S, M)$  policy, optimizing the final order size and the dynamic order-up-to levels is a very cumbersome task, because all parameter values are interdependent. Instead, we propose the use of fairly simple heuristics for determining close-to-optimal solution parameters. The main idea behind our approach is to facilitate parameter determination by performing it in three consecutive steps which allow us to account for the main dependencies between policy parameters.

The remanufacture-up-to levels  $M_t$  constitute a group of parameters which only weakly depend on the others, because remanufacturing is preferred over extra production and has an immediate impact on period cost due to zero lead time. So, these parameters will be determined at first. The extra production decisions depend on the remanufacturing strategy so that the produce-up-to levels  $S_t$  will be calculated second, using the  $M_t$  values as input. Eventually, the final order will be determined because it depends on both the remanufacturing and extra production strategy while the final order itself only affects the inventory position of serviceables, but not the economic target inventories  $S_t$  and  $M_t$ .

For the determination of the order-up-to levels for remanufacturing and extra production in each period a myopic newsvendor approach is chosen. Kiesmüller and Minner (2003) showed that this type of approach works very well, at least in a stationary situation with infinite horizon. However, a newsvendor model cannot be satisfactorily applied for approximating the optimal final order size, because it does not sufficiently take into account the period-by-period process of changes in stock levels caused by the other procurement options. For this reason, a marginal cost approach is used to determine the size of the final order. In the next subsections we will describe parameter determination in detail.

## 4.2 Myopic newsvendor approach for $M_t$ determination

Since there is no lead time to be considered and no dependency on other parameters to be taken into account, remanufacture-up-to levels  $M_t$  are obtained by solving the classic newsvendor problem (see, e.g., Silver et al., 1998, Chapter 10). This requires determining appropriate underage and overage costs. Except for the last period an underage could be satisfied in the subsequent period yielding cost of a backorder  $v$  for a single period. In the last planning period  $T$ , a delayed supply to fill the underage is not possible causing cost of a penalty reduced by saved cost of remanufacturing an item (i.e.  $p - c_R$ ). Therefore, underage cost is given by

$$cu_t^M = \begin{cases} v & \text{for } t < T \\ p - c_R & \text{for } t = T \end{cases}. \quad (14)$$

An overage is dealt with by reducing the next periods remanufacturing quantity, thus causing holding cost for a single period  $h$ . In the last period, remanufacturing cost must be added since there is no further use of the item that was remanufactured in excess. Thus, overage cost is given by

$$co_t^M = \begin{cases} h & \text{for } t < T \\ h + c_R & \text{for } t = T \end{cases}. \quad (15)$$

Determining the critical ratio for the newsvendor solution in the usual way  $\left(\frac{cu_t^M}{cu_t^M + co_t^M}\right)$  and applying it to the period's demand distribution function yields the following term for the remanufacture-up-level  $M_t$

$$M_t =: \arg \min_M \left( P(D_t \leq M) \geq \begin{cases} \frac{v}{v+h} & \text{for } t < T \\ \frac{p-c_R}{p+h} & \text{for } t = T \end{cases} \right). \quad (16)$$

Here and in the sequel we operate with discrete random variables which also will be assumed for later numerical computations. The formulation of parameter calculation from (16) in case of continuous random variables is straightforward.

## 4.3 Adjusted newsvendor approach for $S_t$ determination

Order-up-to levels for extra production must be determined for all periods  $t = 1, \dots, T-l$ . Because of the unit cost differences, we assume that production only takes place in  $t$  if it

is expected that all returns are used up at the point of arrival of the order, i.e. at  $t + l$ . The relevant random net demand variable  $ND_t$  for a newsvendor-like approach under this condition is given by the excess demand over available returns during lead time  $l$ , given by  $ND_t = \sum_{i=t}^{t+l} D_i - \sum_{i=t}^{t+l-1} R_i$ . For estimating underage and overage cost we distinguish between direct and indirect effects. Direct effects refer to events occurring in period  $t + l$  which are directly affected by the extra production decision in period  $t$ . Indirect effects arise from the impact of extra production on remanufacturing decisions beyond period  $t + l$ .

The *direct effect of an underage* is given by the cost  $v$  of a backorder for one period if  $t + l < T$  or by the penalty reduced by extra production cost  $p - c_p$  if  $t + l = T$ , respectively. As an *indirect effect* it might happen that, if there are excess returns after period  $t + l$ , the backorder could be filled from remanufacturing instead of extra production yielding a cost reduction of  $c_p - c_r$ . This event we estimate to take place with probability  $\alpha_{t+l}$ , which represents the probability that after period  $t + l$  (cumulative) returns suffice to cover (cumulative) demand, so that

$$\alpha_{t+l} = P \left( \sum_{i=t+l}^{T-1} R_i > \sum_{i=t+l+1}^T D_i \right). \quad (17)$$

Of course, the indirect effect only is valid for  $t + l < T$ . Thus, underage cost  $cu_t^S$  is approximated as

$$cu_t^S = \begin{cases} v - \alpha_{t+l} \cdot (c_p - c_R) & \text{for } t < T - l \\ p - c_P & \text{for } t = T - l \end{cases}. \quad (18)$$

The *direct effect of an overage* is given by the cost of holding an extra produced item for another period. But this cost could possibly be avoided by adjusting remanufacturing in  $t + l$ . However, such an adjustment is not possible if remanufacturing does not take place in this period, i.e. if the inventory position relevant for remanufacturing exceeds the corresponding order-up-to level ( $IP_{t+l}^R \geq M_{t+l}$ ). According to (11), the inventory position is  $IP_{t+l}^R = I_{t+l}^S + p_t = S_t - I_t^R + \sum_{\tau=t}^{t+l-1} r_\tau - \sum_{\tau=t}^{t+l-1} D_\tau$ . If remanufacturing takes place just in time,  $I_t^R$  can be replaced by  $R_{t-1}$  and  $r_\tau$  by  $R_{\tau-1} \forall \tau = t, \dots, t + l - 1$ . Then, the probability of holding cost caused by  $S_t$  and occurring in  $t + l$ , denoted by  $\omega_t$ , is approximated by

$$\omega_t(S_t, M_{t+l}) = P \left( S_t + \sum_{\tau=t}^{t+l-2} R_\tau - \sum_{\tau=t}^{t+l-1} D_\tau \geq M_{t+l} \right). \quad (19)$$

In this way the interdependency between extra production and remanufacturing decisions (or in other words, between their respective parameters) is taken into account.

