WORKING PAPER SERIES

Marriage Regimes

Dirk Bethmann

Working Paper No. 29/2011

OTTO VON GUERICKE
UNIVERSITÄT
MAGDEBURG

FACULTY OF ECONOMICS
AND MANAGEMENT

Impressum (§ 5 TMG)
Herausgeber:
Otto-von-Guericke-Universität Magdeburg
Fakultät für Wirtschaftswissenschaft
Der Dekan

Verantwortlich für diese Ausgabe:
Dirk Bethmann
Otto-von-Guericke-Universität Magdeburg
Fakultät für Wirtschaftswissenschaft
Postfach 4120
39016 Magdeburg
Germany

http://www.fww.ovgu.de/femm

Bezug über den Herausgeber
ISSN 1615-4274

Marriage Regimes*

Dirk Bethmann†
Otto-von-Guericke-Universität Magdeburg

December 23, 2011

Abstract

Marriage regimes exist in many guises and forms. Economists have studied monogamy and polygyny, the two most commonly encountered types, and pointed to various benefits that can explain why and which individuals form conjugal unions in each regime. However, many of these same benefits should favor polygamy over monogamy more generally, including polyandrous and cenogamous marriages, which are only rarely observed in practice. We show that human reproductive technology in combination with regime-specific potential for conflict among parents of the same and opposite sex over resources devoted to own children can explain why monogamy is most common, polygyny frequent, polyandry rare, and cenogamy virtually non-existent. Within-wives conflicts over resources also provide an alternative explanation for why polygyny has historically been less common than monogamy and why the former has declined in many parts of the world over the last century.

Keywords: Marriage Regimes, Monogamy, Polygyny, Polyandry, Paternal Uncertainty, Reproductive Capacity.

JEL Classification: J12, J13, D02.

*My work has benefited from useful comments by Rudolf Kerschbamer and seminar participants at Queensland University of Technology. My thanks go also to participants of the XXIIth Econometric Society Australasian Meeting in Canberra (Australia). All remaining errors are my own.

†Dirk Bethmann, Otto-von-Guericke-Universität Magdeburg, Faculty of Economics and Management, Postfach 4120, 39016 Magdeburg, Germany; email: dirk.bethmann@ovgu.de.

1 Introduction

The most basic and commonly used classification of marriage regimes is dichotomous and distinguishes between monogamy and polygamous forms. The latter includes polygyny (one houseband, several wives), polyandry (one wife, several husbands), and cenogamy (several wives and several husbands). Monogamy is by far the most common marriage regime encountered, both today and historically. However, polygyny is still practised in many cultures and regions, notably in Sub-Saharan Africa. Polyandry, in contrast, is rarely observed, and cenogamy is virtually not practised anywhere.[1]

Economists have devoted most of their research to monogamy, and to a lesser extent to the study of polygyny, but neglected by and large polyandry and cenogamy. At first glance, the restriction in focus seems natural, given the predominance of monogamy, the commonness of polygyny, the rareness of polyandry, and the virtual absence of cenogamy. However, in several important respects this restriction is unsatisfactory and also potentially misguiding our understanding of how marriage markets operate. First, it begs the question why marriage, or socially-sanctioned inter-personal unions more generally, have been through history almost exclusively organized only in the aforementioned regimes of monogamy and polygyny, and not also in other possible arrangements, including polyandry, cenogamy, or same-sex marriages.[2] Second, and related, many of the very benefits that are suggested in the literature as prime motives for individuals to form a monogamous (or polygynous) conjugal union are not only asexual in kind, i.e. apply equally to same-sex marriages, but should also with equal force compell individuals to form same-sex, cenogamous, or polyandrous marriages. In fact, most of these motives, including risk sharing, the exploitation of economies of scale and the specialization in household, respectively market work, or the production and consumption of household

[1] Monogamy is the only permitted form of marriage in 14.7% of societies. Polyandry is practiced in only 0.3%. No society has adopted cenogamy as its preferred marital structure. Polygyny is commonly practiced in 46.4% of the world societies, and occasionally practiced in another 35.8% (calculations based on Murdock's Ethnographic Atlas Codebook, 1998, World Cultures 10(1): 86-136.). However, even in polygynous societies the majority of marriages are monogamous (see for example Wright, 1994, p. 91).

[2] Same sex marriages, or equivalent formal arrangements, have historically never been widely practised. Apart from the more recent trend in postmodern societies to allow homosexual marriages (Weston 1997), incidences of formal same-sex marriage have traditionally been practised only in confined ethnic groups. Examples include the woman-woman-marriage among the Nuer, Igbo, and other African societies (cf. Cadigan, 1998), where a barren woman marries another woman to become pater (social father) of her children. The arbitrarily chosen biological father (genitor) plays no social role. The social character is also true for the man-man marriages among the Kwakiutl (cf. Adams, 1981), where "chiefs without heirs would turn their leg or a side of their body into a fictive 'daughter' and marry it off to a son-in-law in order to pass on the legitimizing privileges."

public goods should actually favor polygamous over monogamous marriages.[3] The simple reason is that for each of these motives, the respective benefit that accrues to a single member of a conjugal union is likely to increase (at least over some range) in the number of individuals to that same conjugal union. To cite but one example, consider the risk sharing motive for marriage (cf. Weiss, 1997). If one man and one woman marry and form a single household (pool their resources), they can benefit from reduced income variance, if their respective individual income processes are not perfectly correlated. But, following this line of argument, both can do even better by expanding their conjugal union to a polygamous marriage (of any kind) to reduce the variance of household income still further. Risk sharing but also the other motives mentioned above clearly cannot explain why monogamy predominates, or why among polygamous regimes polygyny occurs much more often than polyandry or cenogamy. Nor can they explain, more generally, why conjugal unions have been traditionally almost exclusively inter-sexual in nature. Finally, the restricted focus in the economics literature on the analysis of monogamy and polygyny potentially carries the risk to miss out on important factors that may underlie observed trends or shifts over time in the relative occurence of particular marriage regimes, the most notable example being the trend decline in polygyny and the concomittant rise in monogamy in many parts of the world (see, for example, Gould, Moav, and Simhon, 2008).

The reasons for these shortcomings, we believe, stem from three related and basic, yet major omissions in most of the economics literature on marriage: the importance of fertility considerations; the importance of sex differences; and the importance of own children (as opposed to children that are not biologically one's own). Regarding fertility considerations, the ubiquitous inter-sexual nature of conjugal unions, be they monogamous or polygamous of any kind, and the traditionally intimate relationship between childbearing and marriage underscore that reproduction must be of central importance for individual marriage decisions in all marriage regimes. Further support for this assertion is found in the little-appreciated fact that own offspring is really the only household good that requires inputs of both sexes for its production. No other household good, nor indeed any of the other aforementioned motives for marriage identifed in the economics literature, necessarily requires conjugal unions to be intersexual in nature. It is surprising that these two facts have been so little appreciated in economic theories of marriage. The bearing and rearing of children, no doubt, has always been mentioned in the economics literature

[3]See the chapter by Weiss (1997) in the Handbook of Family Economics for a review of these major motives.

as a primary motive for marriage. But until fairly recently, most economic theories of marriage simply lacked an explicit account of fertility decisions in their analyses (Weiss, 1997).

Sex differences, well researched in other disciplines, such as evolutionary biology, sociology, anthropology, and psychology, have long been recognized as important determinants of both individual and group behavior. Not so, or to a far lesser extent only, in economics. The human (or more generally, the mammalian) mode of reproduction is characterized by internal fertilization, prolonged gestation, and life birth of offspring. These features give rise to two important asymmetries between women and men that are likely to heavily influence their respective mating and parenting behavior, i.e. the reproductive strategies of the two sexes. First, women can be absolutely sure about their motherhood (maternal certainty), but men have to discount the risk that a kid born to one of their partners has been fathered by another man (paternal uncertainty).[4] This asymmetry is important, for misattributed fatherhood constitutes a major threat to the reproductive success of men and entails significant economic costs in terms of both time and resources expended for parenting. Risks of cuckoldry hence tend to adversely affect male parenting levels, for the more uncertain paternity is, the less willing men are, on average, to invest in their alleged progeny (e.g. Alexander, 1974). At the same time, risks of cuckoldry make promiscuity more attractive for men, both as a means to increase their expected number of offspring (Bateman, 1948) and to diversify these same risks. Second, internal fertilization and gestation, as well as a shorter female than male lifespan of fertility, significantly constrains the reproductive capacity of women relative to that of men. In fact, for men, number of own offspring is virtually limited only by their access to fertile mating partners. In the non-economics literature, both asymmetries have been long documented to exert a qualitatively similar influence on the respective mating behavior of women and men and on the amount of parental investment each sex is willing to devote to offspring, i.e. the respective reproductive strategies pursued by the two sexes. In their strive for reproductive success, a function of both the quantity and the quality of own children, men seek to maximize offspring quantity rather than quality, with the reverse holding true for women (Trivers, 1972; Symons, 1979). In other words, men more than women tend to aspire to multiple partners and women tend to contribute more than men to the rearing of children.[5] These

[4] The cross-cultural and historic ubiquity of concerns about paternity uncertainty are vividly underscored in proverbs such as "mama's babay, papa's maybe", the english equivalent to the ancient roman dictum "mater certissima, pater semper incertus".

