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Quantum Risk Preferences in a Laboratory Experiment

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This paper presents a quantum model of risk preferences that seeks to provide an explanation of the experimental results reported in Berninghaus, Todorova & Vogt (2012). The finding that subjects choose the risk-dominant strategy in a 2×2 coordination game, on the average, more often, when they have previously completed a risk questionnaire, is not anticipated by the standard economic theory. The model presented in this paper demonstrates that the coordination game and the risk questionnaire can be analyzed as two decisions situations that do not commute and predicts that the order in which decisions are made will influence behavioral choices.

Keywords quantum mechanics · uncertain preferences · coordination game · risk questionnaire

1. Introduction

Expected utility theory (von Neumann & Morgenstern 1947) is the standard economic model of decision making under risk. It is based on a set of assumptions that specify how a rational decision maker will choose between risky alternatives. Empirical evidence suggests, however, that decision making is more complex than what is assumed by the rational choice framework, as people are found to act differently in repeated decision situations involving equivalent choice problems (e.g., Camerer 1989; Starmer & Sugden 1989; Hey & Orme 1994). Changes in risk preferences are found to be induced by non-normative factors, such as framing (Tversky 1969), compatibility effects (Lichtenstein & Slovic 1971), elicitation procedures (Hershey & Schoemaker 1985; Bostic et al. 1990), and game relativity (Vlaev & Chater 2006).

The mathematical formalism of quantum mechanics has already been applied to extend the analysis of the classical economic theory on the equilibrium selection problem in games (Meyer 1999, Eisert; Wilkens & Lewenstein 1999). Meyer (1999) proves that if players in zero-sum games, and more precisely in the matching pennies game, are allowed to use quantum strategies, then they can always earn payoff at least as good as the one obtainable by application of standard strategies (pure or mixed). Wilkens & Lewenstein (1999) study nonzero-sum games, and in particular the prisoner's dilemma. They show that the dilemma can be avoided if both players use quantum strategies. Khrennikov (2008) offers a quantum-like representation of the well-known from cognitive psychology Shafir–Tversky statistical effect (Shafir & Tversky 1992). Another application of the mathematical formalism of quantum mechanics in social sciences is discussed in Lambert-Mogiliansky, Zamir & Zwirn (2009). The basic idea behind the quantum approach of Lambert-Mogiliansky, Zamir & Zwirn (2009) is that there exists an intrinsic indeterminacy of the type of an individual. It is suggested that when asked to make a decision choice, people actualize one of the many coexisting types.

Inspired by the advancement in the literature to extend the analysis of equilibrium selection in economic games with insights from quantum mechanics, I, in this paper, apply a model of quantum preferences to the results of a laboratory experiment on strategy selection in a 2×2 coordination game (Berninghaus, Todorova & Vogt 2012 [from now on BTV]). The aim of this study is to demonstrate that by extending players' risk preferences to the quantum domain, the perplexing in the light of the standard economic theory experimental results in BTV, receive a plausible explanation.

BTV present results from an experiment designed to study the effect produced on strategy choices when a subject reports risk preferences on a risk questionnaire before engaging in a 2×2 coordination game. BTV's main finding is that the act of stating one's own risk preferences significantly alters strategic behavior in the coordination game. In particular, subjects tend to choose the risk-dominant strategy more often when they have previously stated their attitudes to risk. This finding is not anticipated by the internal consistency of preferences assumption of expected utility theory, stating that in theoretically equivalent situations people will always make the same behavioral choices. The model of uncertain preferences presented in this paper offers an explanation to the experimental results of BTV.

The rest of the paper is organized as follows. Section 2 presents the 2×2 coordination game used in BTV. Section 3 summarizes the experimental design and results of BTV. Section 4 introduces the basic mathematical tools of quantum mechanics. The model of uncertain preferences is developed in Section 4. Section 5 concludes.

2. The Game

Coordination games are non-cooperative, common interest games with multiple, usually Pareto-ranked, Nash equilibria. In a 2×2 coordination game, the two players have to simultaneously choose either Strategy A or Strategy B. Subjects' payoffs are determined by the combination of their strategies—see Figure 1. The game depicted in Figure 1 has two Pareto-ranked pure strategies Nash equilibria ([A, A] and [B, B]) and one equilibrium in mixed strategies. In this game, (A, A) is the Pareto-dominant equilibrium and (B, B) is the risk-dominant equilibrium (Harsanyi & Selten 1988).