If we cannot decrease extra production later on, there is an additional burden from extra producing one unit too much ( $c_p - c_r$ ) because this would have been remanufactured for otherwise. This *indirect effect* is assumed to take place again with probability  $\alpha_{t+l}$ . Thus, overage cost  $co_t^S$  is approximated by

$$co_t^S(S_t, M_{t+l}) = \begin{cases} \omega_t(S_t, M_{t+l})h + \alpha_{t+l} \cdot (c_p - c_r) & \text{for } t < T - l \\ \omega_t(S_t, M_{t+l})(h + c_p) + (1 - \omega_t(S_t, M_{t+l}))(c_p - c_r) & \text{for } t = T - l \end{cases}. \quad (20)$$

Produce-up-to levels  $S_t$  ( $t = 1, \dots, T - l$ ) are determined by solving

$$S_t = \arg \min_S \left( \Psi_t(S) \geq \frac{cu_t^S}{cu_t^S + co_t^S(S, M_{t+l})} \right). \quad (21)$$

where  $\Psi_t(\cdot)$  represents the probability distribution function of the net demand  $ND_t$ . Following this approach, it turns out that different from the standard newsvendor formulation the order-up-to-level to be determined also affects the critical ratio. However, since both  $\Psi(S)$  and  $co_t^S(S, M_{t+l})$  are increasing functions in  $S$ , there exists a unique solution for (21). For finding  $S_t$  a simple procedure can be used that starts with a low level of  $S$  and incrementally increases  $S$  until the inequality in (21) holds.

#### 4.4 Marginal cost approximation for final order $y$

**Relevant cost elements.** For determining the final order size  $y^+$ , we develop an approximation of its marginal impact on the total cost function and search for that size of  $y$  where this marginal cost  $c(y)$  starts to become greater than zero. Behind this approach stands the supposition that the total expected cost TEC is a unimodal function w.r.t. the final order if we apply an order-up-to level policy for extra production and remanufacturing. This supposition is true in the case of equal lead times where it is found from Inderfurth (1997) that the total cost is a convex function of the starting inventory which is equivalent to the final order size in our case. Unfortunately, this property must no longer hold if lead times for production and remanufacturing differ. Numerical tests indeed revealed that the total cost function can lose unimodality if the order-up-to levels  $S_t$  and  $M_t$  are fixed in a completely arbitrary way. However, for the (near-optimal) parameter levels determined according to our heuristic procedure in (16) and (21), extensive tests

(including all problem instances presented in the next section) support our unimodality supposition. So we determine the final order  $y^+$  by

$$y^+ = \arg \min_y (c(y) \geq 0) \quad (22)$$

where the marginal cost is approximated by

$$c(y) = c_F + \theta(y, M_1, \dots, M_T) \cdot h - \pi(y, S_1, \dots, S_{T-1}) \cdot c_P - \beta(y, M_1, \dots, M_T, S_1, \dots, S_{T-1}) \cdot c_R - \gamma(y) \cdot v. \quad (23)$$

Here  $c(y)$  includes both the direct cost effect  $c_F$  of increasing the final order by one unit and approximate cost effects on the quantities held in stock in each period as well as on extra production/remanufacturing decisions and backorders. We do not consider the effect of penalty costs, since these can be avoided by later extra production or remanufacturing. Next, it will be shown how the probabilities necessary to estimate the expectations of the various cost effects depend on both the final order size as well as on the various order-up-to levels.

**Approximate holding time  $\theta$ .**  $\theta$  denotes the expected number of periods a marginal item added by the final order decision remains in stock. This is given by that period, after which one of the two relevant inventory positions falls below the respective order-up-to level for the first time. Since remanufacturing is the preferred procurement option under normal conditions (i.e. for not too long lead times and not too small return rates) an undershoot of a remanufacture-up-to level is assumed to limit the final order holding period. In order to calculate the expected duration of this time interval we weight all possible holding periods with the respective probabilities and sum them up. Approximating the probability that the final order holding time will be  $t$  periods or more by  $P\left(y - \sum_{i=1}^{t-1} D_i \geq M_t\right)$ , the expected holding time can be determined by

$$\theta(y, M_1, \dots, M_T) = \sum_{t=1}^T P\left(y - \sum_{i=1}^{t-1} D_i \geq M_t\right). \quad (24)$$

**Approximation of the probabilities ( $\pi$ ,  $\beta$ ) to reduce extra production and remanufacturing.** Increasing the final order by one unit will also have an impact on remanufacturing and extra production cost since it might lead to a reduction of these procurement activities in later periods. To this end it has to be analyzed if an additional

unit ordered at the beginning will result in a future saving of a remanufactured unit or an extra produced unit, or if there will be no saving at all.