[5] Male competition for females, a characteristic feature of the human mating system noted already by Charles Darwin in his work on the descent of man (Darwin, 1871)

behavioral differences between the two sexes have direct implications for the kind of marriages that men, respectively women, if otherwise unconstrained, would want to engage in and would want to avoid. Incomplete information on individual paternity gives rise to externalities only in polyandrous and cenogamous marriages, but not in monogamous or polygynous ones. In other words, in the former, within-marriage "cuckoldry" is possible, which carries the risk for a polyandrous or cenogamous husband to misattribute parental investment to offspring that is not his own but of another husband. This is not the case in the latter. Men should therefore, and generally, strive for monogamous or polygynous conjugal unions. At the same time, limited female reproductive capacity should induce men to aspire more to polygynous than to monogamous marriages. Cenogamy, in turn, like polygyny, provides a man with access to more wives than does monogamy, but unlike polygyny it also carries the risk of cuckoldry because of the presence of co-husbands, a risk that is all the greater in polyandry which has several husbands but only one wife.

The importance of own children has long been noted by economists, but largely neglected in formal theoretical analysis of marriage.[6] Becker, for example, anticipates that the establishment of paternity limits the extent to which statements about the incidence of polygyny can be mirrored on the occurrence of polyandry. He argues that "own children are strongly preferred to children produced by others". He also mentions the fact that most polyandrous marriages are fraternal and states that "children of relatives are preferred to those of strangers" (Becker, 1991, p. 102). Despite these insights, Becker's theoretical approach is not differentiating between own and other offspring. Individuals derive utility from being with children and since there is no market where children can be bought they have to be sired. Edlund stresses in her work the universality of the paternity presumption in marital arrangements across societies (Edlund, 2005) and is consequently modelling marriage as a contract that transfers custodial rights from women to men (see also Edlund and Korn, 2002). Cox (2003) considers paternal uncertainty in the context of intervivos gifts and Saint-Paul (2008) shows that paternity uncertainty of males and limited reproductive capacity of females lead to positive assortative matching and potentially less human capital for offspring. The first formal treatment of the importance of own children in the context of marriage can be found in Bethmann and Kvasnicka (2011).[7] There, the existence of marriage as an institution is attributed to the need to curb cuckoldry for the purpose of paternity certainty and biparental investment in offspring. It is shown that

[6] However, more and more economists seem to think that the inclusion of evolutionary approaches in the economics of the family might provide new insights. See Cox and Fafchamps (2007) for a discussion.

[7] The earliest preprint of this paper is Bethmann and Kvasnicka (2005).

unregulated mating leads to market failure and that a stringent marriage regime is able to realize a second best outcome. Francesconi et al (2010) use a similar argument in a model with overlapping cohorts and mate guarding among men. Own children is not only the only household good that requires inputs of both sexes for its production, as noted above, it is also the only household good whose utility from consumption depends on who produced the good. This simple fact is of utmost importance in polygamous unions because children can no longer be classified as household collective goods as would be the case in the traditional economics literature. In polygamous marriages, a child, which by nature has only one mother and one father, ceases to be a household collective good for any third party (male or female) in that marriage (only parents by definition derive utility from own kids). This creates conflict between parents and third parties about the individual allocation of resources, including possibly common household resources.

We believe that fertility considerations, sex differences, and the greater valuation of own children are key for understanding and explaining the rank order of marriage regimes in terms of their relative occurences, and by extension, also their changing fortunes over time. The rank order, both stable over time and ubiquitously encountered, can by definition only be caused by equally stable and ubiquitously encountered factors. Fertility considerations, sex differences, and the importance of own children are such stable and ubiquitously encountered factors, as none is confined to particular cultures, times, economic systems, or regions of the world. Marriage has always been intimately linked to fertility, and asymmetries in offspring recognition and in reproductive capacity between the two sexes are instrinsic features of the human mode of reproduction. Similarly, a greater valuation of own offspring is a deeply ingrained evolutionary drive. As none of the aforementioned factors is in any way confined to particular cultures, times, economic systems, or regions of the world, these reproductive concerns clearly qualify as potential candidate explanations. What is more, ample corroborative evidence underscores their pivotal and ubiquitous importance for mating markets and their societal regulations. Examples include inferior investment in offspring by patrilineal rather than matrilineal kin (e.g. Gaulin et al., 1997), greater investment of brothers into the children of their sisters in societies where own paternity is highly uncertain (Alexander, 1979), the dependence of rules of inheritance of a man's property across societies on paternal confidence (Gaulin and Schlegel, 1980; Hartung, 1985), and the relationship between average levels of paternal uncertainty and rules governing consanguineous marriages or inbreeding in societies (Greene, 1978). Moreover, the once exclusive definition of adultery with respect to the marital status of women in many societies (e.g. Bullough, 1976) and the general importance attached to female premarital chastity (e.g. Buss, 1989) both bear witness to the pivotal role of own paternity.

In our analysis, child quality has the character of a collective good among parents, i.e. the father benefits from the mother's contributions to the quality of her child and vice versa. Because individuals do not make decisions as one generic actor but instead behave non-cooperatively, they do not internalize the effects on their partners' utility when contributing to the quality of a child. Samuelson's condition for the efficient provision of a collective good (cf. Samuelson, 1954), therefore, will not be met. However, it turns out that polygamous unions even have difficulties to ensure a minimum constraint on technical efficiency where the ratio of female and male utility losses stemming from their marginal child quality contributions is equal to the technical rate of substitution between female and male contributions. This observation is particularly true for those unions with multiple husbands. Monogamy, in contrast, achieves this technical efficiency. In principle, polygyny can achieve it, too. However, not necessarily so, and what is even more competition for male support among cowives might lead to welfare losses. Polyandrous arrangements lead to inefficient child quality contributions and are therefore quite unattractive. Consequently, these but also cenogamous arrangements carry some inherent instability.

Intuitively, these results are easily understood. Monogamy completely removes information asymmetries stemming from paternal uncertainty. Polygyny also ensures that males can be certain about their fatherhood. However, the allocation of the husband's quality contributions among his children has the potential for jealousy and conflict among his cowives. Under polyandry, paternity cannot be established with certainty such that males only maximize expected utility. Male quality contributions, as a consequence, are only suboptimal. A cenogamous marriage is unstable because there is often one couple that could form a monogamous union that provides a higher level of utility.

2 The Model

The following analysis compares the different marriage regimes in terms of their respective costs and benefits. We gain insights into the internal mechanisms such that we can explain the widespread occurrence of polygyny, the rareness of polyandry, and the nonappearance of cenogamy. For simplicity of exposition, we will assume that marriage is a necessary pre-condition for the bearing and rearing of children. That is the costs of out-of-wedlock childbearing are assumed to be infinitely high. The interested reader is referred to Willis (1999) for an excellent paper on the theory of out-of-wedlock child bearing. The reproductive nature of our definition of marriage requires any marital arrangement to consist of a male and a female part (husband and wife). Depending on whether a role is filled with one or several individuals, one can distinguish between monogamy, polygyny, polyandry, and cenogamy. Our focus is on the different implications that these marital structures have on the efficiency of human reproduction.

Furthermore, we restrict our analysis to marital arrangements with at most two members of the same sex in the same union. As with the perfect enforcement of marriage as a pre-condition for child birth this assumption merely simplifies our exposition. The results, however, can of course be generalized. Table 1 summarizes the composition of the four marital arrangements.

Table 1: Types of Marital Structure

Marital arrangement	Husbands	Wives
Monogamy	one	one
Polygyny	one	two
Polyandry	two	one
Cenogamy	two	two

2.1 The Fertile Population

Consider a society populated by single-period lived fertile women and men with equal preferences, where the two sexes differ with respect to their reproductive technology. More formally, the fertile population consists of two disjoint subsets. First, the set $\female$ of female individuals with typical elements f, g, and h:

$$f, g, h \in \female \tag{1}$$

Second, the set $\male$ of male individuals with typical elements m and n:

$$m, n \in \male \tag{2}$$

Finally, let $\circ = \female \cup \male$ denote the fertile population with typical element i:

$$i \in \circ \tag{3}$$

All individuals are rational and act non-cooperatively. They can spend their economic resources on two competing uses: own non-reproductive consumption and parental investment in children. Economic resources summarize an individual's endowment with wealth, time, and other means. The respective level of endowments with these resources may well vary among individuals.

