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The Effect of Material Price and Product Demand Correlations on Combined Sourcing and Inventory Management

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Abstract. Both material sourcing and inventory management are important competitiveness factors, and it is a significant challenge to integrate the two areas. In sourcing, combined strategies using long-term contracts and the spot market received increasing attention recently, typically concentrating on the financial effects. However, there is limited research on the consequence of combined sourcing considering both purchasing and inventory effects from an operations point of view. In this paper, we analyze the effect of uncertainty on the combined sourcing decision under stochastic demand and random spot-market-price fluctuations and exploit the benefits of forward buying in periods with low spot-price realizations, but also of intended backordering in case of a high spot price. Since the decision on capacity reservation has to take into account the short-term utilization of each source which in turn depends on the available long-term contract capacity, decision making faces highly complex interactions between long-term and short-term decisions.

From finance research, we find scarce evidence that the spot prices of commodities evolve independently over time. Rather, price correlation across time periods is found, and a popular way to describe these price dynamics is to model it as a mean reverting process. Thus, in this contribution we will respectively extend common i.i.d. price models from operations management studies and will additionally consider the effect of correlation between demand and price. In this paper, we provide a managerial analysis showing the effects of demand and spot market price correlations on the optimal procurement policy and provide managerial insights. We model the combined sourcing problem as a stochastic dynamic optimization problem and analyze the optimal procurement strategy by means of stochastic dynamic programming. The behavior of the optimal policy confirmed several previous assumptions, though some interesting and important managerial consequences arise due to demand and price correlations. Based on the policy analysis, a numerical study will reveal to which extent inobservance or misspecification of an existing level of correlation might result in performance losses in operational decision making. These observations play an important role under the trend of increasing volatility and dynamic changes on the spot market but also in the customer's behavior.

Key words: Capacity Reservation, Spot Market, Price Correlation, Mean Reverting Process, Stochastic Dynamic Programming, Managerial Analysis

1 Introduction

For manufacturing companies, the procurement of raw materials and components is becoming a more important and challenging issue because of decreasing margins, new sourcing options, and growing uncertainties in today's supply chains. A larger variety and volume outsourced increases total procurement expenses and global sourcing extends the number of sourcing alternatives, e.g. by including spot markets. These are capable to reduce average procurement cost but also add to supply uncertainty as they often exhibit considerable price variability. Demand variability also becomes larger with the proliferation of products and increased market competition. A further challenge in procurement management is to simultaneously account for different functions of inventory, e.g. to serve as a buffer against demand and supply uncertainty but also as a speculation stock in case of a temporary low purchase price instances. Under these conditions, procurement becomes a strategic activity requiring more information and better decision support being able to cope with uncertainty from both demand and price side.

In the purchasing practice, traditional fixed commitment contracts are often replaced by more flexible capacity reservation contracts but also by spot-market procurement in order to increase flexibility and exploit market opportunities. Leading companies in several industries are combining capacity reservation contracts and spot-market purchases to reap the benefits of the alternative sources. Applications include chemicals, commodity metals, raw materials, oil, liquefied gas, and semiconductors. For instance, Vukina et al. (2009) analyze a case from the food packaging industry using (among other combinations) a forward contract in addition to spot-market purchases, Nagali et al. (2008) describe how Hewlett-Packard uses forward, option and spot market portfolio procurement depending on the risk level. A multiple-sourcing strategy is also used in LNG purchasing (Yacef, 2010). Lian and Stanwey (2011) describe that Chinese steel makers use quarterly contracts with the major ore producers and adjust their ore inventories from the spot market. Recent reports of electricity trading practices combining contracts and spot market procurement include Benth et al. (2012), Gulpiar and Oliveira (2012), and Ruiz et al. (2012).

In our research, beside the spot-market sourcing, we consider a capacity reservation contract in which a reservation price, proportional to the reserved quantity, has to be paid for the option of receiving any amount per period at a fixed contract price up to the reservation quantity. We study the *combined sourcing* problem in a make-to-stock environment wherein the capacity level is to be fixed for some time interval with the contract supplier, which then serves as a real option providing protection against high spot-market price incidents. Then, it has to be decided - period by period - which quantities to procure from the two sources. Orders are released after observing the spot price but before knowing the demand of the subsequent period and simultaneously have to account for demand uncertainty and uncertainty in future price development. According to the make-to-stock situation under consideration, purchased material is immediately processed and stocks are only kept in form of finished goods.

Therefore, the spot market is only used for purchasing materials and not for reselling them to take advantage of spot price fluctuations as would be the case for a merchant (see, e.g., Secomandi, 2010).

The combined procurement strategy has to protect against risks of insufficient demand fulfilment and exploits the benefits of forward buying in periods with low spot price by keeping speculative inventories. The decision on capacity reservation has to take into account the short-term capacity utilization of each source which itself depends on the available long-term capacity reservation level. Thus, we face a highly complex interplay between long-term and short-term decisions under uncertainties in demand and spot-market price.

Even advanced studies in this field of combined sourcing under price and demand risks, including those, which integrate capacity reservation aspects like in Inderfurth et al. (2013), only use very simple models to describe how spot market prices will develop over time. The standard assumption is that spot prices are i.i.d. distributed. In many cases, however, this assumption is not sufficiently realistic to explain how prices might evolve in future and, therefore, might result in unfavorable purchasing decisions. In our contribution, we consider two types of interdependencies between current states and future prices. First, the current demand of a group of products impact expectations on the future price development of materials that go into these products. Thus, high today's product demand can tend to result in a high tomorrow's commodity spot price (demand/price correlation) as we know from studies, e.g. by Issler et al. (2014), which consider the respective effects of demand variability (including demand shocks). Second, as is deeply analyzed in financial literature, it is often found in commodity markets that the spot prices depend on previous prices of the same commodity (see, e.g., Ma et al., 2013). This means that price autocorrelation exists so that the observation of the current price has to be taken into account when estimating the probability distribution of future prices. An accepted way to model realistic price dynamics is to employ a mean-reverting pricing model. In this context, the main goal of this paper is to evaluate the effect of stochastic dependencies in price and demand on the optimal sourcing strategy for the case of using jointly the spot market and capacity reservation contract sourcing. In particular, we answer the following research questions:

- How does the policy structure change when integrating demand-price correlation and price autocorrelation and what is the impact on optimal policy parameters?
- How large is the performance loss when ignoring or misspecifying demand-price correlation and price autocorrelation in procurement decisions?

To this end, we incorporate respective pricing models with correlation in a stochastic multi-period optimization model for joint procurement and capacity reservation decisions (see Inderfurth et al., 2013). After discussing the relevant literature in Section 2, we provide the extended model and main results on the optimal policy structure in Section 3. A numerical analysis put forth in Section 4 shows the effects of demand-price correlation and spot price autocorrelation on optimal policy parameters and assesses the performance loss when ignoring (auto-) correlation for a wide range of parameter situations.

2 Literature Review

The impact of stochastic prices on procurement decisions with a single source has been analyzed since the seminal work by Kalymon (1971) which proves the optimality of price dependent parameters of an (s,S) policy in procurement decisions including variable and fixed cost. Golabi (1985) shows how forward buying is used in situations with i.i.d. stochastic prices and deterministic demand. Price thresholds are determined for the numbers of periods for which to satisfy demand by forward buying. Berling and Martínez-de-Albéniz (2011) consider different schemes for price evolution with intertemporal correlation and show the optimality of a base stock policy. They further show that their policy yields considerable improvements compared to approaches ignoring price correlation. As optimal policy parameters are difficult to obtain, Berling and Xie (2014) develop close-to optimal heuristics for this situation.

There are various variants of inventory control models combining spot-market sourcing with other procurement options where spot-market sourcing profits from either a lead time advantage (used to reduce stock keeping) or capacity flexibility. A comprehensive review of the literature up to 2007 is provided by Haskoz and Seshadri (2007). Examples of works published subsequently include Goel and Gutierrez (2012), Zhang et al. (2011), and Chen et al. (2013). Goel and Guiterrez (2012) combine periodic forward price procurement and continuous spot market sourcing. Zhang et al. (2011) consider a contract in which a total order quantity over the commitment period is fixed. The optimal policy is characterized by order-up-to levels for each sub-period that depend on the spot price, on-hand inventory, and remaining commitment quantity. Chen et al. (2013) consider a contract where a minimum quantity needs to be purchased and additionally consider setup cost for spot-market purchases.

