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Combined sourcing and inventory management using capacity reservation and spot market

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Abstract

Leading companies in several industries purchase materials with the combined use of capacity reservation contracts and spot market. We analyse the optimal and a simplified policy for making long-term capacity reservation and periodic ordering/inventory decisions using the above two sources under stochastic demand and random spot market price fluctuations. In a numerical study we assess the effects of demand and spot market price uncertainties and of other parameters on both the optimal and simplified policy. We provide insights into the interaction of capacity reservation decision, demand uncertainty induced safety inventory, and inventory resulting from forward buying on the spot market.

Keywords: Supply chain management, Capacity reservation, Spot market

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Introduction and literature review

Purchasing based on capacity reservation contracts and buying on the spot market are two sourcing practices. Spot market purchasing is beneficial in case of low spot market prices or insufficient reserved capacity, and the capacity reservation contract can be used as an operational risk hedging tool for high spot market price incidents. We consider a capacity reservation contract in which a reservation price, proportional to the reserved quantity, has to be paid for the option of receiving any amount per period for the contract price up to the reservation quantity.

Leading companies in several industries are combining capacity reservation contracts and spot market purchases to reap the benefits of the alternative sources. Applications include chemicals, commodity metals, raw materials, oil, liquefied gas, and semiconductors. For instance, Vukina et al. (2009) analyze a case in food packaging industry using forward buying combined with spot purchase. A multiple sourcing strategy is also used in LNG purchasing (Yacef, 2010). Recent reports of electricity trading practices applying combining contracts and spot market include Benth et al. (2012), Gulpiar and Oliveira (2012), and Ruiz et al. (2012).

We consider the *combined sourcing* wherein the capacity level is to be fixed for a longer time interval with the contract supplier which serves as a real option providing sufficient protection for high spot market price incidents. Then, it has to be decided - period by period - which quantities to procure from the two sources. The order release is after observing the spot price but before knowing the demand of the subsequent period. The combined procurement strategy has to protect against risks of insufficient demand fulfilment and exploit the benefits of forward buying in periods with low spot price levels and keeping speculative inventories. We don’t deal with the resale of inventories; they are used only for covering future demand. The decision on capacity reservation has to take into account the short-term capacity utilization of each source which itself depends on the available long-term capacity reservation level. Thus, we face a highly complex interdependence of long-term and short-term decisions under uncertainties in demand and spot market price.

This problem was first studied in the inventory literature in Serel et al. (2001), considering a simple capacity reservation/order-up-to policy, and disregarding the spot market price uncertainty. Serel (2007) extended the research to random demand and a spot market with random capacity but no price uncertainty. Li et al. (2009) developed a stochastic dynamic programming model without the inventory policy and replenishment decisions. Zhang et al. (2011) consider two supply sources but the reservation contract is not flexible. Several papers deal with a single-period decision with combined sourcing including Fu et al. (2010 and 2012); Arnold and Minner (2011) extend this approach to a two-period problem. Adilov (2012) deals with the interaction of forward contract and spot market from the point of view of the supplier.

Most of the relevant publications, including the finance literature, disregard the inventory considerations and assume that the replenishment decision in each period can be *postponed* until demand is realized. In a production context this situation would refer to a make-to-order environment. In our approach, however, we consider a make-to-stock environment where, as typical for production/inventory problems with lead time and stochastic demand, period-by-period procurement decisions have to be made *before* demand is known. So the replenishment decisions in our problem setting simultaneously have to account for demand uncertainty and uncertainty in future price development. In this problem environment Inderfurth and Kelle (2009) derived properties of the optimal decision structure. In a subsequent paper Inderfurth and Kelle (2011) established simple analytical expressions for determining optimal parameters of a simplified policy with base stock ordering. In another paper Inderfurth et al. (2013) propose an advanced heuristic approach to calculate parameters of the optimal capacity reservation/ordering policy and compare it with several simple heuristic approximations. However, none of these papers provide detailed insights into the interaction of capacity reservation decision, demand uncertainty induced safety inventory, and inventory resulting from forward buying on the spot market.

The remainder of this paper is structured as follows. After summarizing our previous results, we provide a managerial analysis showing the individual and joint effects of demand and spot market price uncertainties and other parameters on the optimal policy. The goals are to develop managerial insights from the behaviour of the optimal policy and to evaluate the performance of the simplified base-stock policy used in Inderfurth and Kelle (2011).