2.2 Preferences, Endowments, and Reproductive Technology

Male and female individuals have economic resources Y_i and derive separable utility W from consumption C_i and reproductive success, which is a function of both the number K_i and the quality Q of own biological offspring:

$$W(C_i, K_i, Q) = U(C_i) + V(K_i, Q) \tag{4}$$

As Q enters into the utility functions of both sexes, maternal investment in children affects paternal utility and vice versa. Child quality, therefore, has collective good character (see among others Smith, 1977, or Weiss and Willis, 1985). Function U is assumed to be twice differentiable and increasing at a diminishing rate. The same assumptions hold for function V with respect to Q the (continuous) quality of children, i.e.

$$\frac{\partial^2 U(C)}{\partial C^2} < 0 < \frac{\partial U(C)}{\partial C} \qquad \text{and} \qquad \frac{\partial^2 V(K,Q)}{\partial Q^2} < 0 < \frac{\partial V(K,Q)}{\partial Q} \tag{5}$$

With respect to K, the (discrete) number of children, we still assume that V is increasing and strictly concave, but differentiability is not necessary. Furthermore, we assume that utility from reproduction is equal to zero if the quantity and/or quality of own offspring is zero:

$$V(0, Q) = V(K, 0) = 0 \tag{6}$$

We want to stress that sex is an important individual characteristic in our model. In particular, we focus on two differences with respect to reproductive technologies. Extended times of gestation and lactation, as well as a shorter life-time span of fertility, constrain the reproductive capacity of females (limited reproductive capacity, see, for example, Trivers, 1972). Following Willis (1999), we will assume that this limitation is always binding and normalize the number of births per woman to one, i.e.

$$K_f = 1 \tag{7}$$

Moreover, because of internal fertilization and life birth women are always certain about their motherhood (maternal certainty). Male reproductive capacity is only limited by the number of female mating partners, i.e.

$$K_m \in \{0, .., |F_m|\} \tag{8}$$

where F_m is the set of females a man has mated and $|F_m|$ denotes the number of women in this set. Note that it is possible that a man fathers no child at all even though he has mated with several women. In case his partners have mated with other men, he has to account for the possibility that a child is fathered by someone else. In fact the probability of biological fatherhood (δ) is inversely related to the number of men his partner has mated:

$$\delta_f = \frac{1}{|M_f|} \tag{9}$$

where M_f is the set of men female f has mated and $|M_f|$ denotes the number of men in this set. As a consequence, men may only maximize their expected utility from reproductive success in (4).

Contributions to child quality have their price. We denote by a_s, with $s \in \{♀, ♂\}$, the marginal cost at which additional contributions to child quality can be obtained. Individual contributions to the quality of a child are given by q_{if}, where i stands for the individual that invests and f represents the beneficiary child with mother f.[8] Thus female and male individuals face the following budget constraints:

$$Y_f = C_f + a_{♀} q_{ff} \qquad \text{and} \qquad Y_m = C_m + a_{♂} \sum_{f \in F_m} q_{mf} \tag{10}$$

By choosing q_{if}, individuals maximize their (expected) utility (4) subject to their budget constraints (10).

2.3 Parental Investment and Child Quality

Child quality $Q = Q(q_{♀}, q_{♂})$ is determined by an aggregator function with maternal and paternal investments as arguments. We assume that both the female and the male contributions display positive but diminishing marginal returns to the quality of a child:

$$\frac{\partial^2 Q(q_{♀}, q_{♂})}{\partial q_{♀}^2} < 0 < \frac{\partial Q(q_{♀}, q_{♂})}{\partial q_{♀}} \qquad \text{and} \qquad \frac{\partial^2 Q(q_{♀}, q_{♂})}{\partial q_{♂}^2} < 0 < \frac{\partial Q(q_{♀}, q_{♂})}{\partial q_{♂}} \tag{11}$$

Furthermore, we assume that function Q is characterized by constant returns to parental investments taken together:

$$Q(tq_{♀}, tq_{♂}) = tQ(q_{♀}, q_{♂}) \tag{12}$$

Maternal certainty means that the biological mother is the only female that contributes to the quality of a child. With paternal uncertainty, in contrast, all partners the mother

[8]Since we assume that mothers bear exactly one child, the subscript f uniquely identifies a child.

has mated qualify as potential fathers of her child. As a result, all of them will invest in the quality of their putative offspring. Total male contributions ($q_{\male f}$) to the quality of the child mothered by female f, therefore, are the sum of the individual male contributions:

$$q_{\male f} = \sum_{m \in M_f} q_{mf} \tag{13}$$

Because of the collective good character of child quality, therefore, the true but unknown biological father may benefit from contributions of other men to the quality of his child. As a consequence, total quality of the child mothered by female f is given by:

$$Q_f = Q(q_{\female f}, q_{\male f}) \tag{14}$$

where we have used $q_{\female f} = q_{ff}$.

A man who has mated more than one female partner needs to be discussed in greater detail. Since his female partners and hence their offspring might not be identical (because we allow for heterogeneous endowments), the question arises what his reproductive success with respect to quality is. We argue that men care about some measure of average child quality. To ensure that our results are as general as possible we assume the following constant elasticity of substitution aggregation procedure:

$$\bar{Q}\Big(\bigcup_{f \in F_m} Q_f\Big) = \Big[\sum_{f \in F_m} \frac{1}{|F_m|} Q_f^{\frac{\phi-1}{\phi}}\Big]^{\frac{\phi}{\phi-1}} \tag{15}$$

where Q_f is the quality of a child mothered by female f. Since the numbering of children should not influence a man's utility, we use the same share parameter for each child ($1/|F_m|$). The parameter ϕ measures the degree of substitutability, such that its inverse relates to the degree of complementarity. If $\phi \to \infty$ holds, the quality of one child is a perfect substitute for the quality of another child and males simply focus on the arithmetic mean of offspring quality. In case $\phi \to 0$ holds, the quality of one child is a perfect complement for the quality of another child. In this case, the child with the lowest quality determines $\bar{Q}$ and males tend to favor child quality investments that eliminate differences between their children. In between those extremes, our specification allows for the whole range. Note for example that average child quality $\bar{Q}$ refers to the geometric mean when the degrees of substitutability and complementarity are the same ($\phi = 1$).

2.4 Reproduction and Marital Structure

In the following analysis, the bearing and rearing of children requires that agents are married. Each marital structure (summarized in Table 1), however, has certain implications on individual reproductive success. The sets of mating partners of monogamous and polygynous women consist of only one element, such that monogamous and polygynous men can be sure about their fatherhood (see equation (9) above). Polyandrous and cenogamous women, in turn, mate with two men such that polyandrous and cenogamous men have to account for the possibility that the child is fathered by their cohusbands (paternal uncertainty). In the case of polyandry, the expected number of children is one half, $\mathbb{E}[K_m|\text{polyandry}] = \frac{1}{2}$, in the of cenogamy, $\mathbb{E}[K_m|\text{cenogamy}] = 1$. In polygynous and cenogamous unions, a man invests in two children. Monogamous and polyandrous men, in contrast, invest in a single child.

3 Analysis

In this section we analyse female and male behavior under the different marriage regimes. Under monogamy, both spouses will have only one child and they will both be absolutely certain about their parenthood. As a consequence, their utility maximization problems are identical and can therefore be studied simultaneously (cf. Section 3.1). Moreover, given their husbands' decisions, the optimization problems of polygynous, polyandrous, and cenogamous wives are identical to the monogamous case such that we can refer to the female strategy under monogamy. Optimal male strategies in contrast do depend on the respective regime (see Sections 3.2, 3.3, and 3.4 for the polygynous, polyandrous, and cenogamous cases).

3.1 Monogamy

Under the monogamous marriage regime, individuals have exactly one partner of the opposite sex. On the one hand, this reduces the natural reproductive capacity of males. The only limit nature sets is access to female partners. On the other hand, any uncertainty about biological fatherhood is eliminated under monogamy. As a result, male and female individuals face the same optimization problem. Spouses choose their optimal contribution to the quality of their common child q_{sf}, $s \in \{♀, ♂\}$, to maximize utility:

$$\{q_{sf}\} = \arg\max \{U(Y_i - a_s q_{sf}) + V(1, Q)\}, \qquad s = \begin{cases} ♀, & \text{if} \quad i = f \in ♀ \\ ♂, & \text{if} \quad i = m \in ♂ \end{cases} \tag{16}$$

Their first order necessary conditions read as follows:

$$a_s \frac{\partial U(C_i)}{\partial C} = \frac{\partial V(1, Q)}{\partial Q} \frac{\partial Q(q_{♀f}, q_{♂f})}{\partial q_s}, \qquad s = \begin{cases} ♀, & \text{if} \quad i = f \in ♀ \\ ♂, & \text{if} \quad i = m \in ♂ \end{cases} \tag{17}$$

where C_i denotes the non-reproductive consumption of individual i. Optimal contributions to child quality equate the marginal losses in non-reproductive utility to the marginal increase in utility due to higher child quality. Because of the collective good character of child quality, however, the wife's decision influences the husband's and vice versa. By dividing their respective optimality conditions, we obtain the following expression:

$$\frac{a_{♂}}{a_{♀}}\frac{\partial U(C_m)/\partial C}{\partial U(C_f)/\partial C} = \frac{\partial Q(q_{♀f}, q_{♂f})/\partial q_{♂}}{\partial Q(q_{♀f}, q_{♂f})/\partial q_{♀}} \tag{18}$$

Hence, the ratio of female to male marginal child quality costs multiplied by the shadow values of lowering the respective budget constraints is equal to the technical rate of substitution between female and male contributions to child quality. The case where a marginal male contribution is used to reduce female contributions, so that the quality of the child remains constant, illustrates the basic intuition. The condition simply says that the parents are in optimum when the mother's utility gain equals the father's loss in utility. Note that this ensures some kind of technical efficieny in the generation of child quality. Samuelson's condition[9] for efficiency in the provision of a collective good, of course, is not met here because individuals behave non-cooperatively.[10] In the following we will see that even the minimal requirement of technical efficiency in (18) might fail in polygamous unions.