Several papers deal with single-period decision situations with combined sourcing including Fu et al (2010) and Feng and Sethi (2010) where the stock-market typically is used as quick replenishment alternative to avoid lost sales. For instance, Feng and Sethi (2010) consider a single-period with multiple decision points and spot procurement combined with two types of capacity arrangements: dedicated capacity and overall capacity. Under a dedicated capacity arrangement, the manufacturer reserves a capacity for each adjustment order in the contract. Under an overall capacity arrangement, she keeps the flexibility of using the reserved capacity within the given period for possibly multiple adjustments. Jörnsten et al. (2013) showed that a mixed contract is superior to option contract for a single-period decision. However, the single period model formulation disregards the role of keeping inventory as safety stock for the coming period or using forward buying in low price instances in the case of periodic ordering as is often applied in industry procurement processes.

In our research, we are dealing with a capacity reservation contract alongside the spot-market procurement where a manufacturer pays a predetermined amount at each period which is proportional to the capacity reservation level to the supplier, and the supplier guarantees input availability up to a predetermined level of volume at a given price. In such a multi-period problem environment with periodical procurement but without lead time advantage for any of the both sources, Serel et al. (2001) considered a simple capacity reservation/order-up-to policy and Serel (2007) extended this approach to random demand and a spot market with random capacity but disregarding the spot market price uncertainty. Inderfurth and Kelle (2009) considered the case with random spot-market price and unlimited spot market capacity and derived properties of the optimal decision structure using stochastic dynamic programming. In a subsequent paper, Inderfurth and Kelle (2011) established simple analytical expressions for determining optimal parameters of a simplified policy with base stock ordering. In another paper, Inderfurth et al. (2013) propose an advanced heuristic approach to calculate parameters of the optimal capacity reservation/ordering policy and compare it with several simple heuristic approximations. However, the above approaches rest their analysis on the assumption of an i.i.d. spot-price process. Our contribution to the field of combined spot-market and capacity reservation procurement is the integration of advanced pricing models for spot markets in a periodic review setting with stochastic demand. Our approach thus provides detailed insights into the effects of different forms of price correlation on procurement and capacity reservation decisions.

Concerning the modeling of stochastic price processes with intertemporal price dependency, extensive literature from finance theory is dealing with this issue (see, e.g., Meade, 2010, for an overview on different pricing models used in the literature concerning crude oil prices). The two major groups of stochastic models of commodity price behavior rest either on the geometric Brownian motion or on the Ornstein-Uhlenbeck process of mean reversion. In the discussion which process being better suited in a specific situation, Bessembinder et al. (1995) find that a forward-looking analysis of the commodities futures data implies mean reversion. In the case of agricultural commodities and crude oil the magnitude of the estimated mean reversion is large; for example, 44 percent of a typical spot-price shock for crude oil reverses over the subsequent eight months. Empirical studies of historical data have found that mean-reverting models appropriately capture the evolution of commodity prices (e.g., Schwartz, 1997). An extension is a two-factor model of the mean reversion as a short-term effect and the Brownian motion as long-term effect (Schwartz and Smith, 2000). In the case of our sourcing decisions, the short-term effect seems to be prevalent and modelling the price process considering mean reversion is appropriate. Most approaches, however, model the price process in continuous time that does not fit well with periodic review procurement as is often applied in practice. A common modeling approach for discrete time commodity price development is data driven by applying discrete time ARMA type models (Meade, 2010). We restrict our analysis to stationary prices (no log price approach), therefore the error terms have no changes in variance. Under these conditions an AR(1) process is equivalent to the mean reversion (Summers, 1986). In the supply chain management literature, the AR(1) process is often used to describe the demand process (see, e.g. Lee et al., 2000) but it has also been applied to model price dynamics in Ma et al. (2013). As an alternative, some papers in the supply chain literature model the random spot price process as a Markov process (e.g. Feng and Sethi, 2010, and Chen et al., 2013).

In many industries fluctuations of prices for commodities on the input side go hand in hand with fluctuations of the production volume of output goods, at least under a short-term perspective. This is not only theoretically plausible but exhibits also empirical evidence. Based on monthly and quarterly data, it is empirically verified in Issler et al. (2014) that there exists a positive correlation between variation of industry production and price variation of metal commodities. Thus it seems quite likely that demand variability on the product side which triggered by demand randomness in our make-to-stock environment might have an impact on the spot market price level of materials. Xu et al. (2015) consider demand and spot price correlation in multiple sourcing of forward, option and spot market in a single period setting. In the multi-period extension they disregard the correlation and the forward buying advantages.

Based on these research contributions on spot market pricing, we have addressed the aspect of price correlation by using price models where the current period's price depends on the previous period's demand or on the previous period's price realization in form of an AR(1) mean-reverting process. In order to keep the overall procurement optimization problem manageable we restrict our analysis to linear pricing models.

3 Sourcing Model and Optimal Procurement Policy

The model rests upon a formulation without correlation that has been presented in Inderfurth et al. (2013). After summarizing the basic model and its main properties in Section 3.1, the framework is subsequently extended to incorporate intertemporal relationships between demands and prices (Section 3.2) and between subsequent prices (Section 3.3).

3.1 Problem Description and Model Formulation without Correlation

As it is common in the relevant literature, in the no-correlation scenario we consider independent and identically distributed (i.i.d) random demands, $\tilde{x}$, as well as random component spot-market price, $\tilde{p}$, and use the following notation:

- $F(x)$, $f(x)$, μ_x, σ_x cumulative distribution, density function, expected value and standard deviation of **demand** $\tilde{x}$ and
- $G(p)$, $g(p)$, μ_p, σ_p the same distribution characteristics for the **spot-market price** $\tilde{p}$.

The spot-market price consists of a stationary base level μ_p and an (i.i.d.) noise term ε_t yielding the following functional relationship:

$$p_t = \mu_P + \varepsilon_t \quad \text{where} \quad E(\varepsilon_t) = 0 \quad \text{and} \quad VAR(\varepsilon_t) = \sigma^2 . \tag{1}$$

The standard deviation of the spot-market price equals the standard deviation of the noise term, i.e. $\sigma_P = \sigma$. Procurement quantities from the spot market are assumed to be unrestricted given the respective spot price of a period.

We consider a periodic decision process involving different levels of knowledge in time. The **first decision** is on the capacity reservation quantity with the long-term supplier, R, that must be *fixed for a long time horizon* (assumed to be infinite) based on the following stationary cost factors:

c	the unit purchase price charged by the long-term supplier,
r	the capacity reservation price per period for a unit of capacity reserved,
h	the inventory holding cost per unit and period,
v	the backorder cost per unit and period.

The **next decision** is at the *beginning of each time period* about

$Q_{L,t}$	order quantity from the long-term supplier, and/or
$Q_{S,t}$	order quantity from the spot market

at the beginning of each period t, knowing

I_t	inventory level at the beginning of the period and
p_t	the current spot market price realization,

but without knowing the respective demand for that period. All shipments arrive before demand occurs. The period's total cost is charged at the end of the period after demand has realized. All unsatisfied demand is backordered. Costs are discounted at a factor α $(0 < \alpha \leq 1)$.

Given this problem description, the optimization problem over a planning horizon T is characterized as follows:

$$\text{Min } C = E_{\{p_1,\dots,p_T\},\{x_1,\dots,x_T\}} \left\{ \sum_{t=1}^{T} \alpha^{t-1} \left(rR + cQ_{L,t} + p_t Q_{S,t} + h\left[I_{t+1}\right]^+ + v\left[-I_{t+1}\right]^+ \right) \right\} \tag{2}$$

with inventory balance equation $I_{t+1} = I_t + Q_{L,t} + Q_{S,t} - x_t$,

initial inventory $I_1 = \overline{I}$, and constraints on sourcing decisions $0 \le Q_{L,t} \le R$, $0 \le Q_{S,t}$.

The structure of the optimal policy can be determined by using a stochastic dynamic programming approach. In order to develop the recursive equations of dynamic programming we introduce the following additional notation:

$D_t(I_t,R,p_t)$	minimum expected cost from period t to T for a starting inventory I_t and a given capacity reservation level R, **after** realization of spot market price p_t,
$C_t(I_t,R)$	minimum expected cost from period t to T for a starting inventory I_t and a given capacity reservation level R, **before** spot market price p_t realizes.

