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Modeling Correlation in Vehicle Routing Problems with Makespan Objectives and Stochastic Travel Times

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Abstract

The majority of stochastic vehicle routing models consider travel times to be independent. However, in reality, travel times are often stochastic and correlated, such as in urban areas. We examine a vehicle routing problem with a makespan objective incorporating both stochastic and correlated travel times. We develop an approach based on extreme-value theory to estimate the expected makespan (and standard deviation) and embed this within a routing heuristic. We present results that demonstrate the impact of different correlation patterns and levels of correlation on route planning.

Keywords: Routing, Correlation, Stochastic, Makespan, Extreme-value theory

1 Introduction

Much of the existing research on stochastic routing problems assumes that the travel times on arcs in the network are independent of one another. Empirical studies, though, show that correlation is evident in many locations. Park and Rilett (1999) analyze data obtained from the Houston freeway system and found considerable correlation between some of the arcs of the freeway, with correlation coefficients as high as 0.75. Travel time correlation was also identified in truck routing data from Sydney, Australia (Figliozzi et al., 2007). He et al. (2002) simulated the Irvine, California corridor network using GPS-based vehicle records and found statistically-significant correlation in travel times, with both positive and negative correlation coefficients. Guo (2006) suggests arc travel times may be correlated due to weather effects and secondary incidents and their impact on the behavior of roadway users, and notes that assuming independent travel times can lead to biased solutions. Eliasson (2007) also found positive and negative correlations using an automatic travel time measurement system in Stockholm. Jenelius (2012) notes that poor weather conditions can affect travel times on several arcs. Nicholson (2015) suggests that negative correlation can occur when a bottleneck in one location results in increased speeds on the downstream locations. Fan et al. (2005) note that travel time correlation might also be an important aspect to consider in routing emergency vehicles through networks after a natural disaster.

In combining correlation with stochastic travel times, the sum of the expected travel times is not directly impacted by the correlation of the travel times (Rostami et al., 2017), but the expected makespan of the routes is. The makespan is the cost, typically time or distance, to complete the longest route. Bertazzi et al. (2015) note that this is a better objective in a number of situations, such as driver workload balance, computer networks, and disaster relief efforts. In terms of workload balance, the makespan represents the completion time of the longest driver route. If one driver has a much longer route than other drivers, this can promote job dissatisfaction and impact driver retention, which is a known business challenge (Ulmer et al., 2017). In disaster relief, the makespan reflects the time when all destinations affected by the disaster have been served.

The purpose of our research is to investigate a makespan routing problem with stochastic and correlated travel times. A makespan objective with stochastic and correlated travel times requires the use of a complex objective function evaluation or the use of extensive simulations. In this paper, we present a way to approximate the expected makespan and the standard deviation of makespan utilizing extreme-value theory. We evaluate the quality of these approximations and show them to be comparable with simulation with a drastically reduced computation time. We then embed these approximations within routing heuristics to solve for the routes that minimize the expected makespan. We also examine minimizing the makespan plus one standard devation to capture the potential variability. For example, when considering the workload balance example, a driver who has the longest expected route length will likely be dissatisfied if this route also has high variation in travel time. The addition of a measure of variability to the objective has been considered in the study of shortest path problems with correlation (e.g. Zockaie et al. (2014)) and in min-sum stochastic routing problems (e.g. Rostami et al. (2017)). We perform computational experiments for both objectives to help develop an understanding of how correlation changes the routes. The datasets we use are based on Solomon instances with the addition of different arc correlation patterns based on ideas from literature on correlation.

A brief review of related literature is presented in the next section. In Section 3, we discuss how correlation can impact the expected makespan. In Section 4, we describe a procedure using extreme-value theory to approximate the makespan and its standard deviation. We evaluate the accuracy of these approximations through simulation. These approximations are then incorporated into a vehicle-routing algorithm in Section 5, and computational experiments are presented in Section 6. Finally, we conclude the paper and present suggestions for future research in Section 7.

2 Literature Review

This review of related routing problems consists of three parts. First, we present a review of the research on min-max objectives for both node-routing and arc-routing problems. Then, stochastic routing problems are discussed, focusing on problems with random travel times. Finally, path finding and routing problems that incorporate travel time correlation are reviewed.