From equation (18), we can infer that the marriage market outcome would be characterized by positive assortative matching with respect to wealth (Y) because a wealthier man attracts a wealthier woman and vice versa. The fact that the efficiency condition holds provides a rationale for mating market regulations that aim at enforcing monogamous mating.

3.2 Polygyny

In polygynous marital arrangements, men gain access to more than just one female partner as is the case under monogamy. Consequently, male reproductive capacity is enhanced. At the same time, husbands can be certain about their fatherhood under polygyny. Hence, polygynous marriages are favorable for men. The utility maximization problem for a man with two wives reads as follows:

$$\{q_{♂f}, q_{♂g}\} = \arg\max \left\{U(Y_m - a_{♂}q_{♂f} - a_{♂}q_{♂g}) + V\left(2, \bar{Q}(Q_f, Q_g)\right)\right\} \tag{19}$$

[9] Samuelson's condition requires the right hand side of the first order conditions in (17) to be multiplied by two such that child quality will be higher.

[10] Non-cooperative behavior is, of course, crucial in our analysis. Note, however, that the costs of enforcing and controlling collusive agreements are likely to increase with the number of individuals such that cooperative behavior within the different marital agreements comes at different costs.

where Q_f (Q_g) refers to the child mothered by wife f (g). Turning to the husband's first order necessary conditions with respect to his child quality contributions, we obtain for the optimal $q_{♂f}$:

$$a_{♂}\frac{\partial U(C_m)}{\partial C} = \frac{\partial V(2,\bar{Q})}{\partial Q}\frac{1}{2}\left(\frac{\bar{Q}}{Q_f}\right)^{\frac{1}{\phi}}\frac{\partial Q(q_{♀f}, q_{♂f})}{\partial q_{♂}} \tag{20}$$

A comparison of (20) with its equivalent under monogamy (17) reveals two differences. The first difference refers to the quantity of own offspring. The fact that V is evaluated at $K_m = 2$ shows that a polygynous husband benefits from a higher quantitative reproductive success. Note that this is not the case for the polygynous cowives (K_f and K_g are still equal to one). A fact that introduces an asymmetry between spouses under polygyny. The second difference is related to the husband's child quality contributions. Since the two children are mothered by different women, male contributions to child quality might have different marginal effects on average child quality $\bar{Q}$. In fact, equation (20) and its equivalent for $q_{♂g}$ imply that the polygynous husband's marginal utilities stemming from children f and g must be the same. Consider, for example, the case where $q_{♀f} > q_{♀g}$ holds, that is cowife f contributes more to the quality of her child than cowife g. The polygynous husband will choose child quality investments such that

$$\frac{q_{♂f}}{q_{♀f}} \leq \frac{q_{♂g}}{q_{♀g}} \qquad \Leftrightarrow \qquad \frac{q_{♂f}}{q_{♂g}} \leq \frac{q_{♀f}}{q_{♀g}} \tag{21}$$

Hence, male child quality investments are such that the marginal change in the quality of child f is never higher than the marginal change in the quality of child g:

$$\frac{\partial Q(q_{♀f}, q_{♂f})}{\partial q_{♂f}} \leq \frac{\partial Q(q_{♀g}, q_{♂g})}{\partial q_{♂}} \tag{22}$$

In this sense, the husband will never exaggerate child quality differences. However, given $q_{♀f} \geq q_{♀g}$ holds it can be shown that the quality of child f is never below that of child g, that is $Q_f \geq Q_g$. These observations hold independently of the degree of substitutability between the qualities of the two children (ϕ).

Dividing condition (20) by its equivalent for the contributions to the quality of child g helps us understand how the husband allocates resources between his two offspring:

$$\frac{\partial Q(q_{♀f}, q_{♂f})/\partial q_{♂}}{\partial Q(q_{♀g}, q_{♂g})/\partial q_{♂}} = \left(\frac{Q_f}{Q_g}\right)^{\frac{1}{\phi}} \tag{23}$$

Hence, at the optimum the polygynous man contributes to the quality of his children such that the ratio of the marginal changes in each child's quality is equal to the technical rate of substitution between the quality of his two children. Since the man derives utility from the average quality of his children, he only has to consider the marginal effect of his contributions on each child and how this affects the average quality of his offspring. Consequently, functions U and V which specify his preferences play no role. In a sense, one can think of the man's decision as follows. After deciding how many resources to invest in total, he has to decide how to allocate them among his children. In particular, he will ensure that the ratio of the marginal child quality changes with respect to male contributions equals the technical rate of substitution between the qualities of his two children.

At first glance this mechanism seems to be rather technical, but note that there is a huge potential of conflict between the husband and his wives but also among the cowives. One reason why a mother invests in her offspring may be to attract investments by her husband. This is possible since her contributions change the technical rate of substitution between the qualities of his two children, i.e. the right hand side of (23). Therefore, a contribution by one cowife changes the husbands allocation of male contributions among his children which in turn affects the other cowife. The extent to which a wife is able to attract additional male child quality contributions depends on the degree of substitutability (respectively on the degree of complementarity) between female and male child quality contributions.

Anthropologists have long noted that cowives have difficulties to get along (see, for example, Stephens, 1963; Jankowiak, Sudakov, and Wilreker, 2005). Among economists, Shoshana Grossbard has also studied conflict among cowives. In her book (Grossbard-Shechtman, 1993, p. 236/7), she lists four arrangements that aim to reduce frictions in polygynous unions. First, a separate dwelling for each cowife (so-called hut polygyny) which is practiced in more than 62 percent of all polygynous societies (cf. Murdock, 1967). Second, supervisory authority in the hands of the senior cowife. Third, customs where husbands cohabitate with each cowife in regular rotation. Fourth, sororal polygyny where the husband is married to a group of sisters (one in seven polygynous societies prefers sororal arrangements, see again Murdock, 1967). All four arrangements show that the potential for conflict among cowives is taken seriously in polygynous societies. In principle, however, our efficiency criterion can still be met.

The efficiency condition is obtained by dividing the female and male first order necessary conditions:

$$\frac{a_{\male}}{a_{\female}} \frac{\partial U(C_m)/\partial C}{\partial U(C_f)/\partial C} = \frac{\partial Q(q_{\female f}, q_{\male f})/\partial q_{\female}}{\partial Q(q_{\female f}, q_{\male f})\partial q_{\male}} \frac{1}{2} \left(\frac{\bar{Q}}{Q_f} \right)^{\frac{1}{\phi}} \frac{\partial V(2, \bar{Q})/\partial Q}{\partial V(1, Q_f)/\partial Q} \tag{24}$$

As mentioned above, the change in average child quality influences the husband's decisions and because there is no corresponding expression for his wives, it appears in the efficiency condition. Similarly, female and male marginal reproductive success with respect to child quality do not cancel each other out because both sexes' quantitative reproductive success differs. Nevertheless, (close to) efficient outcomes are attainable if the following conditions hold:[11]

$$\frac{1}{2}\left(\frac{\bar{Q}}{Q_f}\right)^{\frac{1}{\phi}} \approx \frac{\partial V(1,Q_f)/\partial Q}{\partial V(2,\bar{Q})/\partial Q} \qquad \text{and} \qquad \frac{1}{2}\left(\frac{\bar{Q}}{Q_g}\right)^{\frac{1}{\phi}} \approx \frac{\partial V(1,Q_g)/\partial Q}{\partial V(2,\bar{Q})/\partial Q} \tag{25}$$

Our analysis of polygynous unions shows that a man's endowments Y_m determine whether he can attract a second wife. In fact, there is huge evidence that wealthy men engage in polygynous mating, whereas men of low economic status may have to remain monogamous, or even single (see, for example, Grossbard-Shechtman, 1986). Polygynous cultures, both past and present, seem to conform well with this characterization (see, for example, Betzig, 1992/95, for evidence on ancient Greece, the Roman Empire, and various European cultures in the Middle Ages; or Borgerhoff Mulder, 1987, and Sellen, 1999, on contemporaneous African societies).

