

**WORKING PAPER SERIES**



**OTTO VON GUERICKE  
UNIVERSITÄT  
MAGDEBURG**

**FACULTY OF ECONOMICS  
AND MANAGEMENT**

Impressum (§ 5 TMG)

*Herausgeber:*

Otto-von-Guericke-Universität Magdeburg  
Fakultät für Wirtschaftswissenschaft  
Der Dekan

*Verantwortlich für diese Ausgabe:*

Otto-von-Guericke-Universität Magdeburg  
Fakultät für Wirtschaftswissenschaft  
Postfach 4120  
39016 Magdeburg  
Germany

<http://www.fww.ovgu.de/femm>

*Bezug über den Herausgeber*  
ISSN 1615-4274

# Judo Economics in Markets with Asymmetric Firms

Daniel Cracau<sup>1a</sup>

*<sup>a</sup>University of Magdeburg, Faculty of Economics and Management*

---

## Abstract

I study a game with one market incumbent and a small entrant in a duopoly with perfectly substitutable products. Firms face a sequential Bertrand competition. Limiting the initial capacity (Judo economics) is a plausible entry strategy for the small firm. If we, however, introduce asymmetry in production cost or product quality, capacity limitation can become obsolete. I derive thresholds as regards the cost and quality differences for the entrant's choice to voluntarily limit the production capacity in equilibrium. I study a market entry game with price competition and perfectly substitutable products. Limiting the initial capacity (Judo economics) is a plausible entry strategy. I show that under asymmetry in production cost or product quality, capacity limitation can become obsolete.

*Keywords:* Sequential Bertrand Competition, Judo Economics, Asymmetric Firms, Cost, Quality

*JEL:* D43, L11

---

## 1. Introduction

Judo economics as an entry strategy was first introduced in Gelman and Salop (1983). They show that a market entrant can use a capacity limitation to successfully survive in a Bertrand competition with homogeneous goods. For the market incumbent, accommodating entry turns out to be the profit maximizing response towards a Judo entrant. Rather than cutting down prices for the whole market, the incumbent allows the new entrant to serve

---

<sup>1</sup>Correspondence to: Daniel Cracau; mailing address: University of Magdeburg, Faculty of Economics and Management, Chair in E-Business, Universitätsplatz 2, 39106 Magdeburg, Germany; telephone: +49-391-67-11359, e-mail: Cracau@ovgu.de

a small niche but sustains the power to serve the residual market as a monopolist.

Meanwhile, the idea of Judo economics has been further elaborated in the economic literature. Empirical work has reported that Judo entrants are more likely to survive competition and incumbents respond less aggressively towards Judo entrants (Thomas, 1999). Also experimental results have shown that Judo economics works in the original setting (Cracau and Sadrieh, 2013). Theoretical work has focused on Judo Economics in more or less symmetric settings (Sørgard, 1995; Allen et al., 2000; Díaz et al., 2009). If any, only a cost advantage for the incumbent is considered. However, it is arguable that in some industries, entrants emerge with a technological innovation that gives them a cost or a quality advantage. The purpose of this article is to apply the idea of Judo Economics to a wider range of settings. I will allow for asymmetries in the firms' cost structure and the consumers' willingness to pay.

## 2. Model preliminaries

I initially study two firms  $i = 1, 2$  in a sequential Bertrand competition with firm 1 being the first moving entrant and firm 2 being the second moving market incumbent.

**Assumption 1.** *Products are perfect substitutes with different quality levels. Consumers' willingness to pay for firm  $i$ 's product is  $w_i$ .*

**Assumption 2.** *Total market demand  $D$  is fixed.*

In the first stage of the game, the entrant decides on his price  $p_1 \leq w_1$  and a capacity limitation  $0 < k \leq D$ . Then, the incumbent decides on her price  $p_2 \leq w_2$ , adjusting output accordingly.<sup>2</sup> We, thus, assume that the incumbent has no capacity limitation.

**Assumption 3.** *Consumers' preferences are lexicographic. Consumers buy from the firm which offers the highest consumer net benefit  $\beta_i = w_i - p_i$ . If firms offer the same net benefit, consumers buy from the incumbent.*

---

<sup>2</sup>W.l.o.g. I use male pronouns for the entrant and female pronouns for the incumbent.

Under Assumptions 1 to 3 and efficient rationing, firms' sales  $s_i$  depend on the firms' decisions as well as on the market parameter. If the incumbent accommodates the entrant by setting a price  $p_2 > p_1$  then  $s_1 = k$  and  $s_2 = D - k$ . Otherwise,  $s_1 = 0$  and  $s_2 = D$ .