Since the solution structure gives a higher priority to remanufacturing than to extra production we first calculate whether remanufacturing takes place at all, i.e. if the stock on hand (without considering later procurement) would fall below remanufacture-up-to level  $M_t$  in any period  $t$ . If it never does, as might be the case for a very large final order, increasing the final order would not have any impact on remanufacturing or extra production. The corresponding event approximately happens with probability  $\rho$

$$\rho(y, M_1, \dots, M_T) = \max_{t=1..T} \left\{ P \left( y - \sum_{i=1}^{t-1} D_i < M_t \right) \right\}. \quad (25)$$

We choose the maximum value in (25) because under general conditions of dynamic demand and return streams during the planning horizon the highest  $M_t$ -undershoot probability might be related to another period than the last one.

When increasing the final order, lowering remanufacturing only seems to be favorable as long as it is not possible to reduce extra production. A future production unit can be replaced if the serviceables' inventory position will drop below the respective produce-up-to level in any period  $t$ . This will also depend on  $y$ , and the probability  $\pi$  of this event can be approximated by

$$\pi(y, S_1, \dots, S_{T-l}) = \max_{t=1, \dots, T-l} \left\{ P \left( y - \sum_{i=1}^{t-1} (D_i - R_i) < S_t \right) \right\}. \quad (26)$$

The maximum value in (26) is chosen for the same reason as in (25). In case of stationary demand and return flows the maximum values for the probabilities in (25) and (26) will always occur in the final period  $T$  and  $T - l$ , respectively (also due to  $p > v$ ).

Summarizing, due to profitability of saving production cost first, the expected cost reduction in case of an incremental final order increase will be  $\pi c_P$ . A cost saving concerning remanufacturing only takes place if extra production cannot be reduced, but remanufacturing can. This probability is estimated by the difference between  $\rho$  and  $\pi$ . However, there is a certain (small) possibility that extra production might take place, but remanufacturing will not. This can happen if at the beginning of the planning period a production order is triggered whereas remanufacturing will not take place later on. This is only found in situations with delayed inflow of returns or when the final order is smaller than the first order-up-to level  $S_1$  and larger than the respective remanufacture-

up-to level  $M_1$ . In these cases we could find  $\pi > \rho$  so that the probability for saving a remanufacturing unit is set to

$$\beta(y, M_1, \dots, M_T, S_1, \dots, S_{T-l}) = \max \{\rho - \pi, 0\}, \quad (27)$$

resulting in a respective cost saving of  $\beta c_R$ .

**Approximation of the expected duration  $\gamma$  of a backorder situation.** Because of extra production lead time and limited return availability a shortage of serviceable spare parts can occur at the beginning of the planning horizon if the final order size is very small. This backorder situation can last for more than one period. The probability of a backorder situation in period  $i < l$  is estimated by  $P\left(y - \left(\sum_{\tau=1}^i D_\tau - \sum_{\tau=1}^{i-1} R_\tau\right) < 0\right)$ . Using this probability we can determine the expected duration  $\gamma$  of such a shortage by

$$\gamma(y) = \sum_{i=1}^l P\left(y - \left(\sum_{\tau=1}^i D_\tau - \sum_{\tau=1}^{i-1} R_\tau\right) < 0\right), \quad (28)$$

resulting in a backorder cost of  $\gamma \cdot v$ .

Calculating the durations and probabilities in (24)-(28) is a quite easy numerical task. The computational effort is even more reduced if demand and returns are treated as continuous random variables which are (approximately) normally distributed. In this case the convolutions of distributions in (24)-(28) can be performed analytically. Together with the simple calculation of the remanufacturing and extra production parameters of the applied  $(S, M)$  procurement policy in (16) and (21), the heuristic offers a fairly simple approach to solve the complex stochastic dynamic planning problem under consideration. The attractiveness of this approach, however, depends on its cost performance which is investigated in the next section.

## 5 Numerical Investigation

The purpose of this numerical investigation is twofold. We start with a detailed performance analysis, which shows how the heuristic performs in a great variety of problem settings. To this end, the results from the heuristic are compared with the optimal solution obtained by stochastic dynamic programming. In a first step we investigate two demand/return scenarios and use of a full factorial design. In a second step, while limiting the number of parameter combinations, a wide range of demand/return scenarios is

Table 1: Expected demand and returns in the considered scenarios.

	Period $t$	1	2	3	4	5	6	7	8	9	10	$\Sigma$
<b>static</b>	$E(D_t)$	6	6	6	6	6	6	6	6	6	6	60
<b>scenario</b>	$E(R_t)$	3	3	3	3	3	3	3	3	3	0	27
<b>dynamic</b>	$E(D_t)$	2	3	5	7	8	8	6	4	2	1	60
<b>scenario</b>	$E(R_t)$	1	2	3	5	5	4	4	3	2	0	27

considered. Further on, it is tested which part of the heuristic’s performance loss is due to the application of the simplified decision rule given by the  $(S, M)$  policy and which part is caused by the heuristic approach for parameter determination. For this numerical study parameter values are selected in such a way that they reflect relations we found in practical situations. To provide benchmark solutions, however, the planning horizon as well as demand/return distributions and the periods of extra production lead time are chosen such that the resulting problems still can be solved to optimality.

## 5.1 Experimental design

For a first test, we considered two demand and return scenarios: a static and a dynamic one with an identical total number of expected demands and returns over a planning horizon of ten periods. The dynamic scenario is characterized by the unimodal life cycle-type shape of expected spare parts demand and used product returns that we often find in practice. The static scenario serves as benchmark case for studying the impact of deviations from static conditions on the performance of our heuristic. The return ratio, i.e. the quotient of average total returns and average total demand, in both scenarios is  $q_R = 45\%$ . Demand and returns are independent random variables and follow discrete approximations of normal distributions with means as given in Table 1. The level of expected demand and returns is scaled such that a numerical optimization is still possible. It is very likely that the results of the following performance analysis will not be different if respective data would be scaled up.