3.3 Polyandry

As we have seen, polygyny may be viewed as a means of men to increase their reproductive success by increasing their number of offspring. Similarly, polyandry might help women to acquire more male quality contributions. However, a polyandrous man (m) is uncertain about his fatherhood, i.e. he can only maximize expected utility:

$$\{q_{mf}\} = \arg\max\left\{U(Y_m - a_{♂} q_{mf}) + \mathbb{E}[V(K_m, Q)]\right\} \tag{26}$$

Since a polyandrous woman mates with two husbands, their probability of fatherhood is only one half. As a consequence, expected male utility from reproduction is given by:

$$\mathbb{E}[V(K_m, Q)] = \frac{1}{2} V(1, Q) \tag{27}$$

Note that equation (6) allows us to drop the expectations operator in (26). The first order necessary conditions of the two husbands under polyandry are as follows:

$$a_{♂}\frac{\partial U(C_m)}{\partial C} = \frac{1}{2}\frac{\partial V(1,Q)}{\partial Q}\frac{\partial Q(q_{♀f}, q_{mf} + q_{nf})}{\partial q_{♂}} = a_{♂}\frac{\partial U(C_n)}{\partial C} \tag{28}$$

[11] Note that these conditions can be true for both wives, for one, but also for none.

where n denotes the second husband. In contrast to the monogamous case in equation (17), marginal utility from reproduction in (28) is divided by two which is due to the fact that men maximize only expected utility. Apart from that, the interpretation of condition (28) is similar to that of equation (17). Note, however, that the true but unknown father free rides on his cohusband's child quality contributions. Division of the husband's first order necessary condition (28) by his wife's leads us to:

$$\frac{a_{♂} \partial U(C_m)/\partial C}{a_{♀} \partial U(C_f)/\partial C} = \frac{1}{2} \frac{\partial Q(q_{♀f}, q_{mf} + q_{nf})/\partial q_{♂}}{\partial Q(q_{♀f}, q_{mf} + q_{nf})/\partial q_{♀}} \tag{29}$$

This expression is necessarily suboptimal (cf. equation (18)) and its deviation from the efficient outcome is caused by paternal uncertainty. Female contributions to child quality are too high, or male contributions too low, or both.[12] Moreover, if the husbands' endowments are not equal, the wealthier husband contributes more to the quality of his putative offspring:

$$Y_m > Y_n \qquad \Leftrightarrow \qquad q_{mf} > q_{nf} \tag{30}$$

In fact, the poorer husband may even completely free ride on his cohusband's contributions in case his resources are much lower than those of his cohusband:

$$a_{♂} \frac{\partial U(C_m)}{\partial C} \leq \frac{\partial U(Y_n)}{\partial C} \qquad \Leftrightarrow \qquad q_{nf} = 0 \tag{31}$$

In this case, it can be shown that the wealthier husband and his wife would have been better off in a monogamous union. Such a polyandrous union, therefore, is likely to be dissolved, or better: it is very unlikely to be formed in the first place. This result and the case where the poorer cohusband does contribute to the child's quality are summarized in our first proposition.

Proposition 1 *Consider a polyandrous marriage with two cohusbands (m and n). Without loss of generality, assume that $Y_m \geq Y_n$ holds. Cohusband m would be better off in a monogamous union with a noninvolved female g that contributes as much resources to child quality as his polyandrous wife f, that is $q_{♀f} = q_{♀g}$. In case cohusband n restrains totally from contributing to child quality ($q_{nf} = 0$), the polyandrous spouses m and f would have been better off in a monogamous union.*

The Appendix provides a formal proof of this result.

[12] This result constitutes an example of the Hamilton rule (cf. Cox, 2003).

3.4 Cenogamy

Cenogamy occassionally occurs in polyandrous societies (Ingoldsby, 2006, p. 109). A second wife may be brought into a polyandrous arrangement when one or more cohusbands are unhappy with their marriage, with barrenness of the wife being one but not the only reason. Nancy Levine (1988, p. 149) writes that cenogamous marriages "continue to be initiated - quite deliberatedly - and the proximate reason is to ensure equity in marriage for all brothers." The polyandrous wife faces male expectations of a son from marriage and has to satisfy domestic labor obligations. Consequently, Levine's observations suggest that the probability of turning fraternal polyandrous arrangements into cenogamous marriages mainly depends on two factors. Cenogamy is more likely to occur the higher the number of married brothers. The same is true, in case the brothers do not share the same parentage. From a societal point of view, however, cenogamous arrangements are "strongly discouraged because bringing more than one childbearing woman into the marriage makes serious domestic discord more likely, breaks down fraternal solidarity by creating separate families, and lays the foundations for partition" (Levine, 1988, ibidem).

From an analytical point of view, cenogamy bears features of polygyny and polyandry. On the one hand, men invest in offspring born to several wives like in a polygynous union. On the other hand, children maybe fathered by a cohusband like in a polyandrous union. The optimization problem of a man living in cenogamy is therefore given by:

$$\{q_{mf}, q_{mg}\} = \arg\max \{U(Y_m - a_{♂} q_{mf} - a_{♂} q_{mg}) + \mathbb{E}[V(K_m, Q)]\} \tag{32}$$

where q_{mf} (q_{mg}) denotes cohusband m's contributions to the quality of the child born to cowife f (g). Note that paternal uncertainty implies the following for his expected utility from reproductive success:

$$\mathbb{E}[V(K_m, Q)] = \frac{1}{4} V(1, Q_f) + \frac{1}{4} V(1, Q_g) + \frac{1}{4} V\left(2, \bar{Q}(Q_f, Q_g)\right) \tag{33}$$

As a result, the cenogamous husband contributes to the quality of the child mothered by cowife f according to the following first order necessary condition (first equality):

$$a_{♂} \frac{\partial U(C_m)}{\partial C} = \frac{1}{4} \frac{\partial Q(q_{♀f}, q_{♂f})}{\partial q_{♂}} \left[\frac{\partial V(1, Q_f)}{\partial Q} + \frac{\partial V(2, \bar{Q})}{\partial Q} \frac{1}{2} \left(\frac{\bar{Q}}{Q_f} \right)^{\frac{1}{\phi}} \right] = a_{♂} \frac{\partial U(C_n)}{\partial C} \tag{34}$$

where $q_{♂f}$ denotes *total* male investments to the quality of the child mothered by wife f, that is $q_{♂f} = q_{mf} + q_{nf}$. Because of the undetermined paternity and the collective good character of child quality, the expected marginal increase of reproductive success stemming from an additional child quality contribution is the same for both cohusbands. This is of course true with respect to both children. As a consequence, the marginal utility from non-reproductive consumption is the same for both cohusbands (second equality). We are led to the following observation:

Observation 1 *Let $q_{♀f} > q_{♀g}$ hold, that is cowife f contributes more to the quality of her child than cowife g. Then the two cohusbands will choose child quality investments such that*

$$\frac{q_{♂f}}{q_{♀f}} \leq \frac{q_{♂g}}{q_{♀g}} \qquad \Leftrightarrow \qquad \frac{q_{♂f}}{q_{♂g}} \leq \frac{q_{♀f}}{q_{♀g}} \tag{35}$$

holds. Hence, male child quality investments are such that the marginal change in the quality of child f is never higher than the marginal change in the quality of child g:

$$\frac{\partial Q(q_{♀f}, q_{♂f})}{\partial q_{♂f}} = \frac{\partial Q(1, \frac{q_{♀f}}{q_{♂f}})}{\partial q_{♂f}} \leq \frac{\partial Q(1, \frac{q_{♀g}}{q_{♂g}})}{\partial q_{♂}} = \frac{\partial Q(q_{♀g}, q_{♂g})}{\partial q_{♂}} \tag{36}$$

In this sense, the two cohusbands will never exaggerate child quality differences. The quality of child f, however, is still never below the quality of child g, that is $Q_f \geq Q_g$. These observations hold independently of the degree of substitutability between the qualities of the two children (ϕ).

In case that the two cohusbands' differ with respect to their economic wealth, the left hand side of equation (34) implies that the wealthier cohusband contributes more to the quality of the two children:

$$Y_m > Y_n \qquad \Leftrightarrow \qquad q_{mf} + q_{mg} > q_{nf} + q_{ng} \tag{37}$$

The exact shares a man devotes to his two putative offspring, however, can not be determined. But because of the symmetry of (34) between the two husbands and across the two children, there is no source for conflict among cohusbands. A closer look at the cohusbands' conditions for optimal child quality contributions helps us to understand how they allocate resources between their two putative offspring:

$$\begin{aligned} &\frac{\partial Q(q_{♀f}, q_{♂f})/\partial q_{♂}}{\partial Q(q_{♀g}, q_{♂g})/\partial q_{♂}} \\ = \; &\left(\frac{\partial V(1, Q_g)}{\partial Q} + \frac{\partial V(2, \bar{Q})}{\partial Q}\frac{1}{2}\left(\frac{\bar{Q}}{Q_g}\right)^{\frac{1}{\phi}}\right) \Big/ \left(\frac{\partial V(1, Q_f)}{\partial Q} + \frac{\partial V(2, \bar{Q})}{\partial Q}\frac{1}{2}\left(\frac{\bar{Q}}{Q_f}\right)^{\frac{1}{\phi}}\right) \end{aligned} \tag{38}$$

Note the difference to equation (23). In contrast to the polygynous man, the child quality contributions of a cenogamous man are not necessarily such that the ratio of the marginal changes in each child's quality is equal to the technical rate of substitution between the quality of his two children. The reason is that the cenogamous man deals with putative offspring. As a consequence, the concavity of function V comes into play. Nevertheless, like their polygynous counterparts cenogamous cowives might also try to attract male child quality contributions which in turn can affect the other cowife.

Finally, we study the efficiency condition obtained by dividing the male and female first order necessary conditions:

$$\frac{a_{\male}\partial U(C_m)/\partial C}{a_{\female}\partial U(C_f)/\partial C} = \frac{1}{4}\frac{\partial Q(q_{\female f}, q_{\male f})/\partial q_{\male}}{\partial Q(q_{\female f}, q_{\male f})/\partial q_{\female}}\left[1 + \frac{\partial V(2,\bar{Q})/\partial Q}{\partial V(1,Q_f)/\partial Q}\frac{1}{2}\left(\frac{\bar{Q}}{Q_f}\right)^{\frac{1}{\phi}}\right] \tag{39}$$

Again, we have the technical rate of substitution on the right hand side. As in polyandry, the true but unknown father free rides on his cohusbands child quality contributions and men only maximize expected utility from reproduction. Like in polygyny husbands have to consider that investments in one child only slightly change average child quality (third term). Also, female and male marginal reproductive success due to changes in average child quality appear but - as already mentioned as a polyandrous feature - only in expected value for the male expression.