$C_1(\overline{I},R)$ corresponds to the minimum cost from optimizing the procurement decisions over all periods under a given reservation level R. Thus, the optimal capacity level, R, can be calculated by solving the single-variable optimization problem

$$\min \; C_1(\overline{I},R)\,. \tag{3}$$

The $C_1(\overline{I},R)$ function results from the solution of the stochastic dynamic procurement problem. For determining the optimal procurement decisions in period $t=1,...,T$, we evaluate the dynamic programming recursive relations which (suppressing the time index, t, for all variables for sake of simplicity) can be expressed by

$$D_t(I,R,p) = \min_{R\ge Q_L\ge 0, Q_S\ge 0}\left\{ rR + cQ_L + pQ_S + L(I+Q_L+Q_S) + \alpha\cdot\int_0^\infty C_{t+1}(I+Q_L+Q_S-x,R)f(x)dx \right\} \tag{4}$$

with $C_t(I,R) = \int_0^\infty D_t(I,R,p)g(p)dp$

and $C_{T+1}(I,R)\equiv 0$ as final cost condition for all I and R.

The function $L(I) = h\cdot\int_0^I (I-x)f(x)dx + v\cdot\int_I^\infty (x-I)f(x)dx$ describes the expected one-period holding and backorder costs.
From analyzing these relationships, one can derive several properties characterizing the optimal dual source procurement policy.

Proposition 1. As shown in Inderfurth et al. (2013), the following properties of an optimal procurement policy hold in the case without correlation:

(a) For a given capacity reservation level R, the optimal procurement policy in each period is of order-up-to type, given by

$$Q_L = \begin{cases} 0 & \text{for } p < c \\ \min\left\{(S_L - I)^+, R\right\} & \text{otherwise} \end{cases} \quad \text{and} \quad Q_S = \begin{cases} \left(S_S(p) - I\right)^+ & \text{for } p < c \\ \left(S_S(p) - \mathrm{R} - I\right)^+ & \text{otherwise} \end{cases}. \tag{5}$$

(b) The order-up-to level, $S_S(p)$, for short-term procurement decreases with increasing spot price. In case of price equality, i.e. $p = c$, both order-up-to levels coincide. Formally described, we have the relationship:

$$S_S(p) \begin{cases} > S_L & \text{if } p_t < c \\ = S_L & \text{if } p_t = c \\ < S_L & \text{if } p_t > c \end{cases}. \tag{6}$$

(c) The minimum cost function $C_1(\overline{I}, R)$ is convex in the capacity reservation level R for each starting inventory $\overline{I}$.

(d) The policy structure remains optimal for both, finite and infinite as well as discounted and undiscounted horizon problems. For the stationary infinite horizon problem the order-up-to-levels are identical for each period.

3.2 Correlation between Demand and Price

Extending the model to include correlation, we first consider the case where the price of the current period is influenced by the past demand level, i.e. we investigate the impact of a correlation between demand in period $t-1$ and the price in t. The functional relationship is assumed to be linear and is described by:

$$p_t = \mu_P + \beta\left(x_{t-1} - \mu_X\right) + \varepsilon_t \quad \text{where} \quad \beta \geq 0, \tag{7}$$

i.e. the deviation (positive or negative) of demand from its expected value in $t-1$ multiplied by a scaling factor β changes the price expectation in t which then is subject to an error term ε_t. In order to directly measure the impact of correlation we replace β by the resulting Pearson coefficient of correlation which is given by $\rho_{XP} = \beta \cdot \frac{\sigma_X}{\sigma_P} = \beta \cdot \frac{\sigma_X}{\sqrt{\beta^2 \sigma_X^2 + \sigma^2}}$. A reformulation of the functional relationship (7) with $\beta = \frac{\sigma}{\sigma_X}\sqrt{\frac{\rho_{XP}^2}{1-\rho_{XP}^2}}$ then yields

$$p_t = \mu_P + \sigma \cdot \sqrt{\frac{\rho_{XP}^2}{1-\rho_{XP}^2}} \cdot \frac{x_{t-1} - \mu_X}{\sigma_X} + \varepsilon_t \quad \text{where} \quad \rho_{XP} \in [0,1). \tag{8}$$

From the formulation in (8) we can directly see how the demand-price-correlation in form of a standardized measure influences the price process. According to what can be expected in practice, the correlation coefficient is assumed to be restricted to positive values. The expected value of the resulting spot-market price is characterized by μ_P and the standard deviation becomes $\sigma_P = \sigma \cdot \sqrt{\frac{1}{1-\rho_{XP}^2}}$. These parameters are used to formulate the demand-dependent price density function $g(p,x)$.

Since demand information in t impacts price in $t+1$, SDP formulation (4) must be adapted as follows:

$$D_t(I,R,p) = \min_{R \geq Q_L \geq 0, Q_S \geq 0} \left\{ rR + cQ_L + pQ_S + L(I+Q_L+Q_S) + \alpha \cdot \int_0^\infty C_{t+1}(I+Q_L+Q_S-x,R,x) f(x)dx \right\}$$

with $C_t(I,R,z) = \int_0^\infty D_t(I,R,p) g(p,z)dp$ and $C_{T+1}(I,R) \equiv 0$.

In spite of these changes, the properties of optimal policy structures provided in Proposition 1 remain valid as stated in Proposition 2. The reason for this result is the (qualitatively) unchanged knowledge of the decision maker when deciding upon the procurement orders.

Proposition 2. In the case of demand-price correlation, all properties of an optimal procurement policy stated in Proposition 1 remain valid.

Proof: Since the current price state p in $D_t(I,R,p)$ does not have an impact on the future cost $C_{t+1}(I+Q_L+Q_S-x,R,x)$ and since the $C_{t+1}(.)$ function is convex for each *x*, the structure of the optimization problem is exactly the same as in the case without correlation so that the policy structure carries over.

3.3 Intertemporal Price Correlation

In a second model extension we investigate price dynamics that are caused by a mean-reverting process, according to the suggestions from financial literature. We assume a process that is characterized as

$$p_t - \mu_p = \beta_1 \left(p_{t-1} - \mu_p \right) + \varepsilon_t \quad \text{where} \quad \beta_1 \in [0,1). \tag{9}$$

The functional relationship forms just a first-order autoregressive process (see, e.g., Zhang and Burke, 2011, Ma et al., 2013) which in general is given by

$$p_t = \beta_0 + \beta_1 \cdot p_{t-1} + \varepsilon_t$$

with $\beta_0 = (1-\beta_1)\mu_P$ and ε_t as error term like above. We further find for the 1-period coefficient of price correlation: $\rho_{PP} = \beta_1$. The expected steady-state spot-market price and its standard deviation are given by $\mu_p = \dfrac{\beta_0}{1-\beta_1}$ and $\sigma_p = \sigma\sqrt{\dfrac{1}{1-\rho_{PP}^2}}$, respectively (Ma et al., 2013). They are input of the conditional price density function $g(p,q)$ where q stands for the spot price of the previous period. Using μ_P and ρ_{PP}, the price dynamics equation can be formulated as follows:

$$p_t = (1-\rho_{PP})\cdot\mu_P + \rho_{PP}\cdot p_{t-1} + \varepsilon_t \quad \text{where} \quad \rho_{PP} \in [0,1). \tag{10}$$

Now, autocorrelation of the prices are incorporated into the SDP-Formulation by considering the previous price realization in the minimum expected cost function **before** spot market price p_t realizes, C_t, and (4) changes as follows:

$$D_t(I,R,p) = \min_{R\geq Q_L\geq 0, Q_S\geq 0}\left\{rR + cQ_L + pQ_S + L(I+Q_L+Q_S) + \alpha\cdot\int_0^\infty C_{t+1}(I+Q_L+Q_S-x,R,p)\,f(x)dx\right\}$$

with $C_t(I,R,q) = \int_0^\infty D_t(I,R,p)g(p,q)dp$ and $C_{T+1}(I,R,p)\equiv 0$.

In this situation the current spot price being known at the time of both procurement decisions has an impact not only on the current but also on the future cost position so that also the contract procurement decision has to account for this price level. Therefore, this change in the information structure also affects the optimal procurement policy as described in the following proposition.