For deriving the total expected costs TEC connected to the heuristic and for solving the Stochastic Dynamic Program (5)-(9), demand and return distributions are discretized in the respective  $\mu \pm 3\sigma$ -interval. All calculations required in our heuristic approach are performed based upon these distributions. Both the evaluation of the heuristic as well

Table 2: Parameter values in the full factorial design ( $c_F = 10$ ).

<b>Parameter</b>	$c_R$	$c_P$	$h$	$v$	$p$	$\rho_D$	$\rho_R$
<b>Low value</b>	12	16	1	25	75	0.1	0.1
<b>High value</b>	16	20	3	75	200	0.4	0.4

as the optimization is performed under use of a computer program we developed on the basis of the C programming language.

With respect to all relevant parameters we used a full factorial design including low and high values for each parameter except for the final order unit cost, which was normalized to 10 (see Table 2). Concerning the other cost parameters, we choose values that are related to the final order cost in such a way as (according to our experience in the automobile industry) we often find in practice. Remanufacturing unit cost exceed the final order cost by 20% or 60%, respectively. Low extra production unit cost is chosen such that there is no cost advantage for remanufacturing ( $c_P^{\text{low}} = c_R^{\text{high}} = 16$ ). High extra production cost is set to twice the unit cost of final order ( $c_P^{\text{high}} = 20$ ). The holding cost parameter ranges between a low value of 10% and 30% of final order unit cost. These relatively large values are chosen because of the scaled down planning horizon of 10 periods which might be interpreted as years in a real-life planning situation. Backorder cost rate ranges between  $v^{\text{low}} = 25$  and  $v^{\text{high}} = 75$  and the penalty at the end is either  $p^{\text{low}} = 75$  or  $p^{\text{high}} = 200$ .

The levels of uncertainty of demand and return streams are characterized by their respective coefficients of variation  $\rho_D$  (for demand) and  $\rho_R$  (for returns). These coefficients are assumed to be time-independent, each taking a low value of 0.1 and a high value of 0.4. Such values might be realistic for the smoothed aggregate spare part demands faced by an OEM at a central warehouse level, even though the need for a spare part is highly erratic for a single customer. The lead time for extra production is varied between 0 and 2 periods where the two-period case represents the upper limit for which an optimization is possible from a computational point of view. Given these alternatives, we face a total number of 768 parameter combinations that are investigated.

## 5.2 Performance analysis

For each instance we determine the relative costs deviation  $\Delta TEC$  of our heuristic approach from the optimal solution. By appropriately grouping the instances, a sensitivity

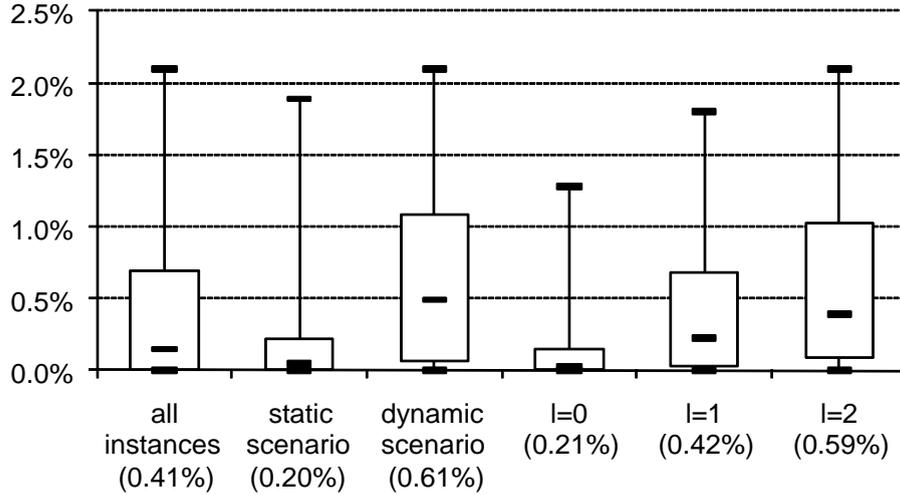


Figure 1: Overall performance of the heuristic and influence of scenario type and production lead time.

analysis of the average performance of the heuristic with respect to the two considered demand/return scenarios as well as for parameters are performed. The results are illustrated using box plots which are supplement by average deviations in brackets and detailed in Table 5 in the Appendix.

In Figure 1, the performance of the heuristic over all instances is shown. Additionally, the influence of the demand/return scenario type as well as of the production lead time is demonstrated. The results reveal that with a value of 0.41% the average cost deviation of the heuristic over all scenarios is extremely small. Even better, also the worst case performance is within a 2.5% limit so that the heuristic seems extraordinary promising. In the static scenario the heuristic has a mean cost deviation of only 0.2%. With an average deviation of 0.61% the performance remains still very good in the dynamic scenario, even though the approximations used for the heuristic approach to some extent refer to static conditions. Aside the scenario structure mainly variations in the lead time of extra production  $l$  and in the demand variability  $\rho_D$  seem to have a significant impact on the heuristics' performance (see Figures 1 and 2).