Similarly to the polyandrous case, it can be shown that the wealthier husband and the wife that contributes more to the quality of her child could be better off in a monogamous union under the assumption that differences among wives are not too big.

Proposition 2 *Consider a cenogamous marriage with two cowives (f and g) and two cohusbands (m and n). Without loss of generality, assume that $Y_m \geq Y_n$ and $q_{\female f} \geq q_{\female g}$ (and hence $Y_{\female f} \geq Y_{\female g}$) hold. Cohusband m would be better off in a monogamous union with a noninvolved female h that contributes as much resources to child quality as his cenogamous wife f, that is $q_{\female f} = q_{\female h}$.*

In case that the qualities of children f and g are perfect complements in the average child quality function, the cenogamous spouses m and f would have been better off in a monogamous union. The same is true, in case cohusband n's sum of contributions to the qualities of children f and g is such that the following holds:

$$q_{nf} + q_{ng} \leq (q_{mf} + q_{mg})\frac{q_{\female g}}{q_{\female f}} \qquad \Leftrightarrow \qquad \frac{q_{\female f}}{q_{\female g}} \leq \frac{q_{mf} + q_{mg}}{q_{nf} + q_{ng}} \tag{40}$$

3.5 The Marriage Market

Having analyzed the different marital arrangements from an individual perspective, we will now investigate a marriage market where all four forms of marital unions coexist.[13] Our first finding concerns the sheer number of possible matchings $M = M(v, w)$. Consider, for example, the case with $v = |♂|$ men and one woman ($w = |♀| = 1$). The consent of each partner provided, this woman can choose between v monogamous unions, $v(v-1)/2$ polyandrous unions, and singlehood:

$$M(v,1) = \frac{v^2 + v + 2}{2}, \qquad v \in \{0, 1, ..\} \tag{41}$$

With two women, there are $v(v-1)/2$ cenogamous and v polygynous arrangements possible. Each of the $v(v-1)/2$ polyandrous unions for one female leaves $M(v-2, 1)$ options for the other female. Similarly, each of the v monogamous unions for one woman, leaves $M(v-1, 1)$ possible unions for the other. In case, one woman remains single, there are $M(v, 1)$ possibilities left for the other woman. As a result, we get:

$$M(v,2) = \frac{v^4 - 2v^3 + 9v^2 + 4v + 4}{4}, \qquad v \in \{0, 1, ..\} \tag{42}$$

possible matchings. Using symmetry (i.e. the fact that $M(v, w) = M(v, w)$ holds) and the results in (41) and (42), we obtain a recursive formula for the possible number of matchings:

$$\begin{aligned} &M(v,w) \\ &= \tfrac{2M(v,w-1)+2vM(v-1,w-1)+v(v-1)M(v-2,w-1)+2(w-1)vM(v-1,w-2)+(w-1)(v-1)vM(v-2,w-2)}{2} \end{aligned} \tag{43}$$

where $w, m \in \{2, 3, ..\}$.[14] Note that this recursion grows much faster than the factorial function. Consider, for example, five women and five men such that there are $5! = 120$ purely monogamous matchings. If we allow for all other marital arrangements (plus singlehood) and apply the above formula, we obtain a number of $29,946$ different matchings. With ten women and ten men, there are already more than 10^{12} possible matchings.

In the following, we focus on the smallest possible mating market that can exhibit all four marital unions: a market with two fertile women and men. From our discussion above, we know that there are twelve possible matchings which are summarized in Table

[13] Market clearing can only be guaranteed if we allow for the possibility that individuals remain single. Each individual, as a consequence, will therefore end up either polygamously married or monogamously married or remain single. Utility of a member of a marital union will, of course, not only depend on the specific union but also on the respective partners.

[14] We define: $M(0, 0) = 0$

2. Females $\{f\}$ and $\{g\}$ can be matched to males $\{m\}$ and $\{n\}$ but also to the pair $\{m, n\}$. Of course they might as well remain single (i.e. matched to the empty set). The analogous reasoning is true for men. Consider, for example, matchings $M_3 = \{\{f, n\}, \{g\}, \{m\}\}$ and $M_{10} = \{\{f, g, m\}, \{n\}\}$. In matching M_3, f and n form a monogamous union while g and m remain single. In matching M_{10}, m and the two women are married polygynously while n remains single.

Table 2: Possible Matchings

Union ($\bigcup$)	$\{f\}$	$\{g\}$	$\{f, g\}$	$\{\}$
$\{m\}$	M_2, M_6	M_4, M_7	M_{10}	M_1, M_3, M_5, M_{11}
$\{n\}$	M_3, M_7	M_5, M_6	M_{11}	M_1, M_2, M_4, M_{10}
$\{m, n\}$	M_8	M_9	M_{12}	–
$\{\}$	M_1, M_4, M_5, M_9	M_1, M_2, M_3, M_8	–	–

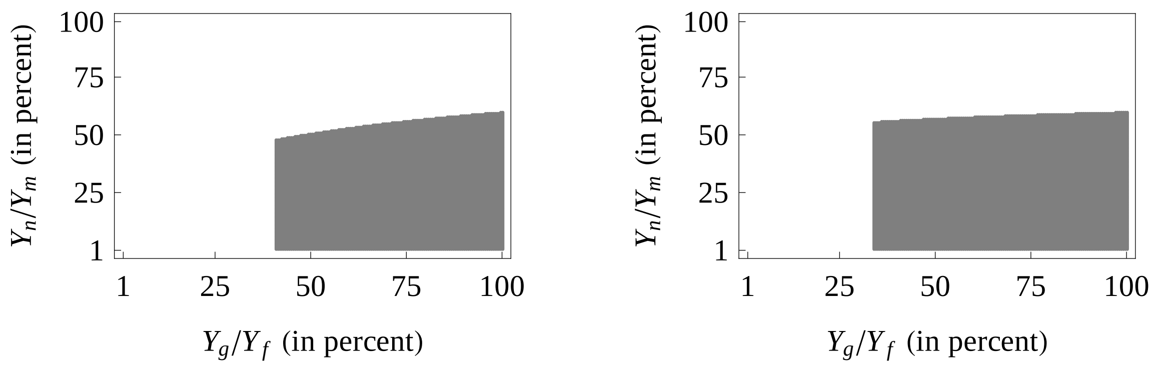
As there is no algorithm that leads us to the set of stable matchings, we will simply check each of the twelve possible matching for a blocking marital union.[15] For this purpose, we parameterize the model and solve it numerically. To be precise, we assume logarithmic preferences over non-reproductive consumption. Utility from reproduction is assumed to be Cobb-Douglas in the quality and quantity of own children. Child quality is determined by the geometric mean of maternal and paternal contributions. Finally, average child quality for a man with several wives is the result of a constant elasticity of substitution aggregation procedure with parameter ϕ. Of greatest interest is the influence of individual commands over economic resources on the formation of marital unions. For this reason, we consider whole ranges for the endowments Y_g and Y_n of the poorer agents while we have normalized the richer agents' endowments Y_f and Y_m to unity.

Our results suggest that there are no stable matchings that include a polyandrous or a cenogamous marriage. Stable matchings with monogamous marriages, in contrast, can be found for all resource distributions considered. With respect to polygynous marriages, results are mixed. The grey areas in Figure 1 display the regions where stable polygynous marriages occur. As can be seen, a certain degree of inequality among men is necessary for the occurrence of polygyny. Only in case of a sizeable wealth gap between men, can the richer man contribute enough to the qualities of both of his children to prevent the formation of a blocking coalition that involves the poorer man. With respect to women, however, the opposite is true. A more equal distribution of resources among

[15] A generalization of the Gale-Shapley (1962) or related algorithms is not straightforward as individuals might form elusive coalitions with members of their own sex.

women is a prerequisite for the establishment of polygny as an alternative to other stable matchings (most notably the assortative monogamous matching). When the wealth gap between the two women is too big, the polygynous man's utility gain from having two children is more than offset by the utility loss due to a lower average child quality. As a consequence, he (and his wealthier wife) would actually prefer a monogamous union. The differences between the left and right diagrams underscore this intuition. A bigger value of ϕ in the right diagram makes men more tolerant toward quality differences among their children and increases the region where polygynous unions are stable in two directions. An increased wealth gap among women now also becomes consistent with polygyny. The same holds for some smaller wealth gaps among men.

Figure 1: Occurrence of Stable Matchings that include Polygynous Marriages.



NOTES: The grey areas indicate wealth distributions where stable matchings exist that include polygynous unions. The left (right) diagram displays the results for a relatively low (high) elasticity of substitution in the aggregation procedure with respect to average child quality (ϕ is equal to one half in the left and equal to two in the right diagram). Stable matchings with monogamous traits occur over the whole area. Polyandrous and cenogamous matchings are unstable throughout.

4 Conclusion

Among others, the institution of marriage serves economic, educational, and psychological purposes. First and foremost, however, it is a reproductive agreement. With only a few (recent) exceptions, this fact has been rarely addressed in the economic literature on marriage. This paper views marriage as a solely reproductive union between males and females.