The exact optimal policy and a simplified policy

The overall objective is to choose the long-term capacity reservation level before the first period starts and, after that, to select in each period of the planning horizon the spot market and reservation based order quantities in such a way that the total expected cost is minimized. We modelled the above decision problem as a stochastic dynamic optimization problem and analysed the optimal procurement strategy by means of stochastic dynamic programming.

We use the following notation:

$F(x), f(x), \mu_x, \sigma_x$ cumulative distribution, density function, expected value and standard deviation of *demand* $\tilde{x}$ and

$G(p), g(p), \mu_p, \sigma_p$ the same distribution characteristics for the *spot market price* $\tilde{p}$.

We consider a periodic decision process involving different level of knowledge in time. The *first decision* is on

R the capacity reservation quantity

that must be *fixed for a longer time horizon* based on the random demand and spot market price distribution and the following stationary cost factors:

r the capacity reservation price per period for a unit of capacity reserved,

c the unit purchase price charged by the long-term supplier,

h the inventory holding cost per unit and period,

v the backorder cost per unit and period.

The *next decision* is at the *beginning of each time period* about

$Q_{L,t}$ order quantity from the long-term supplier, and

$Q_{S,t}$ order quantity from the spot market at the beginning of each period, t, knowing

I_t inventory level at the beginning of the period and

p_t the realized current spot market price.

We were able to prove that for the backorder case the optimal procurement decisions are governed by a quite complex three-parameter policy with a fixed order-up-to level, S_L, for ordering from the long-term supplier and price-dependent order-up-to levels, $S_S(p)$, for short-term spot market procurement. The third policy parameter, R, is the capacity reservation level (see in Inderfurth and Kelle, 2009, and Inderfurth et al., 2013). The numerical optimization method is based on the value iteration of stochastic dynamic programming with discretized state and decision space. However, even though it is possible to exploit the known policy structure, numerical optimization is a highly cumbersome computational task except for small problem instances. Inderfurth et al. (2013) therefore provide an efficient and quite accurate heuristic to approximate the optimal policy parameters.

For practical applicability it often makes sense to consider a more simple policy structure that is easier to manage and where the optimal parameters can be derived analytically. In Inderfurth and Kelle (2011) a simple base-stock policy is considered where both short-term - spot market based - and long-term - capacity reservation based - purchasing decisions follow a single order-up-to level S_B which does not depend on the spot market price p. Thus, in this approach a single order-up level is considered with $S_B = S_L = S_S(p)$. The capacity reservation quantity, R_B, and base stock, S_B, of the combined ordering policy can be expressed in a simple analytic form:

$$R_B = F^{-1}\left(\frac{\delta - r}{\delta}\right) \tag{1}$$

$$S_B = F^{-1}\left(\frac{v}{h+v}\right) \tag{2}$$

with the cost parameters, r, h, and v defined above, $F^{-1}(.)$, the inverse of the demand distribution, and the *conditional expected gain*, δ, of having the fixed price, c, in case of higher spot price ($p > c$) that can be expressed by

$$\delta = E[\pi - c \mid \pi > c] = \int_c^{\infty} (p-c)g(p)dp\,, \tag{3}$$

where g(p) is the probability density function of the spot price.

Numerical and managerial analysis

This section aims to provide detailed insights into the interaction of capacity reservation quantity, safety inventory hold because of demand uncertainty, and speculative inventory resulted from forward buying on spot market. These observations play an important role under the trend of increasing volatility and uncertainty in demand and spot market prices. In order to assess the specific impact of forward buying related stocks the optimal policy is compared with the simple base stock policy, in which speculative inventory is not considered.

The numerical optimization method is based on the value iteration of stochastic dynamic programming with discretized state space and linear approximation of the value function for extremely high or low net inventory levels. Demand and price distributions are discretized in the $\mu \pm 3\sigma$ interval. The level of expected demand and price is scaled such a way that a numerical optimization takes a reasonable time. We calculate results for a base case scenario and selected deviations from these base case data.

In the numerical experiments the random demand and spot price values are drawn from gamma and normal distributions, respectively. The *base-case* cost and price parameters have been chosen in such a way that

- the long-term contract is less costly than the spot market option, on the average,
- the spot price is lower than the contract purchase price in a considerable number of periods,
- the price variability is sufficiently high and holding cost is sufficiently low that forward buying will occur quite often, and
- the backorder cost plays such a role that safety stock is needed, especially in periods without forward buying.