With an increase in lead time the performance tends to worsen, but still remains at a very high level of far below 1% even for maximum lead time of two periods. Concerning the impact of different levels of risk, it is mostly the demand risk that matters. Figure 2 shows that an increasing demand variability (both alone as well as jointly with a rising return variability) results in a slight deterioration of the heuristic's performance. Additionally, we see that the level of return risk practically has no impact at all. The same

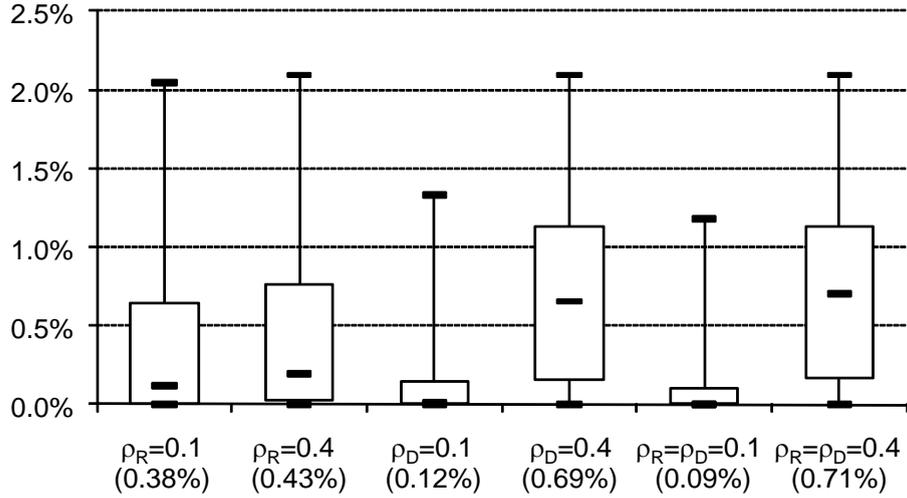


Figure 2: Influence of return and demand risk parameters on heuristic performance.

Table 3: The considered demand and return patterns.

Period		1	2	3	4	5	6	7	8	9	10
demand	falling pattern	13	11	8	7	6	5	4	3	2	1
	unimodal pattern	2	4	7	8	9	9	8	7	4	2
	constant pattern	6	6	6	6	6	6	6	6	6	6
returns	rising pattern	1	1	2	2	3	3	4	5	6	0
	unimodal pattern	1	2	3	4	4	4	4	3	2	0
	falling pattern	6	5	4	3	3	2	2	1	1	0
	constant with $q_R = 15\%$	1	1	1	1	1	1	1	1	1	0
	constant with $q_R = 45\%$	3	3	3	3	3	3	3	3	3	0
	constant with $q_R = 75\%$	5	5	5	5	5	5	5	5	5	0

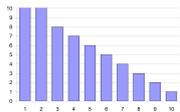
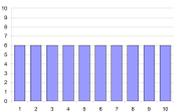
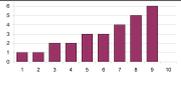
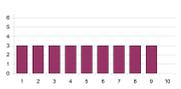
seems to hold for the cost parameters levels (see Figures 4 and 5 in the Appendix).

In order to examine the impact of the dynamic scenario structure we extend the numerical study in a second step and consider in total three demand patterns: a falling, a unimodal, and a constant one; all having a total mean demand of 60. Six return patterns are considered: a rising, a unimodal, and a falling one (all with a return ratio of 45%), as well as three constant patterns with different return ratios  $q_R$ , set either to 15%, 45%, or 75%. Combining all demand and return patterns as depicted in Table 3 yields a total of 18 scenarios. In order to reduce the computational effort a subset of the full factorial design is used by fixing all cost parameters at a specific level (here at their low value) and varying only those parameters with major performance impact, i.e. lead time (with  $l \in \{1, 2, 3\}$  and risk levels (with  $\rho_D \in \{0.1, 0.4\}$  and  $\rho_R \in \{0.1, 0.4\}$ ). This yields 12 instances per scenario.

The results confirm the excellent performance of the heuristic approach showing an average error of 0.31% over all combinations of demand and return patterns. Both average and maximum deviations in each of the 18 scenarios are depicted in Table 4. The highlighted numbers refer to the two basic scenarios investigated in the previous study. The results reveal that also in the considered subset of parameter combinations the mean cost deviations do not change much so that the explanatory power of this second study is not restricted.

Considering the full set of results in Table 4, we first observe that there is no single combination of demand and return patterns for which the heuristic deviates by more than one percent on average from the optimal solution. In detail, there is no clear picture how the pattern of demand and return streams influence the heuristic's performance. It seems that the demand pattern has a major impact. Hereby, it becomes apparent that a unimodal demand pattern is connected with a significantly larger cost deviation than other patterns. The impact of the return ratio also seems to be minor. Interestingly, the cost deviation is highest for the smallest return ratio.

Table 4: Average and maximum error (in parentheses) of the heuristic in 18 demand/return scenarios.

		demand pattern		
				
dyn. ret. pattern		0.31% (0.97%)	0.35% (0.83%)	0.13% (0.40%)
		0.15% (0.33%)	<b>0.69% (1.42%)</b>	0.09% (0.35%)
		0.14% (0.44%)	0.68% (1.78%)	0.11% (0.44%)
static ret. pattern		0.34% (1.06%)	0.88% (2.29%)	0.15% (0.45%)
		0.11% (0.29%)	0.67% (1.28%)	<b>0.07% (0.22%)</b>
		0.20% (0.68%)	0.43% (0.90%)	0.08% (0.25%)

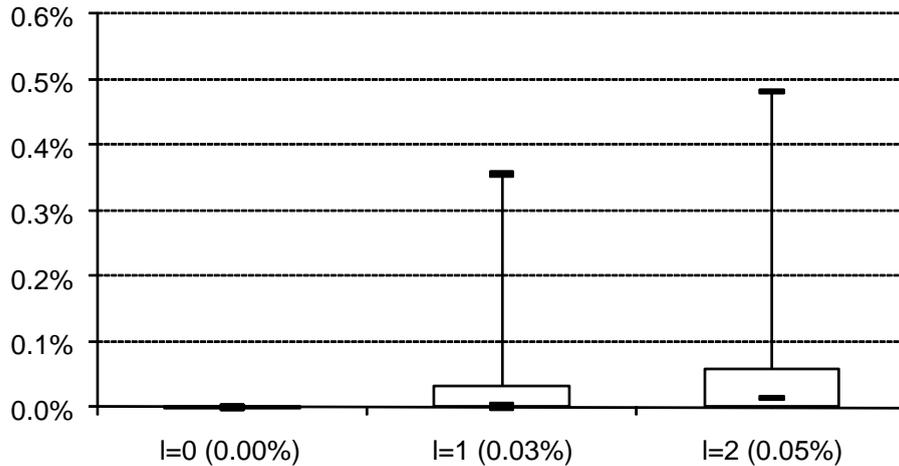


Figure 3: Influence of lead time on the performance of the policy structure (with optimized parameter).