Proposition 3. In the case of autocorrelation between subsequent prices both order-up-to-levels, S_S and S_L, are price-dependent, and properties (a) and (b) of an optimal procurement policy, as stated in Proposition 1, change as follows:

(a) For a given capacity reservation level *R*, the optimal procurement policy under static conditions is given by

$$Q_L = \begin{cases} 0 & \text{for } p < c \\ \min\left\{(S_L(p)-I)^+, R\right\} & \text{otherwise} \end{cases} \quad \text{and} \quad Q_S = \begin{cases} \left(S_S(p)-I\right)^+ & \text{for } p < c \\ \left(S_S(p)-\mathrm{R}-I\right)^+ & \text{otherwise} \end{cases}. \tag{11}$$

(b) Both order-up-to levels, $S_S(p)$ and $S_L(p)$, are price-dependent and generally deviate from each other. Only if spot price and contract purchase price are equal, i.e. $p = c$, both order-up-to levels coincide. Thus we find

$$S_S(p)\begin{cases} > S_L(p) & \text{if } p < c \\ = S_L(p) & \text{if } p = c \\ < S_L(p) & \text{if } p > c \end{cases} \tag{12}$$

For a proof see Appendix A.

4 A Numerical Analysis on the Impact of Correlation

The purpose of this numerical investigation is threefold. First, after introducing the solution methodology in Section 4.1, we show how both demand/price and price/price correlation impacts optimal decisions (reservation quantity and order-up-to levels) in Section 4.2. To this end, we solve the models introduced in Section 3 for a base case and perform a sensitivity analysis. Second, we compare the optimal decisions to those which result from misspecifying the price process by ignoring correlation. This is done by additionally solving the model without correlation but equivalent price variability σ_P (NOCOR-EPV) and using the resulting policy parameters as a simplified approach in the models with correlation. Finally, in Section 4.3 we assess the potential performance loss of wrongly specifying the price model considering a full factorial design of parameter values.

4.1 Solution Methodology and Experimental Design

In order to abstract from planning horizon effects, we dealt with the undiscounted infinite horizon problem where the objective is to minimize expected cost per period. This ensures stability of policy parameters and simplifies our discussion. However, the procedures described below also can be applied to finite horizon problems and to consideration of discounting.

The numerical optimization method is based on the value iteration procedure of stochastic dynamic programming with discretized state space and linear approximation of the value function for extremely high and low inventory levels. Probability distributions of both demand and noise term are discretized in their respective $\mu \pm 3\sigma$ interval. The corresponding levels have been chosen such that a numerical optimization can be performed within reasonable time. For the same reason, the state space has been limited to inventory levels in the interval $[-100,180]$. Spot price dynamics have been limited to the interval $[1,30]$.These intervals allow for sufficient precision in all scenarios from the numerical parameters in Table 1 chosen for our experimental study. Since this restriction as well as rounding in the spot-price dynamics can considerably affect both the average spot-market price and its variability we did simulate the discretized spot-price dynamics over 100,000 periods to assure that in all chosen parameter situations, the differences between the discretized average price and the stated expected price does not exceed 1%. The simulation results have additionally been used to estimate an empirical probability distribution function for the spot-market price to be used when solving the model without correlation but equivalent price variability (NOCOR-EPV).

The optimization procedure exploits results on the policy structure as well as the convexity properties provided in Propositions 1 to 3. The iterative procedure can be sketched as follows. For a given capacity reservation level R, we solve a single-period problem and determine the corresponding order-up-to levels. These order-up-to levels then are used to obtain the one-period value function for all relevant combinations of inventory-levels and spot-market prices. Next, we solved the two-period problem and so on, until both the order-up-to levels no longer change and the difference in average cost per period between the current and the previous iteration falls below 10^{-5}. In our examples, this assures a precision of the objective value of roughly $\pm 0.1\%$ by taking between 1064 and 8942 iterations with an average of 5022. Finally, the procedure is repeated for a capacity reservation level R which is either increased or reduced by one until there is not further improvement in the objective. For computational efficiency, we implemented the optimization procedure using the C programming language.

We executed a full factorial experimental design with 3 levels for all (6) relevant parameters except of the contract price and the average demand that have been fixed. Apart from zero correlation for some comparisons, for the coefficients of positive correlation we used two levels (low and high). This yields $2 \cdot 3^6 = 1458$ instances for each correlation model. In an accompanying sensitivity analysis we selectively varied parameters of a base case scenario consisting of mid-values for all parameters. In all instances, random demand follows a gamma distribution and the (spot-market) noise term is assumed normal. All other parameters are chosen such that there is a large number of instances in which (a) both sources are used in an optimal solution and (b) stock-keeping due to forward buying and safety motivations are present, i.e.

- the long-term contract is on average not more costly than the spot-market option,
- there is considerable probability that the spot price is smaller than the contract purchase price,
- price variability is that high and holding cost are that low that forward buying is present,
- demand variability and backorder cost are so high that safety stock is needed.

For the correlation coefficients values are chosen that cover a broad range of possible cases and are supported by empirical findings (see, e.g. Ma et al., 2013, for an overview on such studies reporting ρ values up to 0.89).

Table 1 provides an overview on the parameter selection.

Table 1: The selected parameters for the experimental design.

Parameters		**Levels**	
	Low	**Mid**	**High**
Contract price c		10 (fixed)	
Reservation price r	.25	.5	1
Holding cost factor h	0.1	0.2	0.4
Shortage cost factor v	4	8	16
Expected demand μ_X		10 (fixed)	
Demand standard deviation σ_X	1	3	5
Expected spot price μ_P	10	12	14
Standard deviation of noise term of price σ	1	2	3
Coefficient of correlation ρ_{XP} / ρ_{PP}	0.4		0.8

For each experiment we did calculate the average total cost per period C, optimal policy parameters, as well as the expected on hand inventory $E(OH)$ and expected backorders per period $E(BO)$ to estimate the speculation effect of forward buying and intended backordering, respectively.

4.2 Impact of Correlation on Policy Parameters

4.2.1 Impact of Demand-Price Correlation

We first discuss the effects of increasing demand-price correlation from a zero level of $\rho_{XP} = 0$ to a high level of $\rho_{XP} = 0.8$ on the optimal solution in a base case scenario consisting of mid-values for all remaining parameters. The corresponding price dependent order-up-to levels are depicted in Figure 1.

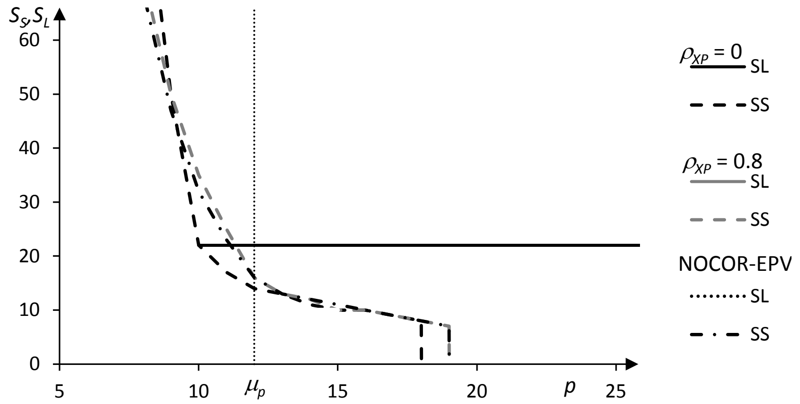

Figure 1: Order-up-to levels in base case scenario for coefficient of correlation $\rho_{XP} \in \{0, 0.8\}$.

In the case of zero correlation (i.e. $\rho_{XP} = 0$) the optimal capacity reservation quantity is $R = 8$ and the order-up-to level for contract supplier procurement is $S_L = 22$. The order-up-to levels for spot market procurement $S_S(p)$ decrease in p reflecting forward buying similar to the findings of Golabi (1985) (high level including multiples of average demand per period and a steep descent for $p < c$), identical order-up-to levels for $p = c = 10$, safety stock considerations (somewhat above average demand per period and slow descent in the interval $p \in [c,18]$), as well as intended backorders (no ordering for $p > 18$). In the absence of correlation between subsequent prices there is a large probability of small prices followed by a high price and vice versa motivating a high level of forward buying and intended backorders.

Increasing the coefficient of correlation to $\rho_{XP} = 0.8$ yields a situation where contract procurement is no longer used, i.e. $R = 0$, because the likelihood of very low spot prices is that high that under medium cost levels the cost of reserving capacities for long-term procurement does not pay off. Thus, S_L becomes irrelevant. In comparison to the previous situation, the order-up-to levels for spot market procurement $S_S(p)$ become smaller for low levels ($p < c$) and larger for $c \le p \le \mu_P$. Since the long-term contract procurement does not play a role, forward buying increases for spot prices between the long-term contract price and the average spot-market price. This cumulative effect of the changes in order-up-to levels yields to an increase in average on-hand inventory from 29.5 to 64.3 (more forward buying). For prices $p > \mu_P$ the order-up-to level remains at a similar value. However, as there is a higher probability of high price incidents, intended backorders start later for $p > 19$. Even so, average backorders increase from 0.05 to 0.09.