		Column Player	
		A	B
Row Player	A	200, 200	0, 125
	B	125, 0	150, 150

FIGURE 1. —The baseline game

In coordination games, achievement of mutual gains is possible only if agents make mutually consistent decisions. The failure to coordinate on a certain action results in disequilibrium and inferior payoffs. Despite the strong incentives to make coordinated decisions, however, strategy selection in coordination games is not a trivial problem because the structure of the game provides no universal device to coordinate actions on any of the multiple equilibria. The difficulty in determining what, if any, equilibrium point will be expected in the game shown in Figure 1, arises from the fact that although the equilibrium point (A, A) is associated with a higher payoff for both players, playing Strategy A is risky. In contrast, playing Strategy B results in a payoff, that albeit lower than the one attainable in the equilibrium point (A, A), is only marginally affected by the other players' actions.

3. The Experiment

BTV present a laboratory experiment with a two-conditions between-subjects design that studies strategy choices in a 2×2 one-shot coordination game and examine how these depend on the act of completing a risk questionnaire. In one condition (Condition G), subjects were asked to select a strategy in the 2×2 coordination game presented in Figure 1. In another condition (Condition Q_G), subjects were instructed to first fill out a questionnaire about their own risk preferences and then to play the 2×2 coordination game presents in Figure 1. The summary data of the distributions of choices in the coordination game from conditions G and Q_C are given in Table 1.

TABLE 1: Distribution of strategy choices in the coordination game

	Condition G	Condition Q_G
Number of participants	56	54
Strategy A chosen	37 (66%)	27 (50%)
Strategy B chosen	19 (34%)	27 (50%)

The questionnaire consisted of three questions. In Question 1, subjects were asked whether they liked taking risks; In Question 2, whether they always tried to avoid risks. Admissible answers were "Agree," "Disagree", and "Neither agree nor disagree." In the third question, subjects were asked to determine their risk preference with greater precision by positioning it on a scale of 0 (most risk loving) to 100 (most risk averse).

Table 2 shows that 34 percent of the subjects chose Strategy B when they played the coordination game right away. When the subjects were asked to first complete the risk questionnaire and then to play the coordination game, 50 percent of all players selected Strategy B. The null hypothesis of equal proportions is rejected at the five percent level of significance (Z-test, z-statistics: 1.7083; p-value: 0.04379).

Within a best-response correspondence framework, this result can be explained by a shift in either beliefs or risk preferences. With the help of two additional conditions, in which players' first order beliefs were elicited (Murphy & Winkler 1970), BTV show that the act of completing the questionnaire exerts no influence on subjects' beliefs. This result implies that subjects should have become, on the average, more risk-averse after they completed the risk questionnaire.

BTV's experimental results contradict the internal consistency of preferences assumption of expected utility theory. Also, the conclusion that subjects become, on the average, more risk-averse after completing the questionnaire cannot be justified by any arguments in a world with deterministic preferences. A model of uncertain preferences, based on the model of Lambert-Mogiliansky, Zamir & Zwirn (2009), which provides an explanation of the BTV's experimental results, is presented below.

4. Mathematical Tools of Quantum Mechanics

Quantum mechanics provides a mathematical and conceptual framework for the development of the laws governing a physical system. The simplest quantum mechanical system is the *quibit*, which is a vector in the two-dimensional complex vector state space. The vectors $|0\rangle$ and $|1\rangle$ form an orthonormal basis in the relevant *Hilbert space*. The state of a quibit is described by the linear combination of the vectors belonging to its basis:

(1) $$|\psi\rangle = \lambda_1|0\rangle + \lambda_2|1\rangle.$$

The numbers λ_1 and λ_2 are complex numbers and are often referred to as *amplitudes*. The state of a quibit is not observable. A measurement of the quibit will result in either 0 or 1 with probabilities $|\lambda_1|^2$ and $|\lambda_2|^2$, respectively. Since the probability must sum up to 1, it follows that $|\lambda_1|^2 + |\lambda_2|^2 = 1$. This is often referred to as the *normalization condition* that should always be fulfilled for a legitimate quantum state. The amplitudes λ_1 and λ_2 could be determined only if infinitely many identically prepared quibits are measured. Another interesting property of a quibit is that its post-measurement state is different from its pre-measurement state. That is, if a quibit is initially in the state $|\psi\rangle$ and the result of a measurement is 0, its state after the measurement collapses to $|0\rangle$. Analogously, if a measurement yields 1, the post-measurement state of the quibit is $|1\rangle$.