Finally, we assume that firms' have linear production cost  $C_i(s_i) = c_i s_i$  with  $0 \leq c_i < w_i$ . Firms maximize their total profit  $\pi_i = p_i s_i - C_i = (p_i - c_i) s_i$ .

### 3. Asymmetry in marginal cost

When considering a cost differential, it is useful to define  $\delta = c_2 - c_1$  as the cost differential between the two firms. A cost differential  $\delta < 0$  then indicates a cost advantage for the incumbent whereas  $\delta > 0$  indicates a cost advantage for the entrant. We can now differentiate two cases.

#### 3.1. No cost advantage for the entrant ( $\delta \leq 0$ )

This case is similar to the original model of Gelman and Salop (1983). It covers the symmetric case as well as the case where the incumbent is at a cost advantage.

**Proposition 1.** *Under Assumptions 1 to 3, the equilibrium price and capacity choices in the symmetric game satisfy  $p_1^{JAsymC} = (w + c_1)/2$ ,  $k^{JAsymC} = (w - c_1)D/(2(w - c_2))$  and  $p_2^{JAsymC} = w$ .*

*Proof.* After observing the entrants' price  $\bar{p}_1$  and capacity  $\bar{k}$ , the incumbent can give two price responses. She may either choose  $p_2 = \bar{p}_1$  and match the entrants' price. In this case, she serves the entire market as the sole firm and earns a profit  $\pi_2^{Match}(p_2) = (\bar{p}_1 - c_2)D$ . Alternatively, she can choose a price  $p_2 > \bar{p}_1$  and accommodate the entrant. In this case, the incumbent acts as a quasi-monopolist for the residual market and earns  $\pi_2^{Resid} = (w - c_2)(D - \bar{k})$ .<sup>3</sup> From that, it is clear that the entrant must choose a price-capacity pair  $(p_1, k)$  so that the condition  $\pi_2^{Match} \leq \pi_2^{Resid}$  is fulfilled, i.e. the incumbent's profit from accommodation must not be smaller than her profit from deterring entry.

Let us now denote  $\lambda(k)$  as the function which assigns the greatest possible price  $p_1$  to each capacity  $k$  that fulfills the condition. The entrant

---

<sup>3</sup>The incumbent will always choose a price  $p_2 = w$  if accommodating, because demand is fixed and thus  $\partial \pi_2^{Resid} / \partial p_2 > 0$ .

then maximize  $\pi_1(p_1, k) = (p_1 - c_1)k$  w.r.t.  $p_1 \leq \lambda(k)$ . For the equilibrium, Gelman and Salop (1983) derive that the constraint holds with equality. The condition for the optimal capacity  $k^*$  can then be derived as  $0 = \lambda(k^*) + k^* \lambda'(k^*) - C_1'(k^*)$ . From that, also equilibrium prices  $p_1^* = \lambda(k^*)$  and  $p_2^* = \text{argmax } \pi_2^{\text{Resid}}(k^*)$  are determined.  $\square$

Equilibrium profits can be derived as  $\pi_1^{\text{JAsymC}} = (w - c_1)^2 D / 4(w - c_2)$  and  $\pi_2^{\text{JAsymC}} = (w + c_1 - 2c_2)D / 2$ . Being the second mover comes along with a strategic advantage and results in higher profits. Moreover, a cost advantage strengthens the incumbent.

### 3.2. Cost advantage for the entrant ( $\delta > 0$ )

This case was excluded by Gelman and Salop (1983) because it can make the Judo limitation obsolete.

**Proposition 2.** *If  $\delta > w - c_2$ , the entrant will not limit his capacity.*

*Proof.* The entrant can limit his capacity and earn Judo profits  $\pi_1^{\text{JAsymC}}$ . If he does not limit his capacity, the incumbent will always match  $p_1$  as long as  $p_1 \geq c_2$ . Setting a price  $p_1^{\text{ForceC}} = c_2 - \epsilon$ , the entrant can force the incumbent to stay out of the market. In this case, any capacity limitation becomes obsolete, i.e. the entrant sets  $k^{\text{ForceC}} = D$ . Using this strategy, the entrant earns  $\pi_1^{\text{ForceC}} = (c_2 - \epsilon - c_1)D$ . The forcing strategy is beneficial for the entrant if  $\pi_1^{\text{ForceC}} \geq \pi_1^{\text{JAsymC}}$ . After some calculations, this condition yields  $\lim_{\epsilon \rightarrow 0} \delta > w - c_2$ . For parameter values such that this condition holds, capacity limitation is no longer the profit maximizing entry strategy.  $\square$

If the cost advantage of the entrant is sufficiently high, he can destroy the strategic advantage of the incumbent by pricing below her marginal cost.