Except for minor numerical differences due to discretization, the optimal policy parameters resulting in a situation with high demand-price correlation are identical to those determined when ignoring correlation but considering equivalent price variability (NOCOR-EPV in Figure 1). This effect is caused by the fact that in case of demand-price correlation the respective demand and price information are available for ordering-decision making at the same point in time (at the beginning of each period). Thus, only the variance enlarging effect of correlation matters and has an impact on the procurement decision.

In order to demonstrate the impact of demand-price correlation on reservation capacity, Figure 2 shows for different levels of the average spot-market price μ_P how reservation capacity R changes when ρ_{XP} increases. First observation reveals that (as expected) long-term reservation quantity R increases with rising average spot-market price μ_P. Furthermore, for very small and for very high values of μ_P, ρ_{XP} does not impact R since under those conditions, sourcing only uses either spot-market or the long-term contract. Under dual sourcing conditions (middle values of μ_P) and small levels of ρ_{XP}, long-term reservation quantity R seems to increase slightly (although this could also be attributed to some numerical effect as the objective function is rather flat in R in the considered instances). For large ρ_{XP} values, R decreases as ρ_{XP} increases. Summarizing, demand-price correlation yields similar effects as can be expected when just increasing spot-price variability (R decreases, forward buying and intended backorders increase), further on referred to as the *variability effect*.

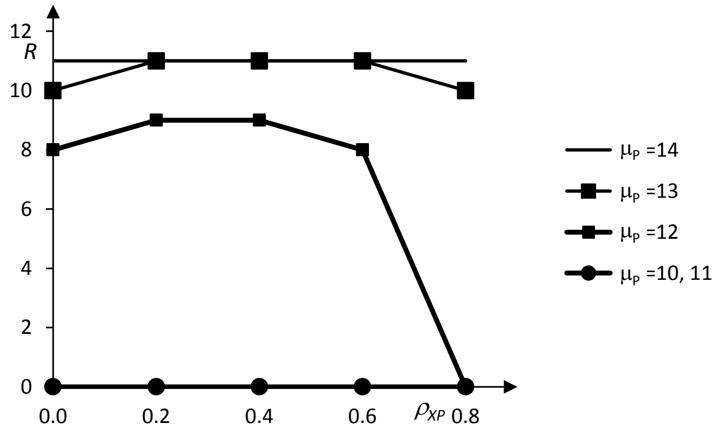

Figure 2: Impact of coefficient of correlation ρ_{XP} on long-term reservation quantity R for different spot-market price levels $\mu_P \in [10,14]$.

4.2.2 Impact of Price Autocorrelation

Similar to the previous situation, introducing price autocorrelation ($\rho_{PP} = 0.8$) into the base case scenario increases spot-price variability σ_P from 2.0 to 3.3. This, however, does not yield a decreasing capacity reservation level R as expected from the *variability effect*. Instead, R increases from 8 to 11. Figure 3 shows that next to the $S_S(p)$ level for spot market procurement now also the order-up-to level for contract supplier procurement is price-dependent. The $S_L(p)$ level, however, does not exceed the S_L value in case of zero correlation. Furthermore, like it is plausible under positive spot price correlation $\rho_{PP} > 0$, the contract order-up-to-level $S_L(p)$ is increasing if the spot price is rising while the spot-market order-up-to-level $S_S(p)$ is decreasing. Interestingly, the $S_S(p)$ values become considerably smaller for $p \le \mu_P = 12$ and larger for $p \ge 14$ than in the case without correlation, and spot-market procurement now also takes place for prices larger than 18.

Despite the considerable escalation of the spot-price variability, these changes result in only a modest increase in forward buying (expected on-hand inventory increases from 29.5 to 29.9) and negligible change in intended backorders (expected backorders remain level at 0.05). This effect can be explained by price dynamics: A spot-price realization below the average spot-market price increases the probability of a small price incident in the next period and reduces the necessity of forward buying. A high spot-price realization shifts the expected price for the next period upwards and reduces the benefit of intended backordering. It further increases the benefit of having contract capacity available which explains the increasing R. This *correlation effect* reduces and even reverses the impact of the increased spot-price variability.

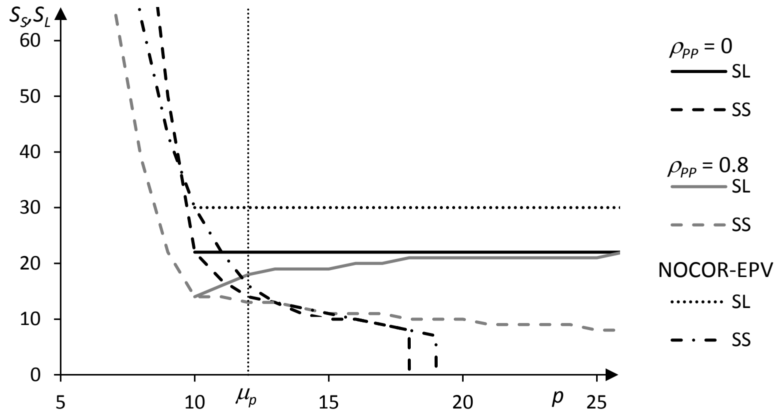

Figure 3: Order-up-to levels in base case scenario for coefficient of correlation $\rho_{PP} \in \{0, 0.8\}$.

Ignoring price autocorrelation yields a solution where contract capacity R decreases to 1 and (price invariant) S_L increases to 30. Spot market order-up-to levels $S_S(p)$ grow larger for $p \le \mu_P$ and are smaller or equal for $p > \mu_P$ than they would be when correctly considering price autocorrelation. Zero spot-market procurement occurs for $p > 19$. Consequently, there is much more forward buying and intended backordering when price autocorrelation is not considered as the expected on-hand inventory is 38.5 units and expected backorder is 0.4 units higher than in case of correctly taking correlation into account.

Figure 4 provides additional insights into the relationship between price autocorrelation and expected on-hand inventory and expected backorders for both cases. While the inventory/backorder level under the optimal solution (opt) is not considerably affected by a rising coefficient of correlation, both expected on-hand inventory and expected backorders in a solution ignoring autocorrelation surmount their respective counterpart by 1.4 (5%) and 0.012 (26%), respectively, for $\rho_{PP} = 0.2$ already. In case of high coefficient of correlation $\rho_{PP} = 0.8$ this gap amounts to 29% and 728%, respectively, and a considerable performance loss of 13% results (deviation from optimal costs when using the price model considering correlation).

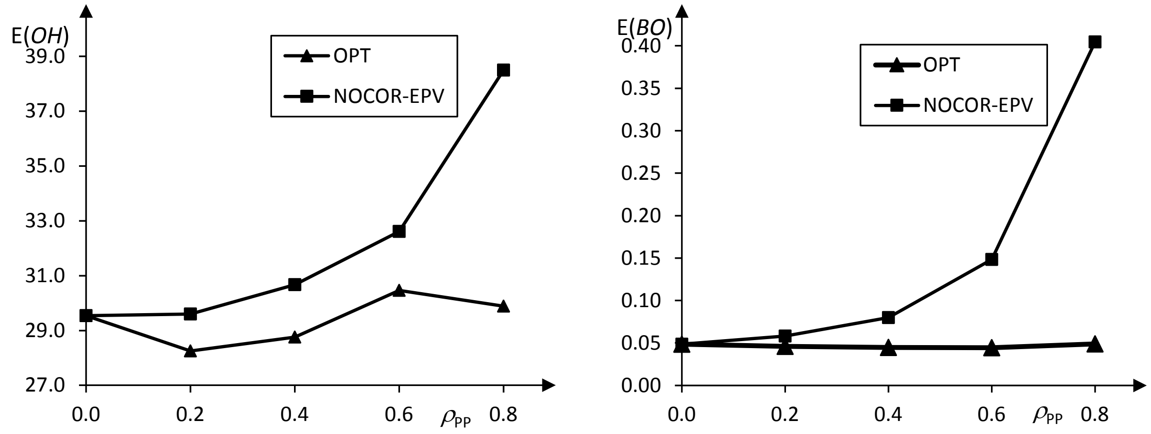

Figure 4: Impact of coefficient of correlation ρ_{PP} on expected on-hand inventory $E(OH)$ (left) and expected backorders $E(BO)$ (right) for different policies (OPT = optimal solution considering price correlation, NOCOR-EPV = results from solution ignoring price correlation but with equivalent price variability).