It is useful to think of a measurement as a linear operator applied to the relevant vector space. In the matrix representation, a linear operator is nothing more than a matrix. When a physical system is measured, the result of the measurement is a value equal to one of the eigenvalues of the operator used. An important for the subsequent analysis question is whether two operators A and B commute. Two operators are said to commute if $AB = BA$. If A and B are *Hermitian operators* (A is a Hermitian operator if its *adjoint* is A) then they can be simultaneously *diagonalized*. That is, we can write $A = \sum_i a_i |i\rangle\langle i|$ and $B = \sum_i b_i |i\rangle\langle i|$, where $|i\rangle$ is some common orthonormal set of eigenvectors for A and B, and a_i and b_i are the eigenvalues corresponding to these eigenvectors. It will be shown later that the property of whether two operators commute has direct implications for the way in which the state of the system is expressed

5. The Model

The evidence that beliefs do not change after completing the risk questionnaire indicates that it is sufficient to model only risk preferences as possessing quantum properties. The following analogy, adopted from Lambert-Mogiliansky, Zamir & Zwirn (2009), is used to link quantum mechanics to uncertain preferences: any decision situation is modeled as an operator, and the behavior observed in the decision situation is viewed as an eigenvalue of the operator. Prior to the decision situation, every player is in a state that is a linear combination of the eigenvectors corresponding to the eigenvalues of the relevant operators. This implies that, in contrast to standard economic theory, stating that the type of a player is deterministic, in the quantum mechanical framework, there is inherent indeterminacy of preferences.

In the BTV's experiment there are two decision situations—the subjects evaluate their risk preferences on a risk scale (Q) (Question 3 of the questionnaire); and the subjects play a one-shot 2×2 coordination game (G). For simplicity, the subjects are characterized as either risk-averse or risk-loving in the questionnaire depending on whether they scored a value on the risk scale above or below 50, respectively (only a negligible number of the subjects scored exactly 50 on the risk scale). Further, I characterize the subjects as either risk-averse or risk-loving in the coordination game depending on whether they played the risk-dominant or Pareto-dominant strategy, respectively. So in effect, there are two, non-repeated and non-strategic, decision situations.

Consider the case when Q and G are two operators that commute. Each of them is characterized by two eigenvalues—a subject is risk-averse (in the questionnaire [ra_Q]; in the 2x2 game [ra_G]) or risk-loving (in the questionnaire [rl_Q]; in the 2x2 game [rl_G]). To each of these eigenvalues, there is a corresponding eigenvector ($\left|ra_Q\right\rangle, \left|ra_G\right\rangle, \left|rl_Q\right\rangle, \left|rl_G\right\rangle$, respectively). Under the assumption that Q and G commute, the initial state of a subject is:

(2) $$\left|\phi\right\rangle = \lambda_1\left|ra_Q ra_G\right\rangle + \lambda_2\left|ra_Q rl_G\right\rangle + \lambda_3\left|rl_Q ra_G\right\rangle + \lambda_4\left|rl_Q rl_G\right\rangle$$

with $\sum_{i=1}^{4}\left|\lambda_i\right|^2 = 1$ under the normalization condition. Now, if one measures first G (that is, subjects play the coordination game right away), the probability of observing ra_G is:

(3) $$\Pr(ra_G) = |\lambda_1|^2 + |\lambda_3|^2 .$$

This probability could be interpreted either as the fraction of subjects who choose the riskless strategy in the game (corresponding to risk-averse behavior) or the probability with which this strategy is chosen by a single player.

Alternatively, one can measure first Q. Then, the probability with which ra_Q is observed is:

(4) $$\Pr(ra_Q) = |\lambda_1|^2 + |\lambda_2|^2 .$$

The resulting post-measurement state (taking into consideration also the normalization condition) of the subject is:

(5) $$\left|\psi_{ra_Q}\right\rangle = \frac{\lambda_1 \left|ra_Q ra_G\right\rangle + \lambda_2 \left|ra_Q rl_G\right\rangle}{\sqrt{|\lambda_1|^2 + |\lambda_2|^2}} .$$

If one now measures G, ra_G will occur with the following probability:

(6) $$\Pr(ra_G / ra_Q) = \frac{|\lambda_1|^2}{|\lambda_1|^2 + |\lambda_2|^2} .$$

The joint probability of ra_G and ra_Q is found by applying the conditional probability formula:

(7) $$\Pr(ra_G, ra_Q) = \Pr(ra_Q)\Pr(ra_G / ra_Q) = |\lambda_1|^2 .$$

The joint probability of ra_G and rl_Q is calculated analogously:

(8) $$\Pr(ra_G, rl_Q) = \Pr(rl_Q)\Pr(ra_G / rl_Q) = |\lambda_3|^2 .$$

Finally, the marginal probability of observing ra_G, when one measures first Q and then G, is given below:

(9) $$\Pr_{QG}(ra_G) = \Pr(ra_G, ra_Q) + \Pr(ra_G, rl_Q) = |\lambda_1|^2 + |\lambda_3|^2 .$$

It is obvious from Equation (9) and Equation (3) that the risk-dominant strategy in the coordination game is played with the same probability regardless of whether subjects have previously completed the risk questionnaire. The same result holds also for the Pareto- dominant strategy. The conclusion is that, when two decision situations commute, the predictions of the standard economic theory and those of the present model of uncertain preferences coincide.