2.1 The Min-Max Routing Problem

Frederickson et al. (1976) were the first to consider min-max, multiple-route routing problems. They show that the multiple traveling salesmen problem (kTSP), the multiple Chinese postman problem (kCPP), and the multiple stacker-crane problem (kSCP) are NP-complete, and they develop heuristics using previously-developed methods from the respective minimum-cost, single-route heuristics. Arkin et al. (2006) also consider several node-routing and arc-routing problems and provide approximation algorithms for each.

Most of the recent focus on the multiple traveling salesmen problem has been on developing efficient solution methodologies using metaheuristics. This research includes the use of tabu search (França et al., 1995), neural networks (Modares et al., 1999), genetic algorithms (Carter and Ragsdale, 2006; Singh and Baghel, 2009), and ant colony optimization (Liu et al., 2009). An extension to this problem is the location-allocation kTSP, where the location of the depot is found in addition to the allocation of customers to routes (Averbakh and Berman, 1997; Nagamochi and Okada, 2004; Xu et al., 2013).

The vehicle routing problem is a generalization of the multiple traveling salesmen problem by including capacities on the vehicles (CVRP), time windows on deliveries to customers (VRPTW), etc. Golden et al. (1997) present an adaptive-memory heuristic for the kTSP and the CVRP with and without the multiple use of vehicles. Modares et al. (1999) suggest the use of neural networks for the CVRP, and Ren (2011) propose a hybrid genetic algorithm. A branch-and-cut search has been developed by Applegate et al. (2002) to optimally solve a 120-customer, 4-vehicle problem using distributed processing. Campbell et al. (2008) note that the min-max VRP is applicable in the routing of critical supplies for disaster relief; they also formulate a second objective in which the average time to each customer is minimized. Bertazzi et al. (2015) conduct a worst-case analysis comparing the min-max objective and the traditional min-sum objective and find that the length of the longest route when solving the min-sum VRP can be as much as k times the longest route in the min-max problem, where k is the number of vehicles, motivating the need to design efficient heuristics for the min-max problem.

Recent research has analyzed several variants of the min-max vehicle-routing problem. Carlsson et al. (2009), Narasimha et al. (2013), and Wang et al. (2015) consider the min-max VRP with multiple depots, in which a vehicle must start and end at the same depot. Xu et al. (2010) investigate the VRP in which a depot must be selected from a given set, and Valle et al. (2011) consider the VRP in which not all customers need to be visited by a vehicle as long as they are close enough to another customer that is on the route. Yakıcı and Karasakal (2013) extend the problem to account for split deliveries and heterogeneous demand.

Relatively little research has been identified on min-max, multiple-route arc-routing problems since the articles of Frederickson et al. (1976) and Arkin et al. (2006) mentioned above (see Benavent et al. (2014) for a review). Lacomme et al. (2004) develop memetic algorithms for the capacitated arc-routing problem. Ahr and Reinelt (2006) solve the kCPP using tabu search. Benavent et al. (2009) and Benavent et al. (2010) investigate the multiple windy rural postman problem, developing a branch-and-cut method as well as a multi-start/local-search heuristic; Akbari and Salman (2017) propose a heuristic for this problem based on an MIP-relaxation and a local-search algorithm. Willemse and Joubert (2012) discuss the application of the kCPP to the patrolling of an estate by security guards; they provide a tabu-search heuristic to solve the kCPP as well as the multiple rural postman problem when a subset of the network is patrolled.

2.2 Routing Problems with Stochastic Travel Times

Most of the stochastic routing research literature is concerned with stochastic customers (each customer has some probability of realizing demand) and/or demands (the demands are random variables); very little considers stochastic travel times for either node-routing problems (Cordeau et al., 2007) or arc-routing problems (Wøhlk, 2008). Shortest path problems have incorporated stochastic times – even with correlations between times – but are only concerned with finding the shortest time between two vertices of a network (e.g., Burton (1993)) or a set of non-dominant paths (Ji et al. (2011)). Various single-route routing problems have also incorporated stochastic travel times, such as the traveling salesman problem (beginning with Leipälä (1978) and Kao (1978)), and the Chinese postman problem (Tan et al., 2005). However, the only multiple-route problems found that incorporate stochastic travel times are in the context of the vehicle routing problem (VRP, recent reviews of stochastic vehicle routing are provided by Ritzinger et al. (2016) and Gendreau et al. (2016)).