The counteracting of both the *variability* and the *correlation effects* is further demonstrated by considering the impact of price autocorrelation on reservation capacity (see Figure 5). As in the case of demand-price correlation, long-term reservation quantity R increases in expected spot-market price μ_P. In the optimal solution, R increases in ρ_{PP} making contract procurement a viable option even in the case of a small expected spot price μ_P. Since forward buying is less beneficial for dealing with high spot-market price situations, long-term contract capacity is used to insure against persisting high spot-price situations. When ignoring autocorrelation, however, long-term reservation quantity R decreases for increasing ρ_{PP}.

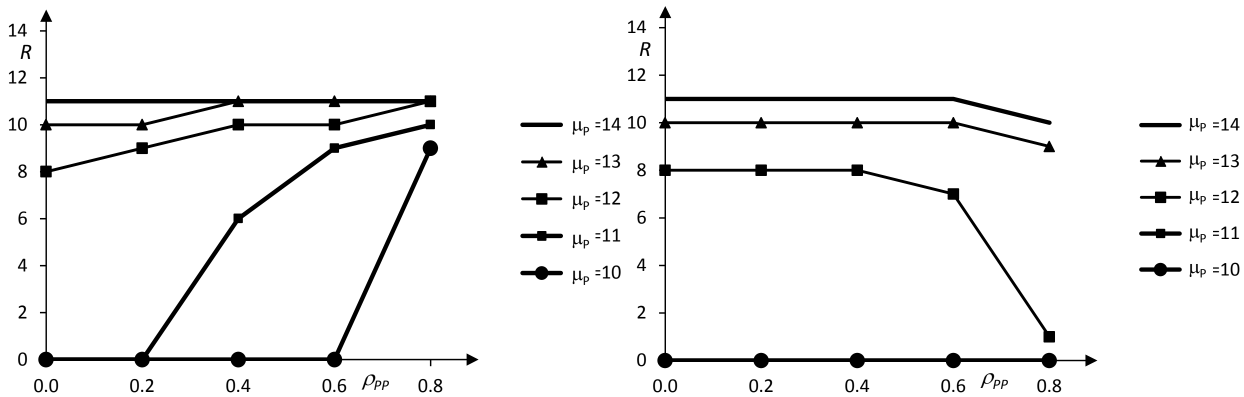

Figure 5: Impact of coefficient of correlation ρ_{PP} on the long term reservation quantity R for the optimal policy (left) and correlation ignoring policy (right) for different spot-market price levels $\mu_P \in [10,14]$

4.3 Impact of Misspecification of the Price Process on Cost

In this section, we report on results regarding the potential performance loss when wrongly specifying the price model in such a way that correlation is ignored although present and its effect is only incorporated by an increase of the price variability (NOCOR-EPV) which is estimated according to the σ_P level under correlation. Since there is no considerable effect in case of demand-price correlation (see above), our discussion concentrates on price autocorrelation. For each instance of the considered full factorial design (see Table 1), we determined the relative cost deviation $\Delta Cost_{rel}$ when using parameters from the solution without price autocorrelation but equivalent spot-price variability as a substitute in the model where price correlation is present.

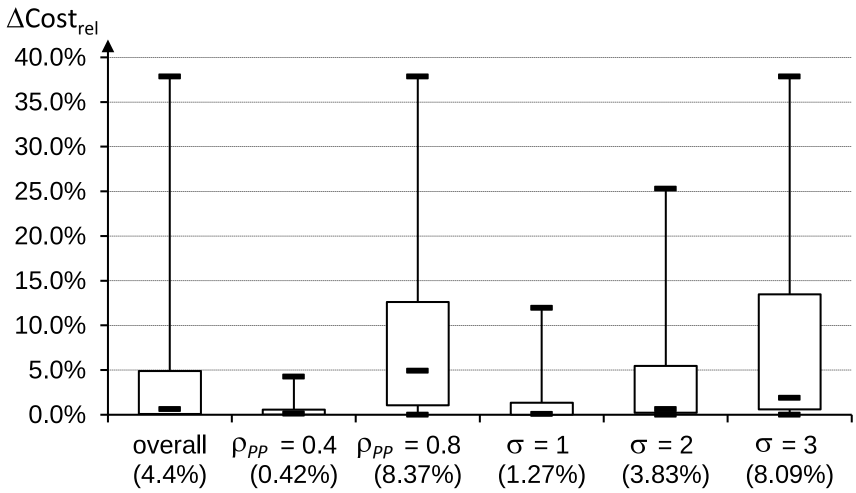

Figure 6: Impact of coefficient of correlation ρ_{PP} and standard deviation of noise term σ on expected cost error of misspecification.

Figure 6 provides a box plot diagram summarizing the general behavior of the approach in case of misspecification. The first column shows the percent cost increases if the NOCOR-EPV parameter values are used instead of the optimal ones. The box plot specifies (in increasing order) the minimum, first quartile, median, third quartile, and maximum of the percent cost increase. The average increase is in parentheses below the description of the graph (numerical values of all box-plot graphs are included in **Appendix B**). Over all 1458 instances, the average cost penalty is 4.4%. The worst-case cost increase is 38% in an instance with parameters: $r = 0.5$ (middle), $h = 0.2$ (small), $v = 4$ (small), $\sigma_X = 1$ (small), $\mu_P = 12$ (middle), $\sigma = 3$ (large), $\rho_1 = 0.8$ (large).

Next, we review the impact of different model parameters on the heuristic performance. In Figure 6, columns 2 to 6 reveal that both the coefficient of correlation ρ_{PP}, but also the variability of the noise term during price evolution σ considerably affects the error. In addition, Table 2 reports on the combined effect of both parameters showing that the largest performance losses are present for parameter combinations where $\sigma = 3$ and $\rho_{PP} = 0.8$. Restricting to these instances we further analyzed the effects of cost and other parameters (see Figure 8 in **Appendix B**). It shows that both average and median cost penalty tends to grow for increasing capacity reservation price r and demand variability σ_X, as well as for decreasing holding and backorder cost rates h and v, respectively. The worst-case behavior only seems to be impacted by the backorder cost rate where an increasing v also decreases the worst-case error.

Table 2: Combined impact of correlation ρ_{PP} and standard deviation of noise term σ on average and maximum (in parentheses) expected cost error of misspecification

	$\sigma = 1$	$\sigma = 2$	$\sigma = 3$
$\rho_{PP} = 0.4$	0.1% (1.1%)	**0.4% (2.5%)**	0.8% (4.3%)
$\rho_{PP} = 0.8$	2.5% (12%)	7.3% (25.3%)	**15.3% (37.9%)**

From what we have seen above, a critical factor in misspecifying price autocorrelation is the selection of the capacity reservation quantity R. In Figure 7 we therefore show the effect of ignoring autocorrelation on both R and the corresponding cost deviation $\Delta Cost_{rel}$ for different levels of ρ_{PP} based on the worst-case instance. It can be seen that starting at $\rho_{PP} = 0.4$ a large gap between the optimal and the heuristic reservation quantity exists that yields performance erosion that is more than proportionally increasing in ρ_{PP}.

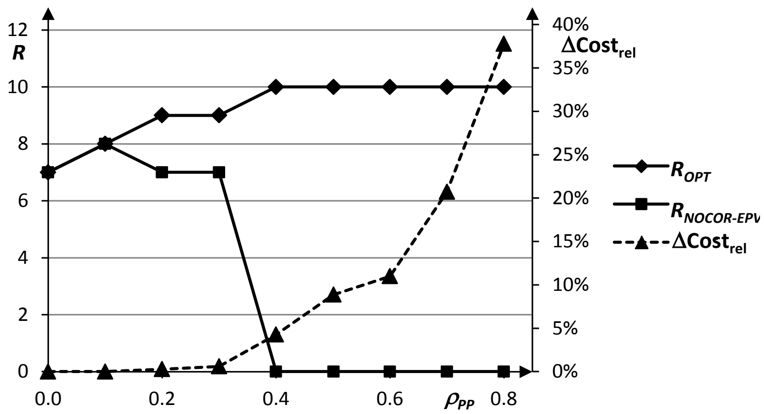

Figure 7: Impact of coefficient of correlation ρ_{PP} on capacity reservation level R and expected cost error of misspecification.

Interestingly, there is no clear effect when changing average spot-market price μ_P. A main driver for this result is that for very large and small values of the average spot-market price sole sourcing becomes the optimal strategy, either in form of exclusive spot market or of contract procurement.