### 5.3 Impact of the policy structure

A more detailed investigation of the reason for cost deviations of our heuristic requires the determination of (close-to) optimal policy parameters for the simple  $(S, M)$  policy structure. In doing so we can assess which part of the heuristic, i.e. either the simplified policy or the parameter determination approach, to a larger extent contributes to the performance loss of our solution approach. This approach provides insights regarding the appropriateness of a simple two-parameter rule for joint manufacturing/remanufacturing decisions in case of lead time differences. This important question is still open and has not been answered in other research contributions addressing related problems like in Kiesmüller and Minner (2003).

For searching optimal order-up-to levels for extra production and remanufacturing in combination with a best final order size, we implemented a local search procedure that uses our heuristic's values for the final order and the parameters  $S_t$  and  $M_t$  as initial solution. The algorithm checks whether the value of the objective function can be reduced by first increasing or decreasing the final order and subsequently proceeding (one period after the other) in the same way with each period's order-up-to level  $S_t$  and remanufacture-up-to level  $M_t$ . Each time an improved solution is found the algorithm restarts. If there is none, it stops. Although the computing time of the algorithm already benefits from a good initial solution it still is quite time consuming and therefore hardly recommendable for real life problems. Since there is no guarantee of finding optimal policy parameters using this search procedure the results only show an upper bound of

the performance loss.

The search procedure was applied to all 768 instances of the first test. From Figure 3 it can be seen, that the simple policy structure yields excellent results and thus only has a very limited responsibility for the suboptimality of our heuristic. As expected, the deviation will rise with increasing lead time, because the optimal policy becomes more and more complex. Nevertheless, the cost deviations for non-zero lead times are extremely small concerning both average and worst case behavior. Both are only very slightly increasing with lead time duration indicating that also for lead times larger than two periods (which unfortunately could not be investigated numerically) the simple  $(S, M)$ -policy will perform very well.

Since the major part of our heuristic's performance loss is due to the parameter determination approach it would be interesting to see how the heuristically found parameter values deviate from (close-to) optimal ones. In many cases where our heuristic leads to optimal or near-optimal solutions, we do not find mentionable parameter deviations. So we restrict our presentation to those 10 out of the 768 problem instances where the heuristic performed worst and where considerable parameter deviations are to be expected. These instances are provided in Table 6 in the Appendix, starting with the worst case as number one and ordered according to increasing performance. In addition to the cost deviation  $\Delta TEC$  (in %) from minimal cost, we report the final order size  $y$ , the order-up-to levels  $S_t$  for production, and remanufacture-up-to levels  $M_t$ .

These 10 worst performing cases all refer to problem instances with highest lead time ( $l = 2$ ) and highest demand variability ( $\rho_D = 0.4$ ). Concerning the other parameters, we find all parameter values under consideration appearing in these instances. 8 of the 10 instances imply the dynamic scenario, only instances #6 and #7 represent the static one. A closer look at Table 6 reveals that the heuristic final order level is somewhat smaller than the one from parameter optimization, but mostly quite close. Parameter optimization more or less yields optimal final order sizes, except for 3 instances (#1, #2 and #10) where we find larger deviations. Interestingly, this does not result in a major cost deviation suggesting that the cost function is quite flat around the optimal final order level. The remanufacture-up-to levels generated by our heuristic are quite close to the optimal  $M_t$  levels. In contrast, the approximation of the  $S_t$  parameters performs somewhat worse. The produce-up-to levels (reported for 8 periods only because

of the two-period lead time) of our heuristic are almost always smaller than the optimal ones. They follow the unimodal shape of the optimal levels, but partly show considerable deviations which seem to be the main reason for the loss of cost performance. Related to all 10 presented instances the cost deviation resulting from the application of the simple  $(S, M)$  policy on average contributes 10% to the total deviation caused by the heuristic. In instance we find the highest value of policy-based contribution (24%) to the overall performance loss.

## 6 Managerial Insights and Conclusions

A major challenge for spare parts management after EOP is the high uncertainty concerning spare parts demand over a long period until EOS. In our study we presented a model formulation to coordinate different procurement options for spare part acquisition, namely final order, extra production, and remanufacturing. We developed an advanced heuristic based upon a quite simple order-up-to decision rule for joint production and remanufacturing. This policy structure turns out to be very effective which is an important finding from a managerial point of view since it confirms that in planning situations with finite horizon and stochastic dynamic demand and return flows of various shapes a simple-to-implement  $(S, M)$  policy can be applied without concern.

We developed a heuristic procedure for parameter determination which takes into account the main stochastic and dynamic interactions when employing the  $(S, M)$  rule, but still consists of quite simple calculations so that it can be applied for problem instances of arbitrary size. Our numerical study revealed that this heuristic performs extremely well under all cost and demand/return conditions so that it can be strongly recommended as a decision support tool for the multi-option spare parts procurement problem under consideration. An important part of this heuristic is the myopic way of determining the order-up-to levels of the policy. The good quality of the heuristic confirms what already has been experienced in other research contributions (e.g. by Morton and Pentico, 1995 or Bollapragada and Morton, 1999), namely that dynamic order-up-to levels can be approximated fairly well using a myopic single-period approach even if we face a multi-period non-stationary situation as in our problem environment. The managerial impact of this finding is that problem complexity can be reduced considerably by confining oneself

to a simple myopic problem sight when parameters of a dynamic policy are determined.