Cook and Russell (1978) use simulation to evaluate the quality of deterministically-generated routes on the VRP with stochastic travel times. Laporte et al. (1992) were the first to incorporate stochastic travel times as part of a VRP model; they present two versions of the problem: (i) chance-constrained programming and (ii) stochastic programming with recourse. They use a branch-and-cut algorithm to solve problems with up to 20 vehicles and travel times that can take on a value from as many as five discrete states. Lambert et al. (1993) develop a model specific to a particular banking context in which travel time is based on a given probability the route is congested. Kenyon and Morton (2003) consider two models with different objective functions: (i) minimizing the expected time that all vehicles will return to the depot and (ii) maximizing the probability of completing the routes by a given deadline. They develop a branch-and-cut approach to solve the problem when the cardinality of the sample space is small and embed a sampling-based procedure for larger sample spaces or continuous random parameters.

The vehicle routing problem with time windows (VRPTW) and stochastic travel times has recently been investigated using genetic algorithms (Ando and Taniguchi, 2006) and tabu search (Russell and Urban (2008); Li et al. (2010); Taş et al. (2013); Zhang et al. (2013)); these models include a penalty in the objective function and/or a constraint limiting the probability of not meeting the time-window constraint. Taş et al. (2014) propose an exact solution approach based on column generation and a branch-and-price method. Lecluyse et al. (2009) also consider the VRPTW, but include the standard deviation of the travel time as part of the objective function. Based on extreme-value theory, Ehmke et al. (2015) approximate chance constraints for the VRPTW and adapt a tabu-search algorithm to ensure a certain level of arriving within customer time windows. Adulyasak and

Jaillet (2015) consider the VRP with deadlines in both the stochastic (known probability distribution) and robust (unknown, but belonging to a family of distributions) cases. Taş et al. (2014) consider time dependency in the modeling of stochastic travel times. Time dependency has also been considered in modeling the dial-a-ride problem with scheduled pickup-drop-off times by Fu (2002), Xiang et al. (2008), and Schilde et al. (2014). Related problems in which the customer service times are stochastic were considered by Sungur et al. (2010), Lei et al. (2012), and Errico et al. (2016). The only min-max routing problem identified in the literature that incorporates stochastic travel times is Kenyon and Morton (2003), and they note that for large problems "Monte Carlo sampling may be the only viable option." And while they note that the travel times may be dependent in their models, they do not explicitly evaluate the effect of correlated travel times.

2.3 Routing Problems with Correlation

We were able to identify several papers that consider stochastic travel times and correlation for path finding or vehicle routing.

Zockaie et al. (2013) consider a shortest path problem with on time arrival probability (SPOTAR) between an origin-destination pair in a network with random travel times. Their motivation is to check the effect of the correlation on reliable path finding. To quantify the impact of correlation, a two-stage algorithm based on Monte Carlo simulations is presented. The authors investigate only spatial correlation between neighboring adjacent arcs. One of the conclusions is that correlation has a complex impact on route choice.

In Zockaie et al. (2014), the minimum path travel time budget problem (MPTTB) on a Chicago-based network is considered. The travel time budget is addressed by a reliability index defined as a linear combination of mean travel time and standard deviation. Two solution methods are implemented and compared: an outer approximation method based on a cutting-plane algorithm and Monte Carlo simulations. Only spatial correlation between neighboring adjacent arcs is investigated.

Zockaie et al. (2016) extend their Monte Carlo simulation approach, developed in Zockaie et al. (2013) and Zockaie et al. (2014) for SPOTAR and MTTBP, for stochastic time varying networks with spatial and temporal travel time correlations. The developed method considers arc travel time correlations by drawing random numbers from a joint arc travel time distribution instead of treating arcs independently. For temporal correlation, consecutive time intervals are assumed to be correlated, and for spatial correlation, instead of considering a correlation only between two neighboring arcs, the authors consider it between three neighboring arcs. We allow for any arcs in the network to be correlated, but we do not explicitly consider different time periods.