In that way we tried to capture all relevant scenarios that are of interest for managerial analysis. The detailed parameter selection for the base case and the main results are included in Table 1 and Table 2.

Table 1 – Parameters in the base case

Parameter	c	r	h	v	μ_X	σ_X	μ_P	σ_P
Base case value	8	0.5	0.2	8	10	3	10	1

Table 2 – Main results in the base case

	Optimal combined sourcing	*Base stock policy sourcing*
Capacity reservation level R	10	12
Order-up-to-level for long term sourcing S_L / base stock level S_B	21	17
Average net inventory E(I)	8.9	7.0
Average fraction of demand filled from long term contract	93%	95%
Expected total cost Z	87.4	88.5
Relative cost deviation from optimal dual sourcing ΔZ	-	1.2%

First we summarize the behaviour of the long-term decision, the optimal reserved capacity, R^*. (See an illustration of the sensitivity results in Figure 1). The long-term purchase price, c, is fixed in our experiments while all the other cost parameters, demand and spot price parameters have been varied around the base-case values.

Reservation means buying a real option that offers a fixed purchase price, c, up to the reserved quantity R in each period for paying a reservation price, $r \cdot R$, in each period. It provides the protection in case of high spot prices. The optimal selection of R depends also on the inventory policy because inventory can also provide protection in case of high spot price. So there is a highly complex interdependence of long-term and short-term decisions under uncertainties in demand and spot market price. We analyse the interaction of *capacity reservation*, *safety inventory* hold because of demand uncertainty, and *speculative inventory* as a result of forward buying on spot market.

As we expect, there is a decrease in the optimal R^* for increasing ***reservation price, r***. The decrease is monotonous first and then R^* drops to zero (see left side of Figure 1) and only the spot market is used. In our case it happens when the total cost of long-term sourcing ($c+r$) reaches 93.8% of the average spot price, μ_p. We found that forward buying in low spot price occasions and holding inventory can provide a more economic protection in case of higher long-term sourcing cost. For the optimal policy the forward buying results in a large increase in the average inventory, $E^*(I)$, as r is increasing (see right side of Figure 1).

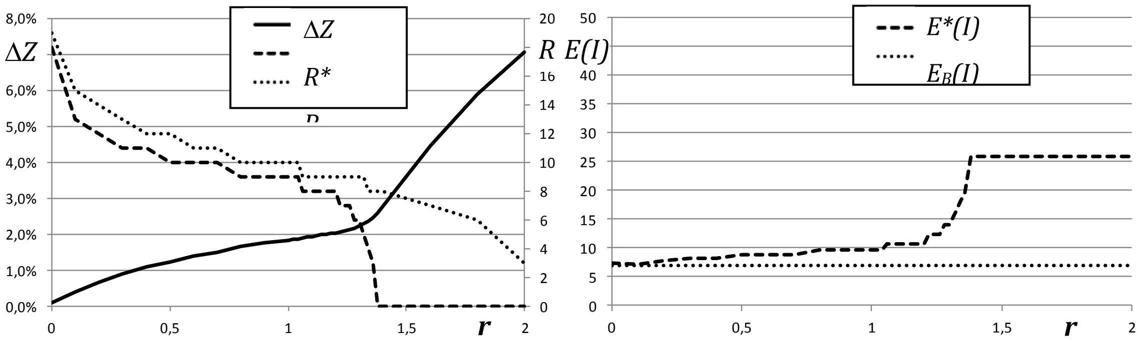

Figure 1 – Effects of changing the reservation price, r, on optimal/base stock reservation quantity, $R^*/R_{B,}$ the cost difference, ΔZ, and on the average inventory, $E^*(I)/E_B(I)$

Since in the base-stock policy the order quantity does not depend on the actual spot price, forward buying is not applied and there is no change in the average inventory, $E_B(I)$, by modifying r. So applying the base-stock policy capacity reservation is suggested ($R_B > 0$) even in cases when the total cost of long-term sourcing is higher than expected spot price (for $c+r>\mu_p$) to provide price protection, though less and less as r increases.