Under those circumstances, the performance loss only arises from errors in selecting order-up-to levels, thus becoming smaller. Since for a given contract-purchasing price per unit the relationship between average spot-market price and capacity-reservation price mostly decides on sole or combined sourcing, we also report on the joined impact of μ_P and r in Table 3. Here, it can be seen that just those combinations of μ_P and r (small/small, medium/medium, large/large) yield largest average and worst case performance losses which make it most likely that combined sourcing will occur.

Table 3: Combined impact of average spot-market price μ_P and capacity reservation price r on average and maximum (in parentheses) expected cost error of misspecification (restricted to $\sigma = 3 / \rho_{PP} = 0.8$ instances)

	$r = 0.25$	$r = 0.5$	$r = 1$
$\mu_P = 10$	**20.9% (36.8%)**	20.8% (35.4%)	13.8% (27.3%)
$\mu_P = 12$	8.8% (35.1%)	**22.2% (37.9%)**	20.9% (33.8%)
$\mu_P = 14$	1.9% (5.8%)	6.5% (23.6%)	**22.3% (37.8%)**

Summarizing, it turns out that we find a considerable number of instances where a major performance loss is present when price autocorrelation is disregarded, even if the price variance blow-up in case of increasing correlation is taken into consideration. The analysis of policy parameters reveals that it is almost always the deviation in the capacity reservation level that drives the performance loss. Specifically, the critical cases of losses in the 20% and 30% area are those where misspecification of correlation results in the choice of a zero reservation level instead of capacity reservation close to expected demand so that long-term procurement is waived while this is not the case if the optimal policy. Interestingly, the computational study shows that the performance loss that can be attributed to deviations in the order-up-to levels resulting from misspecifying the correlation is fairly small. This holds despite the fact that the price-dependency of the long-term procurement level $S_L(p)$ is disregarded completely in case of misspecification (see also Appendix C).

5 Conclusions and Outlook

Most studies that address dual-sourcing procurement problems in which also spot markets with price uncertainty are considered only use simplistic models for incorporating randomness of spot prices. In this paper it is shown that relying on simple i.i.d. price processes can lead to major deviations from optimal procurement decisions and minimum overall costs if inter-temporal dependencies in the evolution of prices play a role. Based on a stochastic and multi-period combined sourcing model with capacity reservation it is possible to analyze the optimal procurement strategy in the case without and with price correlation. This can be done for two different types of well-observed correlation schemes across periods, namely a demand-price and a price-price correlation.

From a computational study that is based on the analytical policy results we learn that these two types of correlation have a significantly different impact on the performance loss when the decision maker misspecifies the correlation and treats its effect just as an increase in price variability. While this is a viable procedure in case of demand-price correlation, it can create huge performance losses if a major spot-price autocorrelation is present. This type of correlation needs to be considered in the capacity reservation decision since, compared to a situation without correlation, it reduces the benefits of forward buying and intended backordering so that in general much more long-term capacity is used to hedge against persisting high spot price situations. Neglecting this relationship can result in cost increases of far more than 30% if a high price autocorrelation is accompanied by a high level of price variability. Managers should be aware of this risk of ignoring price autocorrelation especially in situations where cost and other parameters are such that complete or predominant single-sourcing is not optimal.

Further research should also investigate the effects of an inter-temporal correlation of demands which is not addressed in this study where i.i.d. demands are assumed. Demand autocorrelation complicates the analysis insofar as it enforces the introduction of an additional state variable in the formulation of the dynamic optimization problem and, thus, will change the structure of the optimal policy. Since autocorrelation on the demand side is an effect that is observed in practice, it is not only challenging but also necessary to extend the present research to incorporate this aspect. Furthermore, it would also be interesting and worthwhile to investigate to which extent our results also hold under other procurement options that are used in practice like fixed commitment contracts or forward contracts.

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Appendix A - Proof of Proposition 3

The starting point is given by the dynamic programming recursive equations for $t = 1,...,T$:

$$D_t(I,R,p) = \min_{R \ge Q_L \ge 0, Q_S \ge 0} \left\{ rR + cQ_L + pQ_S + L(I + Q_L + Q_S) + \alpha \cdot \int_0^\infty C_{t+1}(I + Q_L + Q_S - x, R, p) f(x) dx \right\}$$

with $C_t(I,R,q) = \int_0^\infty D_t(I,R,p) g(p,q) dp$ and $C_{T+1}(I,R,q) \equiv 0$.

The **major steps of the proof** include

- proving the optimality of the $(S_L(p), S_S(p))$ policy by complete induction,
- proving that this policy holds for any t if $C_{t+1}(I,R,q)$ is convex in I and R
- proving that $D_t(I,R,p)$ is convex in I and R if this policy is applied,
- proving that this holds for the final period $t = T$, and
- proving that $C_1(\bar{I},R,q)$ is a convex function in R .

The optimization problem in period *t* can be reformulated as

$$D_t(I,R,p) = \min_{R \ge Q_L \ge 0, Q_S \ge 0} \left\{ cQ_L + pQ_S + H_t(I + Q_L + Q_S, R, p) \right\}$$

with $H_t(I + Q_L + Q_S, R, p) \equiv L(I + Q_L + Q_S) + \alpha \cdot \int_0^\infty C_{t+1}(I + Q_L + Q_S - x, R, p) f(x) dx$

By assumption $C_{t+1}(I,R,p)$ is convex in I and R for each p , *thus* $H_t(I,R,p)$ is also convex in I and R for each p due to well-known convexity of $L(I)$. So, under the assumption that a $(S_L(p), S_S(p))$ policy holds, we can analyse the properties of minimum cost functions $D_t(I,R,p)$ and $C_t(I,R,q)$

i. in case of $p \le c$: $D_t(I,R,p) = \begin{cases} p\cdot(S_S(p)-I)+H_t(S_S(p),R,p) & \text{if } I \le S_S(p) \\ H_t(I,R,p) & \text{if } I \ge S_S(p) \end{cases}$

ii. in case of $p \ge c$:

$$D_t(I,R,p) = \begin{cases} c\cdot R + p\cdot(S_S(p)-I-R)+H_t(S_S(p),R,p) & \text{if } I \le S_S(p)-R \\ c\cdot R + H_t(I+R,R,p) & \text{if } S_S(p)-R \le I \le S_L(p)-R \\ c\cdot(S_L(p)-I)+H_t(S_L,R,p) & \text{if } S_L(p)-R \le I \le S_L(p) \\ H_t(I,R,p) & \text{if } I \ge S_L(\text{p}) \end{cases}$$

We can easily show that $D_t(I,R,p)$ is twice continuously differentiable in I and R for each p. Due to convexity of $H_t(I,R,p)$ for each p we have:

$$\frac{\partial^2}{\partial I^2}H_t(I,R,p) \ge 0 \ , \ \frac{\partial^2}{\partial R^2}H_t(I,R,p) \ge 0, \text{ and}$$
$$\frac{\partial^2}{\partial I^2}H_t(I,R,p)\cdot\frac{\partial^2}{\partial R^2}H_t(I,R,p) - \frac{\partial^2}{\partial I\partial R}H_t(I,R,p)\cdot\frac{\partial^2}{\partial R\partial I}H_t(I,R,p) \ge 0.$$

So the Hessian of $D_t(I,R,p)$ is positive-semidefinite for each p, $D_t(I,R,p)$ is convex in I and R for each p, and $C_t(I,R,q) = \int_0^\infty D_t(I,R,p)g(p,q)dp$ is convex in I and R due to $g(p,q) \ge 0$.

Steps of induction:

- **For $t = T$ (start of induction)** we have: $H_T(I,R,p) = L(I)$ independent of R, *thus* $H_T(I,R,p)$ is convex in I and a $(S_L(p), S_S(p))$ policy is optimal for $t = T$.
- **For each** $t < T$ the following holds: From convexity of $C_{t+1}(I,R,p)$ it follows that also $C_t(I,R,p)$ is convex in I and R for each p, *so* $H_{t-1}(I,R,p)$ is also convex in I and R, and consequently for each R and p a $(S_L(p), S_S(p))$ policy is optimal also for $t-1$.

Conclusions on policy structure:

For each R a $(S_L(p), S_S(p))$ policy is optimal for each $1 \le t \le T$.

- Policy parameter $S_{L,t}(p)$ is calculated from $\frac{\partial H_t(S,R,p)}{\partial S} + c = 0$ for each R and p.
- Policy parameter $S_{S,t}(p)$ is calculated from $\frac{\partial H_t(S,R,p)}{\partial S} + p = 0$ for each R and p.