We show that biological asymmetries with respect to offspring recognition and reproductive capacity are sufficient to explain why monogamy is most common and why among the polygamous arrangements polygyny is so widespread, polyandry so rare, and cenogamy virtually absent. Our investigations focus on parental contributions to child quality and we show that monogamous unions meet a minimum constraint on technical efficiency. With some restrictions, polygynous unions can meet this criterion, too. In general, however, polygamous unions (of any kind) turn out to have difficulties to meet the criterion. In this context, risk aversion in the face of paternal uncertainty is one but not the only reason for distortions. Another distortion stems from strategic behavior and free-riding when marital unions comprise more than the biological parents of a child. Both work in favor of monogamy and against polygamy.

Our findings suggest that fertility considerations, differences between the sexes, and the importance of own children are key to understand the foundations of marriage and the shaping of marriage regimes. We therefore argue that reproductive considerations have to be taken seriously when explaining predominant marriage regimes but also when exploring marriage decisions and patterns. Most importantly, ongoing discussions about reforming the institution of marriage should not ignore the interplay of technological change on the one hand and the close link between human reproduction and marital arrangements on the other. For the possibility of genetic paternity testing can eliminate asymmetry in offspring recognition. As a result, the institution of marriage looses its reproductive rationale and other purposes of marriage such as economic and romantic ones no doubt will gain importance.

Appendix

Proof of Proposition 1. The proof is by contradiction:

$$U(C_m) + V\left(1, Q(q_{\female g}, q_{mf})\right) \leq U(C_m) + \mathbb{E}\left[V\left(1, Q(q_{\female f}, q_{mf} + q_{nf})\right)\right] \tag{44}$$

The right hand side refers to the man's expected utility in the polyandrous union and we assume that child quality contributions are chosen optimally (cf. Section 3.3). The left hand side describes the monogamous union with female g and m simply uses his polyandrous child quality contributions. Therefore, we can drop the utility derived from non-reproductive consumption. Moreover, we know that $q_{mf} \geq q_{nf}$ holds such that we can write:

$$V\left(1, Q(q_{\female g}, q_{mf})\right) \leq \frac{1}{2}V\left(1, Q(q_{\female f}, 2q_{mf})\right) < \frac{1}{2}V\left(1, 2Q(q_{\female f}, q_{mf})\right) < V\left(1, Q(q_{\female f}, q_{mf})\right) \tag{45}$$

Note that we have used the following properties: Q has increasing returns with respect to $q_{♀}$ and constant returns with respect to parental contributions taken together. Moreover we have used concavity of V with respect to Q. Since we have assumed that $q_{♀f} = q_{♀g}$ hold, we have indeed constructed a contradiction which proves our claim that m is better off in a monogamous union with g.

Second, we show that f and m are better off if they form a monogamous union in case $q_{nf} = 0$ holds. A neccessary and sufficient condition for this to be the case is that m contributes more to the quality of child f than under polyandry. Let $\tilde{q}_{mf}$ denote his contributions under monogamy and suppose that $\tilde{q}_{mf} \leq q_{mf}$ holds:

$$a_{♂}\frac{\partial U(Y_m - a_{♂}\tilde{q}_{mf})}{\partial C} \leq a_{♂}\frac{\partial U(Y_m - a_{♂}q_{mf})}{\partial C}. \tag{46}$$

Using the optimal female contributions under polyandry, m's first order neccessary conditions under monogamy and polyandry (17 and 28) lead us to a contradiction:

$$\frac{\partial V(1, Q(q_{♀f}, \tilde{q}_{mf}))}{\partial Q}\frac{\partial Q(q_{♀f}, \tilde{q}_{mf})}{\partial q_{♂}} < \frac{1}{2}\frac{\partial V(1, Q(q_{♀f}, q_{mf}))}{\partial Q}\frac{\partial Q(q_{♀f}, q_{mf})}{\partial q_{♂}}. \tag{47}$$

This contradicts our initial assumption and we infer that $\tilde{q}_{mf} > q_{mf}$ holds. Hence, we have indeed shown that f and m are better off under monogamy. ■

Proof of Proposition 2. The proof is by contradiction:

$$U(C_m) + V\left(1, Q(q_{♀h}, q_{mf} + q_{mg})\right) \leq U(C_m) + \mathbb{E}\left[V(K_m, Q)\right]. \tag{48}$$

The right hand side refers to the man's expected utility in the cenogamous union and we assume that child quality contributions are chosen optimally (cf. Section 3.4). The left hand side describes the monogamous union with female h. The man simply contributes the sum of his cenogamous child quality investments (i.e. $q_{mf} + q_{mg}$) to child h:

$$\begin{aligned}
& V\left(1, Q(q_{♀h}, q_{mf} + q_{mg})\right) \leq \tfrac{1}{4}V\left(1, Q(q_{♀f}, q_{♂f})\right) + \tfrac{1}{4}V\left(1, Q(q_{♀g}, q_{♂g})\right) + \tfrac{1}{4}V\left(2, \bar{Q}\right) \\
\Leftrightarrow\ & V\left(1, Q(q_{♀h}, q_{mf} + q_{mg})\right) < \tfrac{1}{4}V\left(1, Q(q_{♀f}, q_{♂f})\right) + \tfrac{1}{4}V\left(1, Q(q_{♀f}, q_{♂g})\right) + \tfrac{1}{2}V(1, \tfrac{Q_f + Q_g}{2}) \\
\Leftrightarrow\ & V\left(1, Q(q_{♀h}, q_{mf} + q_{mg})\right) < \tfrac{1}{2}V\Big(1, Q(q_{♀f}, \tfrac{q_{♂f}+q_{♂g}}{2})\Big) + \tfrac{1}{2}V\Big(1, \tfrac{1}{2}Q(q_{♀f}, q_{♂f}) + \tfrac{1}{2}Q(q_{♀f}, q_{♂g})\Big) \\
\Leftrightarrow\ & V\left(1, Q(q_{♀h}, q_{mf} + q_{mg})\right) < \tfrac{1}{2}V\Big(1, Q(q_{♀f}, \tfrac{q_{♂f}+q_{♂g}}{2})\Big) + \tfrac{1}{2}V\Big(1, Q(q_{♀f}, \tfrac{q_{♂f}+q_{♂g}}{2})\Big).
\end{aligned} \tag{49}$$

Note that we have used $q_{♀f} \geq q_{♀g}$, increasing returns of Q with respect to $q_{♀}$, concavity of V with respect to K, and the fact that $\bar{Q} \leq \frac{1}{2}Q_f + \frac{1}{2}Q_g$ holds for all values of ϕ. Next, let $\bar{q}_{♂}$ denote the average child quality contribution of a single husband, that is we define $\bar{q}_{♂} \equiv (q_{♂f} + q_{♂g})/2$. We obtain:

$$V\left(1, Q(q_{♀h}, q_{mf} + q_{mg})\right) < V\Big(1, Q(q_{♀f}, \bar{q}_{♂})\Big) \tag{50}$$

Because of $q_{♀h} = q_{♀f}$ and $q_{mf} + q_{mg} \geq \bar{q}_{♂} \geq q_{nf} + q_{ng}$, we have again constructed a contradiction and m is indeed better off in a monogamous union with h. Furthermore, we infer that m's quality contributions to the monogamously sired child h exceeds the sum of his contributions to children f and g, that is:

$$\tilde{q}_{mh} > q_{mf} + q_{mg} \tag{51}$$

Second, we show that f and m are better off if they form a monogamous union in case $\phi \to 0$ holds. With respect to the previous result, it sufficies to show that f is better off in the monogamous union with m. Note that $\phi \to 0$ is the Leontieff case meaning that men will not tolerate any quality differences between their children, i.e. we consider equal child qualities $Q_f = Q_g$:

$$Q(q_{♀f}, q_{mf} + q_{nf}) = Q(q_{♀g}, q_{mg} + q_{ng}) \quad \Rightarrow \quad Q(q_{♀f}, q_{mf} + q_{nf}) \leq Q(q_{♀f}, q_{mg} + q_{ng}) \tag{52}$$

The inequality means that $q_{♂f} \leq q_{♂g}$ must hold which in turn implies that $q_{mf} + q_{nf} \leq \bar{q}_{♂}$. Since $Y_m \geq Y_n$ implies that $\bar{q}_{♂} \leq q_{mf} + q_{mg} < \tilde{q}_{mf}$ holds, we know that child f receives more male contributions under monogamy than under cenogamy (in case $\phi \to 0$ holds). Hence, f is better off under monogamy.