Guo et al. (2017) describe and implement a method to generate a valid correlation matrix for spatially and temporally correlated travel times. Given a high dimensionality of a joint spatial-temporal correlation matrix, their approach provides an intuitive and logical method to create a valid correlation matrix. They use a heuristic test to identify whether there are enough simulation scenarios for their numerical results to be valid.

Jiang and Mahmassani (2014) consider a time-dependent vehicle routing problem with time windows in congested urban environments. They use simulation to retrieve information on the dynamics of traffic within the road network. Their solution approach is based on a two-phase set-partitioning-based heuristic. It is applied to two real-world networks, Fort Worth and Chicago, and time-dependent travel times were obtained under various incidents. Numerical results show that different events may significantly affect the quality of the solution. For instance, weather incidents such as heavy snow fall increased the total time of the route plan by more than 45%, and not considering the incidents leads to multiple violations of customer time windows. We use their observations about disruptions in one of our tested correlation patterns.

Rostami et al. (2017) investigate a capacitated vehicle routing problem with correlated stochastic travel times. The objective function is to maximize the travel time reliability, which is represented as a linear combination of travel time and its variance. They model the problem as binary quadratic program and propose two alternative set partitioning reformulations. They solve their problem for two different types of correlation: correlation only between adjacent arcs and correlation between all arcs. Computational results based on well-known Solomon datasets show that travel time variance could be reduced up to 70% for some instances. We mimic their use of Solomon datasets but use different correlation matrices and a makespan objective.

3 Analyzing the Effect of Correlation on Makespan

The model under consideration is an extension of the mixed general routing problem (MGRP) presented by Corberán et al. (2005). Let $G = (V, E, A)$ be a strongly-connected mixed graph with vertex set V, edge set E, and arc set A. Multiple vehicles are available to serve a subset $V_R \subseteq V$ of required vertices. We assume the time (cost) to traverse each arc of the network, $L \in E \cup A$, is a random variable; these times are not required to be independent nor identically distributed (non-IID). The objective is to minimize the expected value of the makespan (min-mean-max) or the expected value plus some multiple of the standard deviation. Our focus is on the travel times so, as long as the time to complete each route can be determined by the sum of the times to traverse a set of arcs and vertices of the network, our analysis should be applicable.

3.1 Effect of Arc Correlation on Route Correlation

It is well known how correlation affects the variance of a combination of variables. The benefits of risk pooling (e.g., for portfolio diversification, supply chain management, insurance) decrease as the correlation between the variables becomes more positive. However, with our min-mean-max objective, it is not straightforward what the impact of correlation will be, so we need to identify the correlation between routes as well as the expected value and variance of route times.

Let the travel time on any arc follow an arbitrary distribution with a mean of μ_i and a variance of σ_i^2. These travel times need not be independent; the correlation of the time to travel arc i and arc k is ρ_{ik}, hence a covariance of $\sigma_{ik} = \rho_{ik}\sigma_i\sigma_k$. A route, then, consists of traveling from the depot, over a set of arcs and back to the depot. The travel time for the entire route will tend toward – but not necessarily achieve – a normal distribution due to the Central Limit Theorem, with a mean of $E(Y) = \sum_{i \in R} \mu_i$ and a variance of $Var(Y) = \sum_{i \in R} \sum_{k \in R} \sigma_{ik}$, where $\sigma_{ii} = \sigma_i^2$ and $\rho_{ii} = 1$. Since the individual travel times between arcs are correlated, the route times will also be correlated.