The case when the two operators Q and G do not commute is considered next. Also in this case, each of the operators is characterized by two eigenvalues and eigenvectors corresponding to these eigenvalues. The difference to the previous case is that the eigenvectors of each of the operators constitute two different bases in the relevant Hilbert space and the state of a subject could be written in terms of only one of these bases. For example, it can be written as follows:

(10) $$|\psi\rangle = \lambda_1 |ra_Q\rangle + \lambda_2 |rl_Q\rangle .$$

Each vector from one of the bases can be expressed as a linear combination of the vectors of the other basis. Consider:

(11) $$\begin{aligned} |ra_Q\rangle &= \mu_1 |ra_G\rangle + \mu_2 |rl_G\rangle \\ |rl_Q\rangle &= \mu_3 |ra_G\rangle + \mu_4 |rl_G\rangle \end{aligned} .$$

Substituting for $|ra_Q\rangle$ and $|rl_Q\rangle$in (10) and rearranging the terms, results in the following state vector:

(12) $$|\psi\rangle = (\lambda_1\mu_1 + \lambda_2\mu_3)|ra_G\rangle + (\lambda_1\mu_2 + \lambda_2\mu_4)|rl_G\rangle .$$

Measuring first G, ra_G is found with probability:

(13) $$\Pr(ra_G) = |\lambda_1\mu_1 + \lambda_2\mu_3|^2 .$$

Alternatively, one can measure first Q and receive ra_Q with probability:

(14) $$\Pr(ra_Q) = |\lambda_1|^2 .$$

After the measurement of Q, the state vector collapses to:

(15) $$\left|\psi_{ra_Q}\right\rangle = \frac{\lambda_1\left(\mu_1\left|ra_G\right\rangle + \mu_2\left|rl_G\right\rangle\right)}{\sqrt{\left|\lambda_1\right|^2}}.$$

Now, if G is measured, ra_G occurs with the following probability:

(16) $$\Pr(ra_G / ra_Q) = \frac{\left|\lambda_1\right|^2 \left|\mu_1\right|^2}{\left|\lambda_1\right|^2}.$$

By applying the conditional probability formula, the joint probability of ra_G and ra_Q is found to be equal to:

(17) $$\Pr(ra_G, ra_Q) = \Pr(ra_Q)\Pr(ra_G / ra_Q) = \left|\lambda_1\right|^2 \left|\mu_1\right|^2.$$

The joint probability of ra_G and rl_Q is calculated analogously:

(18) $$\Pr(ra_G, rl_Q) = \Pr(rl_Q)\Pr(ra_G / rl_Q) = \left|\lambda_2\right|^2 \left|\mu_3\right|^2.$$

The final step is to calculate the marginal probability of ra_G, when one measures first Q and then G:

(19) $$\Pr_{QG}(ra_G) = \left|\lambda_2\right|^2 \left|\mu_3\right|^2 + \left|\lambda_1\right|^2 \left|\mu_1\right|^2.$$

It is obvious that, in the general case, the expression in (19) is different from the expression in (13). In addition, the result for $\Pr_{QG}(ra_G)$ and an analogously calculated result for $\Pr_{QG}(rl_G)$ will not sum up to 1. This observation implies that when two operators do not commute, the probability space changes after each measurement and a joint probability between elements from the two probability spaces is not a defined event. Consequently, behavioral choices will not be independent of the order in which decision situations are encountered.

In relation to the BTV's result, the model of uncertain preferences implies that the risk questionnaire and the coordination game are two decision situations which do not commute. The subjects from Condition G and Condition Q_G play the same coordination game. However, at the time of playing the game, they are in a different sate, which explains the differences in the distributions of their strategy choices.

6. Conclusion

Based on a simple model of uncertain preferences, this paper seeks to provide an explanation of the experimental results reported in BTV. In quantum mechanics framework, the type of player is not deterministic, as stipulated by the standard economic theory. Rather, it is assumed that the subjects possess uncertain preferences. With uncertain preferences, the order in which two (or more) decision situations are encountered will matter (not matter) if the decision situations “do not commute” (“commute”). The BTV’s experimental results could then be interpreted as evidence to show that the risk questionnaire and the coordination game are two decision situations which do not commute, and that the difference between the strategic behavior of subjects who completed the questionnaire before playing the game and that of the subjects who played the game right away should come as no surprise.

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