The effect of the inventory policy on R^* can be observed by changing the ***inventory holding cost, h*** (see Figure 2). With the increase of h forward buying is a less economic protection alternative to capacity reservation, resulting in a decreasing average inventory, $E^*(I)$, and an increase in the optimal R^* level, but after a short steep increase the change is diminishing. This effect cannot be seen on R_B, since it does not depend on h. However, the decrease in safety stock for larger h decreases the average inventory, $E_B(I)$, also for the base-sock policy. Comparing the expected total cost of the optimal and the base-stock policy, the cost difference, ΔZ, is large for high r and low h parameters, in those cases when the benefit of the forward buying is large.

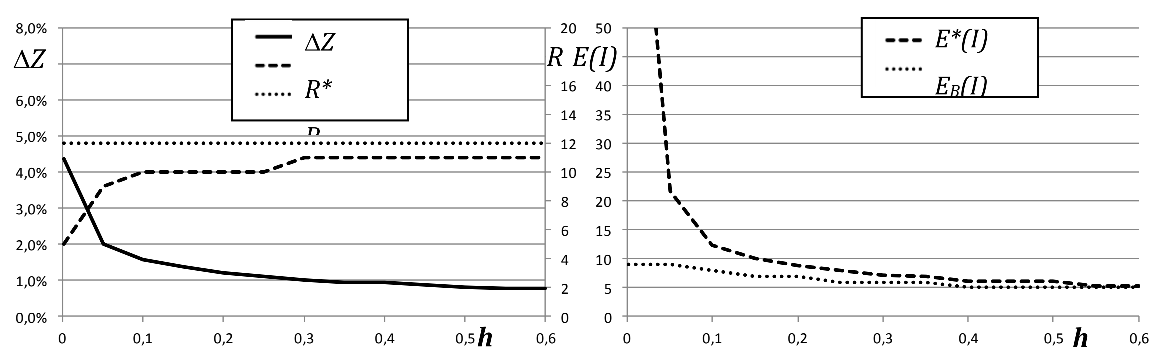

Figure 2 – Effects of changing the holding cost, h, on optimal/base stock reservation quantity, $R^*/R_{B,}$ the cost difference, ΔZ, and on the average inventory, $E^*(I)/E_B(I)$

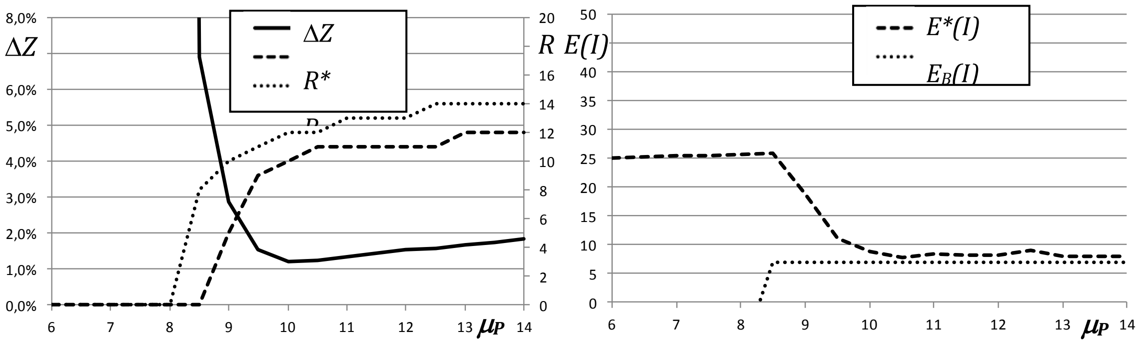

Figure 3 – Effects of changing the expected spot price, μ_P, on optimal/base stock reservation quantity, R^*/R_B, the cost difference, ΔZ, and on the average inventory, $E^*(I)/E_B(I)$

With increasing ***average spot price, μ_p***, the optimal R^* is increasing, as we expect (see Figure 3). However, this increase starts for higher μ_p, and the R^* values are below R_B. In our numerical example the base-stock policy suggests positive R_B as the average spot price is above 94.1% of the long-term sourcing price (c+r) while the optimal R^* gets positive only when the average spot price is above the total long-term sourcing price. The reason is similar as we discussed before: the average inventory for the base-stock policy is lower due to not forward buying, thus the reservation is economic also for higher total reservation cost compared to the expected spot price. Here we can also observe that after a short steep increase in the reservation quantity the change is marginally decreasing with the increase of the average spot price.