Our approach can also be used to check the importance of flexibility of procurement options for the spare parts management problem. For instance, considering all 768 problem instances of the first test, a restriction to a final-order-only strategy results in total expected costs that are between 8% (best case) and 90% (worst case) higher than the costs of using the additional options of extra production and remanufacturing. That means that managers should take care to establish respective procurement contracts and facilities which allow for extra production and remanufacturing activities during the end-of-life service period of a good.

The presented study is restricted to a problem environment for final order determination in combination with extra production and remanufacturing decisions that naturally does not capture all situations that we might face in practice. It would be straightforward to include aspects like initial inventories or the discounting of period costs. It is also quite easy to take into consideration the impact of holding costs for stocking returned items, even if in that case disposal decisions should be taken into account. From previous research (see Inderfurth, 1997) we know that under these conditions the two-parameter  $(S, M)$  policy should be extended by a third parameter which can be interpreted as a dispose-down-to level. Thus, we still can apply a fairly simple decision rule and extend parameter approximations developed for this case (see Kiesmüller and Scherer, 2003).

Our problem description does not account for fixed costs and respective order batching in case of extra production and remanufacturing. This is because we do not consider the problem of short-term production planning, but refer to a medium-term production and remanufacturing strategy within a multiple-year planning horizon formed by a product's service period. However, production might be accompanied by the necessity to produce or order a major quantity due to the specific effort combined with an extra production run, especially if it is performed by an outside supplier of a part. In that case often a minimum production quantity has to be purchased. This aspect will have a major impact on the policy structure and will be left for future research. It is also a matter of future studies to integrate additional procurement options like product swapping or the use of compatible successive products into our approach. Finally, it will be interesting to include options of active acquisition of returns, as e.g. considered in Galbreth and Blackburn (2006) and Kleber et al. (2009), into the spare parts procurement problem so that the flexibility of

the remanufacturing option is increased.

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## A Appendix

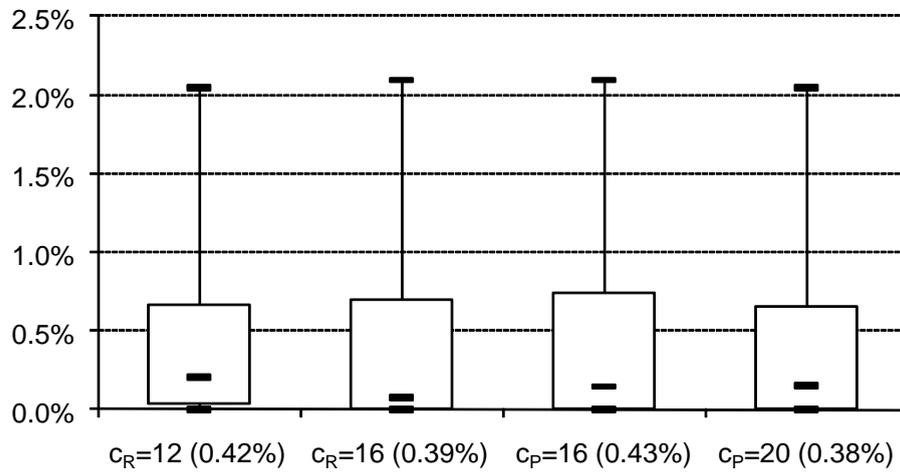


Figure 4: Influence of remanufacturing and extra production unit cost  $c_R/c_P$  on heuristic performance.

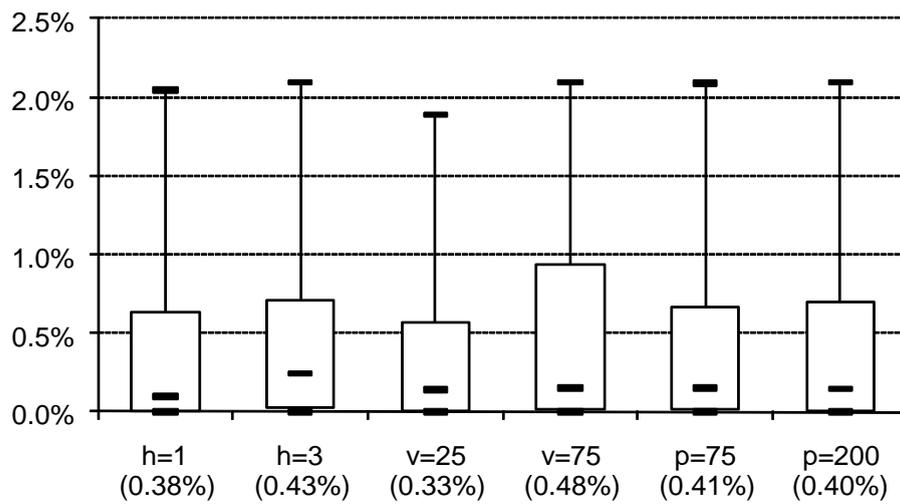


Figure 5: Influence of holding cost  $h$ , backorder cost  $v$ , and penalty  $p$  on heuristic performance.

Table 5: Boxplot data.