The correlation of two linear combinations of variables can be determined as follows (see, e.g., Johnson and Wichern (2007)). Let X_1 and X_2 be two random vectors of size n and m, respectively (hence, we index i and k from 1 to n for X_1 as well as j and l from $n+1$ to $n+m$ for X_2), with variance/covariance matrices of V_{11} and V_{22}, respectively. The covariances between pairs of variables from different sets (cross-correlation) are $Cov(X_1, X_2) = V_{12} = V_{21}'$. Then, we define the linear combination of variables as $Y_1 = a'X_1$ and $Y_2 = b'X_2$. The correlation of these two linear combinations of variables can then be expressed as:

$$Corr(Y_1, Y_2) = \frac{Cov(Y_1, Y_2)}{\sqrt{Var(Y_1)\sqrt{Var(Y_2)}}} = \frac{a'V_{12}b}{\sqrt{a'V_{11}a}\sqrt{b'V_{22}b}}. \tag{1}$$

Our interest is in the sum of the arc travel times for a given route, which is a simple sum of variables. Hence, the coefficient vectors a and b will be vectors of ones. Thus:

$$a'V_{11}a = \sum_{i=1}^{n} \sum_{k=1}^{n} \sigma_{ik} \tag{2}$$

$$b'V_{22}b = \sum_{j=n+1}^{n+m} \sum_{l=n+1}^{n+m} \sigma_{jl} \tag{3}$$

$$a'V_{12}b = \sum_{i=1}^{n} \sum_{j=n+1}^{n+m} \sigma_{ij}. \tag{4}$$

So, from (1), we can calculate the time to complete two routes by calculating the correlation of two linear combinations of variables:

$$
\begin{aligned}
Corr(Y_1, Y_2) &= \frac{\sum_{i=1}^{n}\sum_{j=n+1}^{n+m}\sigma_{ij}}{\sqrt{\sum_{i=1}^{n}\sum_{k=1}^{n}\sigma_{ik}}\sqrt{\sum_{j=n+1}^{n+m}\sum_{l=n+1}^{n+m}\sigma_{jl}}} \\
&= \frac{\sum_{i=1}^{n}\sum_{j=n+1}^{n+m}\rho_{ij}\sigma_i\sigma_j}{\sqrt{\sum_{i=1}^{n}\sum_{k=1}^{n}\rho_{ik}\sigma_i\sigma_k}\sqrt{\sum_{j=n+1}^{n+m}\sum_{l=n+1}^{n+m}\rho_{jl}\sigma_j\sigma_l}}. \qquad (5)
\end{aligned}
$$

To illustrate the effect of arc travel-time correlation on route travel-time correlation, suppose we have two routes with the same number of arcs in each (i.e., n = m) and that the variances of the travel times for all arcs are equal ($\sigma_i = \sigma_j \ \forall i, j$). Further assume that the magnitude – but not necessarily the sign – of the correlation coefficients are the same. Equation (5) then reduces to:

$$
Corr(Y_1, Y_2) = \frac{n\rho_{ij}}{1 + (n-1)\rho_{ik}}. \qquad (6)
$$

Figure 1 illustrates the correlation of route times when the correlations of arc times are positive within a route (ρ_{ij}) and either positive ($\rho_{ik} = \rho_{ij}$) or negative ($\rho_{ik} = -\rho_{ij}$) between routes. If $n = 1$, the route correlation is the arc correlation. However, the correlation of route times approaches one as the number of arcs in the routes increases. And the effect is substantial even for the size of problems typically encountered in routing problems; for a arc correlation of $\rho_{ij} = 0.20$, the route correlation equals 0.60 for $n = 6$ arcs. So even weakly correlated arc travel times can result in strongly correlated route times that we must deal with in designing routes. And when the variances and correlations are not the same for all arcs, we can take advantage of these relationships in designing routes by appropriately assigning arcs to routes to influence the route correlations to our advantage.

3.2 Effect of Route Correlation on the Makespan

Once the arcs have been assigned to routes, the mean and variance of each route as well as their correlation can be calculated as discussed in the previous section. So we now turn our attention to determining the effect of correlated route times on the makespan. The exact distribution of the maximum of two correlated, non-IID, Gaussian random variables, $Z = max\{Y_1 \sim N(\mu_{Y_1}, \sigma^2_{Y_1}), Y_2 \sim N(\mu_{Y_2}, \sigma^2_{Y_2})\}$ is known (Nadarajah and Kotz, 2008). The mean μ_Z and the variance σ^2_Z are:

$$
\begin{aligned}
\mu_Z &= \mu_{Y_1}\Phi[\alpha] + \mu_{Y_2}\Phi[-\alpha] + \sigma_{\Delta Y}\phi[\alpha] \qquad (7) \\
\sigma^2_Z &= (\mu^2_{Y_1} + \sigma^2_{Y_1})\Phi[\alpha] + (\mu^2_{Y_2} + \sigma^2_{Y_2})\Phi[-\alpha] + (\mu_{Y_1} + \mu_{Y_2})\sigma_{\Delta Y}\phi[\alpha] - \mu^2_Z \qquad (8)
\end{aligned}
$$