In the strategic decision on the reservation quantity, R, the most important factor is the uncertainty since it is the driving force of hedging by reservation. Because of the strong interaction, we consider the ***joint effect*** of the ***spot price variability, σ_p***, and ***demand variability, σ_x***. The optimal reservation quantity depends also on the effect of the inventory decisions including safety stock protection and forward buying. The two-way sensitivity results are illustrated in Tables 3-5.

For low demand uncertainty, the increase in price uncertainty has small influence on the optimal reservation quantity. In low price uncertainty case the demand uncertainty has moderate increasing effect on R^*. The expression (3) for the *base stock policy* suggests that with larger *price variability* the relative expected gain, $(\delta-r)/\delta$, is increasing and results in an increasing R_B. This shows the managerial consequence that with increasing spot price uncertainty the reservation quantity should be increased by a marginally higher rate as demand uncertainty is increasing.

Table 3 – Joint effects of changing the spot price variability, σ_p, and demand variability, σ_x, on optimal/base stock reservation quantity, R^*/R_B

R^* / R_B	$\sigma_X = 0$	$\sigma_X = 3$	$\sigma_X = 6$	$\sigma_X = 10$
$\sigma_P = 0$	10 / 10	11 / 12	11 / 13	11 / 14
$\sigma_P = 0.5$	10 / 10	11 / 12	11 / 13	11 / 14
$\sigma_P = 1$	10 / 10	10 / 12	10 / 13	10 / 14
$\sigma_P = 1.5$	10 / 10	10 / 12	9 / 13	8 / 14
$\sigma_P = 2$	10 / 10	8 / 12	7 / 14	4 / 15

However, if we check the behaviour of the *optimal policy*, the above increasing effect is not valid. The optimal reservation quantity is decreasing with the increase of uncertainty (both demand and price uncertainty) and the decrease is progressively larger as the uncertainties grow. This behaviour seems to be counter-intuitive. The explanation is that in the above cases there is a large increase in average inventory resulting from the optimal ordering strategy, so the spot price uncertainty is protected against by the accumulated inventory. What are the reasons for the inventory increase? The increase in *spot price variability* provides more frequent low price occurrences and thus, forward buying takes place more often yielding larger average inventory. If the *demand uncertainty* is increasing the required safety stock will increase helping in price protection, too. Both forward buying and safety inventory is providing a more economic protection than increased reservation as long as the inventory holding cost is not very high.

Comparing the expected total cost of the optimal and the base-stock policy, the difference is large for the combination of high spot price variability and high demand variability (see Table 4) because the base-stock policy does not apply forward buying and the safety stock calculation is not considering appropriately the combination of demand and price uncertainties.

Table 4 – Joint effects of changing the spot price variability, σ_p, and demand variability, σ_x, on the relative cost deviation of the base stock policy from the optimal policy, ΔZ

ΔZ	$\sigma_X = 0$	$\sigma_X = 3$	$\sigma_X = 6$	$\sigma_X = 10$
$\sigma_P = 0$	0,0%	1,0%	2,1%	3,4%
$\sigma_P = 0.5$	0,0%	1,0%	2,2%	3,5%
$\sigma_P = 1$	0,1%	1,3%	2,6%	4,2%
$\sigma_P = 1.5$	1,7%	3,3%	4,9%	7,0%
$\sigma_P = 2$	5,6%	7,6%	9,8%	12,5%

From these investigations we can learn that the increase in the variability of spot price and product demand have a decreasing impact on the capacity reservation decision because the inventory accumulation provides a good hedging as long as the inventory holding cost is not too large. The optimal capacity reservation level is close to or below the expected demand and the inventory is increasing with larger price and demand uncertainty. The sensitivity of the inventory level (via an adjustment of the order-up-to parameters) is significant. The long-term supplier (contract) order-up level, S_L is in average higher than expected demand due to safety stock accumulation. Safety stock motivation is large for high shortage/holding cost rate, high spot price average, and high demand variability. The short-term supplier (spot market) order-up level, $S_S(p)$ is in average higher than S_L because of the forward buying motivation which gets larger for small holding cost factor, small average spot price, and large spot price variability.

An increase in both price and demand variability always leads to a rise of the optimal average inventory (see Table 5) as a combined effect of forward buying and safety stock accumulation. For the base-stock policy the safety stock increases the inventory as the demand variability increases but it is not affected by the price variability as forward buying is not used. A similar impact is found if the backorder cost is increased and/or the holding cost is reduced.