	Q1	Median	Q3	Sample Maximum	Mean
<b>Figure 1</b>					
all instances	0.01%	0.15%	0.69%	2.09%	0.41%
static scenario	0.00%	0.05%	0.22%	1.89%	0.20%
dynamic scenario	0.06%	0.49%	1.08%	2.09%	0.61%
$l = 0$	0.00%	0.02%	0.15%	1.28%	0.21%
$l = 1$	0.03%	0.23%	0.68%	1.80%	0.42%
$l = 2$	0.09%	0.39%	1.03%	2.09%	0.59%
<b>Figure 2</b>					
$\rho_R = 0.1$	0.00%	0.12%	0.64%	2.05%	0.38%
$\rho_R = 0.4$	0.03%	0.19%	0.76%	2.09%	0.43%
$\rho_D = 0.1$	0.00%	0.01%	0.14%	1.33%	0.12%
$\rho_D = 0.4$	0.15%	0.65%	1.13%	2.09%	0.69%
$\rho_R = \rho_D = 0.1$	0.00%	0.00%	0.10%	1.18%	0.09%
$\rho_R = \rho_D = 0.4$	0.16%	0.70%	1.13%	2.09%	0.71%
<b>Figure 3</b>					
$l = 0$	0.00%	0.00%	0.00%	0.00%	0.00%
$l = 1$	0.00%	0.00%	0.03%	0.36%	0.03%
$l = 2$	0.00%	0.01%	0.06%	0.48%	0.05%
<b>Figure 4</b>					
$c_R = 12$	0.04%	0.20%	0.67%	2.05%	0.42%
$c_R = 16$	0.00%	0.08%	0.70%	2.09%	0.39%
$c_P = 16$	0.01%	0.15%	0.75%	2.09%	0.43%
$c_P = 20$	0.01%	0.15%	0.65%	2.05%	0.38%
<b>Figure 5</b>					
$h = 1$	0.00%	0.10%	0.64%	2.05%	0.38%
$h = 3$	0.03%	0.24%	0.71%	2.09%	0.43%
$v = 25$	0.01%	0.14%	0.57%	1.89%	0.33%
$v = 75$	0.01%	0.15%	0.94%	2.09%	0.48%
$p = 75$	0.01%	0.15%	0.67%	2.09%	0.41%
$p = 200$	0.01%	0.15%	0.70%	2.09%	0.40%

Table 6: Parameter values, final order  $y$ , remanufacture-up-to levels  $M_t$ , order-up-to levels  $S_t$ , and relative cost deviation in 10 worst case instances for optimal solutions (Opt), the optimized (M,S)-policy (Pol), and the heuristic approach (Heu).

#	$c_R$	$c_P$	$h$	$v$	$p$	$\rho_R$	$y$	$M_1$	$M_2$	$M_3$	$M_4$	$M_5$	$M_6$	$M_7$	$M_8$	$M_9$	$M_{10}$	$S_1$	$S_2$	$S_3$	$S_4$	$S_5$	$S_6$	$S_7$	$S_8$	$\Delta TEC$	
							Opt 18																				
<b>1</b>	16	16	3	75	200	0.4	Pol	13	3	7	11	13	14	15	13	11	6	3	18	25	32	35	35	29	20	14	0.3%
							Heu	12	3	7	12	14	15	15	14	12	7	3	17	24	29	30	30	28	21	13	2.1%
							Opt 18																				
<b>2</b>	16	16	3	75	75	0.4	Pol	13	3	7	11	13	15	15	13	11	6	3	18	25	32	35	34	29	19	12	0.3%
							Heu	12	3	7	12	14	15	15	14	12	7	3	17	24	29	30	30	28	21	12	2.1%
							Opt 46																				
<b>3</b>	12	20	1	75	75	0.1	Pol	46	4	8	13	15	17	17	15	12	6	3	18	25	30	33	30	25	18	11	0.0%
							Heu	45	4	8	13	15	17	17	15	13	8	3	18	25	30	32	32	29	22	11	2.0%
							Opt 44																				
<b>4</b>	12	16	1	75	75	0.4	Pol	44	4	8	13	15	17	17	15	12	6	3	18	25	31	35	33	27	19	11	0.1%
							Heu	41	4	8	13	15	17	17	15	13	8	3	18	25	31	33	33	30	23	12	2.0%
							Opt 47																				
<b>5</b>	12	20	1	75	75	0.4	Pol	47	4	8	13	15	17	17	15	12	6	3	18	25	31	33	31	26	18	11	0.0%
							Heu	45	4	8	13	15	17	17	15	13	8	3	18	25	31	33	33	29	22	11	1.9%
							Opt 20																				
<b>6</b>	16	16	3	25	200	0.1	Pol	21	9	9	9	9	9	9	9	9	9	9	21	21	22	22	23	23	23	21	0.3%
							Heu	18	9	9	9	9	9	9	9	9	9	9	9	18	18	18	18	18	18	18	18
							Opt 21																				
<b>7</b>	16	16	3	25	200	0.4	Pol	21	9	9	8	9	9	9	9	9	9	9	21	22	23	23	24	24	24	22	0.4%
							Heu	18	9	9	9	9	9	9	9	9	9	9	9	19	19	19	19	19	19	19	19
							Opt 48																				
<b>8</b>	16	16	1	75	75	0.1	Pol	48	4	8	13	15	17	16	15	12	6	3	18	25	30	38	37	29	19	11	0.0%
							Heu	46	4	8	13	15	17	17	15	13	8	3	18	25	30	32	32	29	23	12	1.9%
							Opt 48																				
<b>9</b>	16	16	1	75	75	0.4	Pol	48	4	8	13	15	17	16	15	12	6	3	18	25	31	39	38	30	20	12	0.1%
							Heu	46	4	8	13	15	17	17	15	13	8	3	18	25	31	33	33	30	23	12	1.8%
							Opt 18																				
<b>10</b>	16	16	3	75	75	0.1	Pol	13	3	7	11	13	15	15	13	11	6	3	18	25	32	34	34	28	19	11	0.2%
							Heu	12	3	7	12	14	15	15	14	12	7	3	17	23	28	30	30	27	21	11	1.8%