## 4. Asymmetry in product quality

We assume that product quality is perfectly represented by consumers' WTP and therefore consider  $w_i$  as the quality of firm  $i$ 's product. For the further analysis, we denote  $\eta_i = w_i - c_i$  as the social benefit of firm  $i$ 's product. We also define  $\kappa = \eta_2 - \eta_1$  as the difference in social benefit between firms' products. If  $\kappa > 0$ , the incumbent's product provides a higher social benefit and vice versa.

4.1. No quality advantage for the entrant ( $\kappa > 0$ )

**Proposition 3.** *Under Assumptions 1 to 3, the equilibrium price and capacity choices in the asymmetric demand game satisfy  $p_1^{JAsymW} = (w_1 + c_1)/2$ ,  $k^{JAsymW} = \eta_1 D/2\eta_2$  and  $p_2^{JAsymW} = w_2$ .*

*Proof.* The proof follows a similar reasoning as that of Proposition 1. Knowing the entrant's price  $\bar{p}_1$  and capacity  $\bar{k}$ , the incumbent may either match the social benefit of the entrant's product ( $p_2 = w_2 - w_1 + \bar{p}_1$ ) or accommodate ( $p_2 > w_2 - w_1 + \bar{p}_1$ ). In the first case, the incumbent serves the entire market and earns a profit of  $\pi_2^{MatchW}(p_2) = (w_2 - w_1 + \bar{p}_1 - c_2)D$ . In the second case, the incumbent serves only the residual demand and therefore maximizes her profit  $\pi_2^{ResidW} = (w_2 - c_2)(D - \bar{k})$ . Because the entrant has to ensure that entry is accommodated, he must choose a pair  $(p_1, k)$  that fulfills the condition that the incumbent's profit from accommodation is not smaller than her profit from entry deterrence, i.e.  $\pi_2^{MatchW} \leq \pi_2^{ResidW}$ . Let  $\theta(k)$  denote the function that maps the greatest  $p_1$  that fulfills the condition for each  $k$ . Then, the entrant maximizes  $\pi_1(p_1, k) = (p_1 - c_1)k$  w.r.t.  $p_1 \leq \theta(k)$ . As Gelman and Salop (1983) derive that the constraint holds with equality in equilibrium, one can derive the condition for the optimal capacity  $k^*$  as  $0 = \theta(k^*) + k^*\theta'(k^*) - C'(k^*)$ . This determines equilibrium prices  $p_1^* = \theta(k^*)$  and also  $p_2^* = \operatorname{argmax} \pi_2^{ResidW}(k^*)$ .  $\square$

For the assumptions made, equilibrium profits are  $\pi_1^{JAsymW} = \eta_1^2 D/4\eta_2$  and  $\pi_2^{JAsymW} = (2\eta_2 - \eta_1)D/2$ . We can see that each firm's profit increases in the social benefit of its own product whereas it decreases in the social benefit of the other firm's product.

4.2. Quality advantage for the entrant ( $\kappa < 0$ )

**Proposition 4.** *If  $\eta_1 > 2\eta_2$ , the entrant will not limit his capacity.*

*Proof.* The entrant can limit his capacity and earn Judo profits  $\pi_1^{JAsymW}$ . If he does not limit his capacity, the incumbent will always set a price  $p_2 = w_2 - w_1 + p_1$  to match the net benefit that the entrant's product provides to the consumers. Of course, the incumbent cannot set a price  $p_2 < c_2$ . Setting a price  $p_1^{ForceW} = w_1 - w_2 + c_2 - \epsilon$ , the entrant can force the incumbent to stay out of the market. In this case, any capacity limitation becomes obsolete, i.e. the entrant sets  $k^{ForceW} = D$ . Using this strategy, the entrant earns  $\pi_1^{ForceW} = (w_1 - w_2 + c_2 - \epsilon - c_1)D$ . The forcing strategy is beneficial for the entrant if  $\pi_1^{ForceW} \geq \pi_1^{JAsymW}$ . After some calculations, this condition

yields  $\lim_{\epsilon \rightarrow 0} \eta_1 > 2\eta_2$ . For parameter values such that this condition holds, capacity limitation is no longer the profit maximizing entry strategy.  $\square$

If the social benefit of the entrant's product is sufficiently high, he can destroy the strategic advantage of the incumbent by pricing her out of the market.