Table 5 – Joint effects of changing the spot price variability, σ_p, and demand variability, σ_x, on the average inventory, $E^*(I)/E_B(I)$

$E^*(I)/E_B(I)$	$\sigma_X = 0$	$\sigma_X = 3$	$\sigma_X = 6$	$\sigma_X = 10$
$\sigma_P = 0$	0 / 0	8 / 7	17 / 15	29 / 27
$\sigma_P = 0.5$	0 / 0	8 / 7	17 / 15	28 / 27
$\sigma_P = 1$	1 / 0	9 / 7	18 / 15	30 / 27
$\sigma_P = 1.5$	8 / 0	16 / 7	26 / 15	38 / 27
$\sigma_P = 2$	20 / 0	29 / 7	38 / 15	53 / 27

As we have seen, the optimal reservation quantity is influenced by the inventory policy. On the other hand, the optimal inventory policy is also constrained by the reservation quantity as the maximum amount that can be ordered from the long-term supplier. What is specifically interesting from a managerial point of view is the fact that this combined sourcing policy is using both the speculative and safety motive of stock holding in an optimal manner. From the shape of the spot market order-up-to function, $S_s(p)$ (Figure 4), it becomes visible that for low spot price realizations – compared to the long-term procurement cost – the speculation motive becomes dominant and forward-buying plays a major role. For high spot prices, however, demand uncertainty is the main driver for stock-keeping and the safety motive dominates. The way the procurement decisions are combined is additionally affected by the capacity reservation decision which itself strongly depends on the forward buying behavior. So, all relevant decisions are highly interwoven, and it depends on the choice of the policy parameters if these interconnections are considered appropriately.

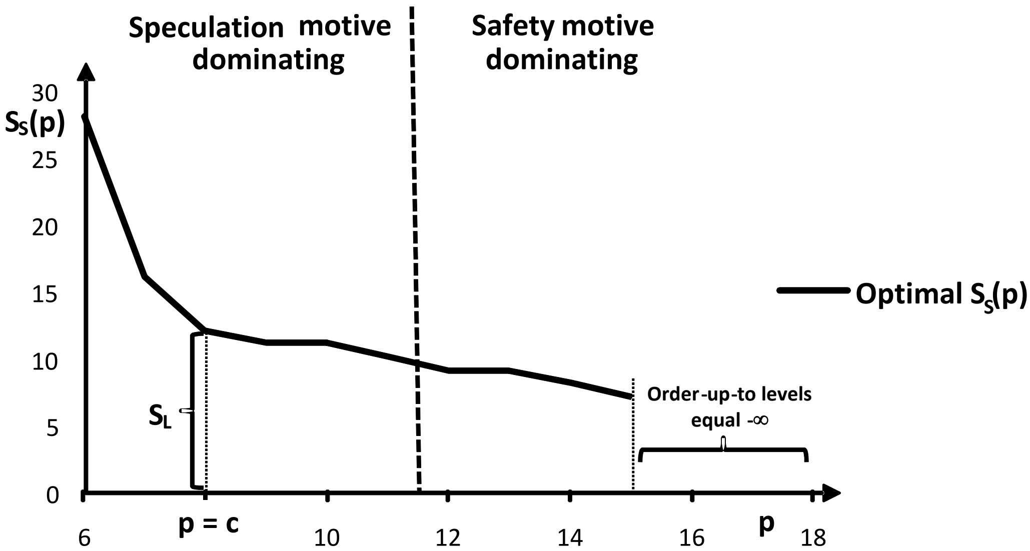

Figure 4 – Typical shape of the optimal order-up-to level function

Conclusion

From the numerical computations in the performance study we can get some insights into the effect of specific problem parameters on the optimal reservation size R^*, on the order-up-to levels and on the average inventory level, $E^*(I)$. To this end we used the stochastic dynamic programming computations and calculated respective results for a base case scenario and selected deviations from these base case data.

Further research should deal with extensions of the underlying sourcing problem. So it would be interesting to analyze if a simple policy structure is still optimal when additional procurement options like fixed commitment contracts or forward contracts are incorporated. Further extensions can also include more sophisticated spot price models from the finance area, covering for instance random walk or mean-reverting price processes. Finally, the issue of long-term contract negotiation regarding the cost and capacity parameters can be considered in the context of contract analysis and design for supply chain coordination among the long-term supplier and the buyer.

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