Third, we show that f and m are better off if they form a monogamous union in case the following condition is satisfied:

$$\frac{q_{mf} + q_{mg}}{q_{nf} + q_{ng}} \geq \frac{q_{♀f}}{q_{♀g}} \tag{53}$$

Condition (53) guarantees that female f does not experience a utility loss in a monogamous union with m. Because it ensures that m's child quality investments $(q_{mf} + q_{mg})$ are not less than the male contributions to the quality of child f. In fact, the joint contributions of the two cohusbands to the quality of child f (i.e. $q_{♂f} = q_{mf} + q_{nf}$) are exceeding the wealthier man's quality contributions $(q_{mf} + q_{mg})$ to both children if and only if:

$$\frac{\partial Q\big(1, \frac{q_{mf}+q_{mg}}{q_{♀f}}\big)/\partial q_{♂}}{\partial Q\big(1, \frac{q_{nf}+q_{ng}}{q_{♀g}}\big)/\partial q_{♂}} > \frac{\frac{\partial V(1, Q(q_{♀g}, q_{nf}+q_{ng}))}{\partial Q} + \frac{\partial V(2,\bar{Q})}{\partial Q}\frac{1}{2}\left(\frac{\bar{Q}}{Q(q_{♀g}, q_{nf}+q_{ng})}\right)^{\frac{1}{\phi}}}{\frac{\partial V(1, Q(q_{♀f}, q_{mf}+q_{mg}))}{\partial Q} + \frac{\partial V(2,\bar{Q})}{\partial Q}\frac{1}{2}\left(\frac{\bar{Q}}{Q(q_{♀f}, q_{mf}+q_{mg})}\right)^{\frac{1}{\phi}}} \tag{54}$$

where we used the fact that partial derivatives of a differentiable linear-homogeneous function are homogeneous of degree zero. Note that $Q(q_{♀f}, q_m) \geq Q(q_{♀g}, q_n)$ holds. Under the assumption (53), the left hand side is not bigger than one while the right hand side cannot be smaller than one such that we have again constructed a contradiction. As a result, we have proven that the husband with at least as much resources as his cohusband and the wife which contributes at least as much child quality units as her cowife are always better off in a monogamous union if $\phi \to 0$ or (53) hold. Then, f and m form a blocking pair that prevents the formation of the cenogamous union. ■

Furthermore, we want to add that the inequality in (54) tends to be violated even when we allow the relative difference between the female child contributions to exceed the male difference or when we allow for some degree of substitutability in the average child quality function. So that cenogamy is indeed very unstable. Moreover, it is hard to think of reasons that cause the poorer female g to enter a union where the amount of male resources that she attracts is smaller than the amount contributed by the poorer cohusband.

References

Adams, John W. (1981): "Recent Ethnology of the Northwest Coast", *Annual Review of Anthropology*, 10, 361-392.

Alexander, Richard D. (1974): "The evolution of social behavior", *Annual Review of Ecology and Systematics*, 5, 325-383.

Alexander, Richard D. (1979): *Darwinism and Human Affairs*, University of Washington Press, Seattle.

Bateman, Angus J. (1948): "Intrasexual selection in Drosophila", *Heredity*, 2, 349-368.

Becker, Gary S. (1991): *A Treatise on the Family*, Harvard University Press, Cambridge.

Bethmann, Dirk and Michael Kvasnicka, (2005): "Paternal Uncertainty and the Economics of Mating, Marriage, and Parental Investment in Children", *Collaborative Research Center 649 Discussion Paper*, 46/2005.

Bethmann, Dirk and Michael Kvasnicka (2011): "The Institution of Marriage", *Journal of Population Economics*, vol. 24(3), 1005-1032.

Betzig, Laura (1992): "Roman Polygyny", *Ethology and Sociobiology*, 13, 309-349.

Betzig, Laura (1995): "Medieval Monogamy", *Journal of Family History*, 20 (2), 181-216.

Borgerhoff Mulder, Monique (1987): "On Cultural and Reproductive Success: Kipsigis Evidence", *American Anthropologist*, Vol. 89, No. 3, 617-634.

Bullough, Vern L. (1976): *Sexual variance in society and history*, Wiley, New York.

Buss, David M. (1989): "Sex differences in human mate preferences: Evolutionary hypotheses tested in 37 cultures", *Behavioural and Brain Sciences*, 12, 1-49.

Cadigan, R. Jean (1998): "Woman-to-woman marriage: Practices and benefits in sub-Saharan Africa", *Journal of Comparative Family Studies*, Vol. 29, No. 1, 89-98.

Cox, Donald (2003): "Private Transfers within the Family: Mothers, Fathers, Sons and Daughters", in Munnell, Alicia H. and Annika Sundén (eds.): *Death and Dollars: The Role of Gifts and Bequests in America*, Brookings Institution Press, Washington, D.C.

Cox, Donald and Marcel Fafchamps (2008): "Extended Family and Kinship Networks: Economic Insights and Evolutionary Directions", in Schultz, T. Paul and John Strauss (eds.): *Handbook of Development Economics*, 4, Ch. 3, North-Holland.

Darwin, Charles (1871): *The decent of Man, and Selection in Relation to Sex*, John Murray, London.

Edlund, Lena (2005): "The Role of Paternity Presumption and Custodial Rights for Understanding Marriage Patterns", *mimeo*, Columbia University.

Edlund, Lena and Evelyn Korn (2002): "A Theory of Prostitution", *Journal of Political Economy*, 110(1), 181-214.

Francesconi, Marco; Ghiglino, Christian; and Motty Perry (2010): "On the Origin of the Family", CEPR Working Paper DP7629.

Gale, David and Shapley, Lloyd S. (1962): "College Admissions and the Stability of Marriage", *The American Mathematical Monthly*, 69(1), 9-15.

Gaulin, Steven J.C.; Donald H. McBurney; and Stephanie L. Brakeman-Wartell (1997): "Matrilineal biases in the investment of aunts and uncles: A consequence and measure of paternity uncertainty", *Human Nature*, 8, 139-151.

Gaulin, Steven J. C. and Alice Schlegel (1980): "Paternal confidence and paternal investment: A cross-cultural test of a sociobiological hypothesis", *Ethology and Sociobiology*, 1, 301-309.

Gould, Eric D.; Moav, Omer; and Avi Simhon (2008): "The Mystery of Monogamy", *American Economic Review*, Vol. 98, No. 1, 333-357.

Greene, Penelope J. (1978): "Promiscuity, Paternity, and Culture", *American Ethnologist*, 5(1), 151-159.

Grossbard-Shechtman, Amyra (1986): "Economic Behavior, Marriage and Fertility: Two Lessons from Polygyny", *Journal of Economic Behavior and Organization*, 7, 415-424.

Grossbard-Shechtman, Shoshana (1993): *On the Economics of Marriage: A Theory of Marriage, Labor, and Divorce*, Westview Press, Boulder, San Francisco, Oxford.

Hartung, John (1985): "Matrilineal inheritance: New theory and analysis", *Behavioral and Brain Sciences*, 8, 661-688.

Ingoldsby, Bron B. (2006): "Marital Structure", in Ingoldsby, Bron B. and Suzanna D. Smith (eds.): *Families in global and multicultural perspective*, Ch. 6, Sage Publications.

Jankowiak, William; Sudakow, Monika; and Benjamin C. Wilreker (2005): "Co-wife Conflict and Co-operation", *Ethnology*, Vol. 44, No. 1, 81-98.

Levine, Nancy E. (1988): *The Dynamics of polyandry: Kinship, Domesticity, and Population on the Tibetan Border*, The University of Chicago Press.

Murdock, George P. (1967): *Ethnographic atlas*, Pittsburgh University Press, Pittsburgh.

Saint-Paul, Gilles (2008): "Genes, Legitimacy and Hypergamy: Another look at the economics of marriage", *Institut d'conomie Industrielle (IDEI) Working Paper No 509*.

Samuelson, Paul A. (1954): "The Pure Theory of Public Expenditure", *Review of Economics and Statistics*, Vol. 36, No. 4, 387-389.

Sellen, Daniel (1999): "Polygyny and Child Growth in a Traditional Pastoral Society: The Case of the Datoga of Tanzania", *Human Nature*, Vol. 10, No. 4, 329-371.

Smith, Maynard J. (1977), "Parental investment: a prospective analysis", Animal Behaviour 25(1), 1-9.

Stephens, William N. (1963): *The Family in Cross-cultural Perspective*, Holt, Rinehart, and Winston: New York.

Symons, Donald (1979): *The evolution of human sexuality*, Oxford University Press, Oxford.

Trivers, Robert L. (1972): "Parental investment and sexual selection", in Campbell, Bernard (ed.): *Sexual selection and the descent of man 1871-1971*, 136-179, Aldine, Chicago.

Weiss, Yoram (1997): "The Formation and Dissolution of Families: Why Marry? Who Marries Whom? And What Happens upon Divorce?", in Rosenzweig, Mark R. and

Oded Stark (eds.): *Handbook of Population and Family Economics*, 1A, Ch. 3, North-Holland.

Weiss, Yoram and Robert J. Willis (1985): "Children as Collective Goods and Divorce Settlements", *Journal of Labor Economics*, 3(3), 268-292.

Weston, Kath (1997): *Families We Choose: Lesbians, Gays, Kinship*, revised edition, Columbia University Press: New York.

Willis, Robert J. (1999): "A Theory of Out-of-Wedlock Childbearing", *Journal of Political Economy*, 107(6), S33-S64.

Wright, Robert (1994): *The Moral Animal: The new science of evolutionary psychology*, Pantheon Books, New York.

Otto von Guericke University Magdeburg
Faculty of Economics and Management
P.O. Box 4120 | 39016 Magdeburg | Germany

Tel.: +49 (0) 3 91 / 67-1 85 84
Fax: +49 (0) 3 91 / 67-1 21 20

www.fww.ovgu.de/femm

ISSN 1615-4274