## 5. Concluding remarks

The results of this study are derived under the main assumptions of perfect substitutability and fixed market demand. The first assumption is crucial for the existence of Judo-type market outcomes. With horizontal product differentiation, a first mover earns positive profits in a sequential price competition even without capacity limitation, see for example Furth and Kovenock (1993). In contrast, the assumption of fixed market demand is non-crucial. Equilibrium characteristics remain the same with linear demand, however, outcomes become analytically less tractable.<sup>4</sup> Finally, as Gelman and Salop (1983) show, the assumption of efficient rationing only alters the distribution of equilibrium profits, but not the structure of equilibrium market outcomes.

The results for the asymmetric settings derived in this article provide a basis for future work. I derive that the social benefit of a firm's product has a strong impact on firms' profits. Therefore, it would be interesting to study a situation comparable to that in Motta (1993), where product quality is not exogenously given but firms determine their product quality by costly investments. Aligned with that, one could model market demand in dependence of products' quality. Bocard and Wauthy (2009), for example, show that quality reduction and capacity limitation can be strategic substitutes. They study a game where an entrant commits to a quality and a capacity before a simultaneous price competition with the dominant market incumbent. By assumption, the incumbent in their model always sets the maximum quality level. The entrant can only choose to reduce quality. Obviously, both capacity limitation and quality reduction relax price competition in this setting. They show that in equilibrium, it is more advantageous for the entrant to limit his capacity and imitate the quality of the incumbent than choosing

---

<sup>4</sup>For settings with asymmetric cost and linear demand, for example, equilibrium prices, capacity and profits can only be derived numerically.

a large capacity but reducing quality. Because my model also considers entrants offering potentially higher quality than the incumbent, the equilibrium result with endogenous quality choice might differ from that of Boccard and Wauthy (2009).

A further model extension might include fixed market entry cost. Entry cost can create incentives for the entrant to turn his first mover position into a strategic advantage by setting limit prices. Typically, limit prices are assumed to be an entry-deterrence option of the incumbent (Milgrom and Roberts, 1982) but in the setting with an advantaged entrant, this idea can be reversed.

### **Acknowledgements**

I am very grateful to Abdolkarim Sadrieh for helpful comments and suggestions.

### **References**

- Allen, B., Deneckere, R., Faith, T., Kovenock, D., 2000. Capacity precommitment as a barrier to entry: a bertrand-edgeworth approach. *Economic Theory* 15, 501–530.
- Boccard, N., Wauthy, X., 2009. Entry accommodation under multiple commitment strategies : judo economics revisited. CORE Discussion Papers 2009050, Université catholique de Louvain, Center for Operations Research and Econometrics (CORE).
- Cracau, D., Sadrieh, A., 2013. Coexistence of small and dominant firms in bertrand competition: judo economics in the lab. FEMM Working Papers 13001, Otto-von-Guericke University Magdeburg, Faculty of Economics and Management.
- Díaz, A. G., González, R. H., Kujal, P., 2009. List pricing and discounting in a bertrand-edgeworth duopoly. *International Journal of Industrial Organization* 27 (6), 719 – 727.
- Furth, D., Kovenock, D., 1993. Price leadership in a duopoly with capacity constraints and product differentiation. *Journal of Economics* 57 (1), 1–35.

- Gelman, J. R., Salop, S. C., 1983. Judo economics: capacity limitation and coupon competition. *Bell Journal of Economics* 14 (2), 315–325.
- Milgrom, P., Roberts, J., 1982. Limit pricing and entry under incomplete information: an equilibrium analysis. *Econometrica* 50 (2), 443–459.
- Motta, M., 1993. Endogenous quality choice: price vs. quantity competition. *Journal of Industrial Economics* 41 (2), 113–131.
- Sørgard, L., 1995. Judo economics reconsidered: capacity limitation, entry and collusion. *International Journal of Industrial Organization* 15, 349–368.
- Thomas, L. A., 1999. Incumbent firms' response to entry: price, advertising, and new product introduction. *International Journal of Industrial Organization* 17 (4), 527–555.



**Otto von Guericke University Magdeburg**  
Faculty of Economics and Management  
P.O. Box 4120 | 39016 Magdeburg | Germany

Tel.: +49 (0) 3 91/67-1 85 84  
Fax: +49 (0) 3 91/67-1 21 20

[www.wv.uni-magdeburg.de](http://www.wv.uni-magdeburg.de)