- Policy parameter R is calculated from: $\dfrac{\partial C_1(\overline{I}, R, \overline{p})}{\partial R} = 0$ for given initial inventory $\overline{I}$ and initial price $\overline{p}$.

- Functions $C_1(\overline{I}, R, \overline{p})$ and $H_t(I, R, p)$ are convex.

From unconstrained optimization we get as optimal inventory levels

- after Q_L-optimization : $S_{L,t}(p_t, R)$ from $\dfrac{\partial H_t(S, R, p_t)}{\partial S} + c = 0$
- after Q_S-optimization : $S_{S,t}(p_t, R)$ from $\dfrac{\partial H_t(S, R, p_t)}{\partial S} + p_t = 0$.

Due to $\dfrac{\partial}{\partial Q_L} H_t(I + Q_L + Q_S, R, p_t) = \dfrac{\partial}{\partial Q_S} H_t(I + Q_L + Q_S, R, p_t)$, and due to restrictions $0 \le Q_L \le R$ and $0 \le Q_S$ we get the policy structure described in *Proposition 3(a)* and find the relevant cost functions to be convex.

From convexity of $H_t(S, R, p)$ and respective optimality conditions for the order-up-to levels it immediately follows that $S_{S,t}(p_t) \begin{cases} > S_{L,t}(p_t) & \text{if } p_t < c \\ = S_{L,t}(p_t) & \text{if } p_t = c \\ < S_{L,t}(p_t) & \text{if } p_t > c \end{cases}$

This is just the relationship described in *Proposition 3 (b)*.

The stationarity property of the policy for infinite horizon problems for problems without discounting can be shown in just the same way as in the case without correlation (see Inderfurth et al., 2013).

Appendix B

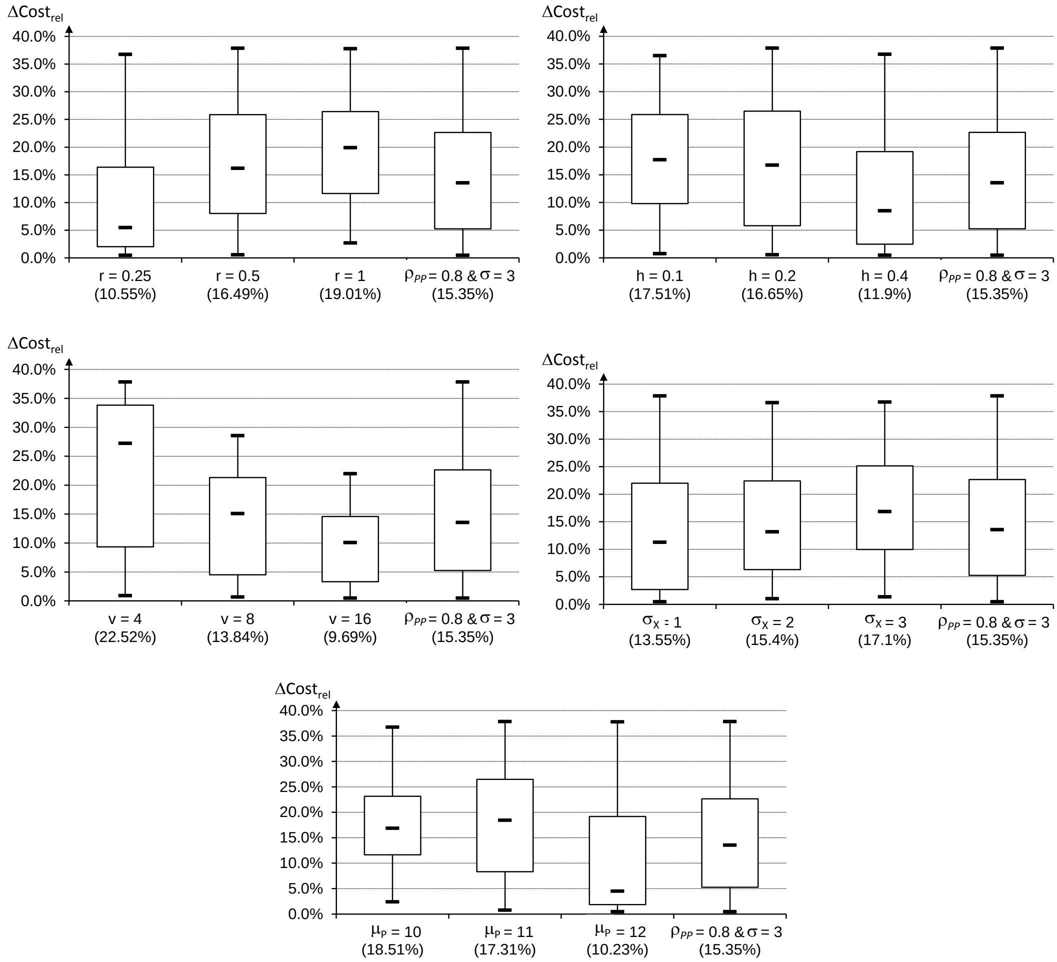

Figure 8: Impact of model parameters capacity reservation price r, holding cost h, backorder cost v, demand standard deviation σ_X, and average spot-market price μ_P on expected cost error of misspecification (restricted to $\sigma = 3$ / $\rho_{PP} = 0.8$ instances).

Table 4: Boxplot Data

	Min	Q1	Median	Q3	Max	Mean
Figure 6						
overall performance	0.00%	0.09%	0.65%	4.90%	37.86%	4.40%
$\rho_{PP} = 0.4$	0.00%	0.02%	0.12%	0.58%	4.26%	0.42%
$\rho_{PP} = 0.8$	0.00%	1.06%	4.92%	12.61%	37.86%	8.37%
$\sigma = 1$	0.00%	0.01%	0.10%	1.33%	11.97%	1.27%
$\sigma = 2$	0.00%	0.23%	0.63%	5.46%	25.32%	3.83%
$\sigma = 3$	0.01%	0.60%	1.90%	13.47%	37.86%	8.09%
Figure 8						
$\rho_{PP} = 0.8$ & $\sigma = 3$	0.49%	5.26%	13.57%	22.66%	37.86%	15.35%
$r = 0.25$	0.49%	2.04%	5.48%	16.38%	36.75%	10.55%
$r = 0.5$	0.57%	8.02%	16.21%	25.86%	37.86%	16.49%
$r = 1$	2.71%	11.64%	19.92%	26.43%	37.78%	19.01%
$h = 0.1$	0.78%	9.79%	17.72%	25.86%	36.51%	17.51%
$h = 0.2$	0.57%	5.82%	16.74%	26.48%	37.86%	16.65%
$h = 0.4$	0.49%	2.48%	8.52%	19.18%	36.75%	11.90%
$v = 4$	0.91%	9.35%	27.25%	33.84%	37.86%	22.52%
$v = 8$	0.69%	4.51%	15.11%	21.33%	28.57%	13.84%
$v = 16$	0.49%	3.33%	10.08%	14.60%	22.01%	9.69%
$\sigma_X = 1$	0.49%	2.71%	11.28%	22.01%	37.86%	13.55%
$\sigma_X = 2$	1.05%	6.32%	13.18%	22.43%	36.63%	15.40%
$\sigma_X = 3$	1.38%	9.97%	16.87%	25.14%	36.75%	17.10%
$\mu_P = 10$	2.40%	11.64%	16.87%	23.14%	36.75%	18.51%
$\mu_P = 12$	0.77%	8.31%	18.44%	26.48%	37.86%	17.31%
$\mu_P = 14$	0.49%	1.86%	4.51%	19.18%	37.78%	10.23%

Appendix C – Impact of Misspecification of Policy Structure in Base Case

We assume a simplified policy structure in which the order-up-to level for long-term supplier procurement is constant, i.e. $S_L(p) = S_L$. Table 5 provides the cost error for different S_L levels in the base case scenario (optimal cost $C = 95.79$) where all other policy parameters are taken from the optimal solution. Fixing the order-up-to level S_L at $S_S(c)$ only yields small error which further reduces when optimizing over S_L. The remaining error could (possibly) be further reduced when appropriately adapting other policy parameters, too.

Table 5: Impact of constant order-up-to level S_L on cost performance in base case scenario.

S_L	C	$\Delta Cost_{rel}$
$14 = S_S(c)$	96.62	0.9%
16	96.06	0.3%
17	95.96	0.2%
18	**95.93**	**0.14%**
19	95.95	0.16%

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