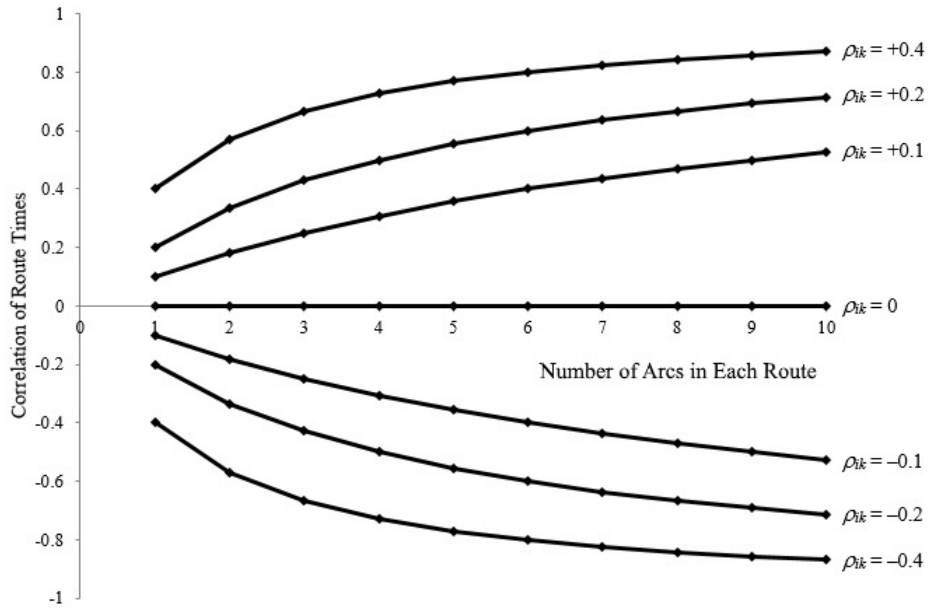


Figure 1: Correlation of Route Times for Identically-distributed Arc Travel Times

where:

$$\sigma_{\Delta Y} = \sqrt{\sigma_{Y_1}^2 + \sigma_{Y_2}^2 - 2\sigma_{Y_1}\sigma_{Y_2}Corr(Y_1, Y_2)} \tag{9}$$

$$\alpha = \frac{(\mu_{Y_1} - \mu_{Y_2})}{\sigma_{\Delta Y}}, \tag{10}$$

and where ϕ and Φ are the density and cumulative distribution functions for the standard normal distribution, respectively, and the correlation between the two routes, $Corr(Y_1, Y_2)$, is obtained from (5).

To isolate the effect of correlated route times on the makespan, suppose the two routes have equal means and variances, $\mu_{Y_1} = \mu_{Y_2} = \mu_Y$ and $\sigma_{Y_1}^2 = \sigma_{Y_2}^2 = \sigma_Y^2$. Then, from Equation 9:

$$\sigma_{\Delta Y} = \sqrt{2\sigma_Y^2[1 - Corr(Y_1, Y_2)]}. \tag{11}$$

For equal means, $\alpha = 0$, and for a normal distribution, $\Phi[0] = 0.5$ and $\phi[0] = \frac{1}{\sqrt{2\pi}}$, so the expected value of the maximum of the two routes will be:

$$\mu_Z = \mu_Y + \sigma_{\Delta Y}\sqrt{\frac{1 - Corr(Y_1, Y_2)}{\pi}}. \quad (12)$$

If the routes are perfectly positively correlated (i.e. $Corr(Y_1, Y_2) = 1$), the expected value of the maximum will simply be the mean of each route, since the two routes will always have the same time. However, as the correlation decreases, μ_Z increases rapidly as shown in Figure 2 ($r = 2$). In designing routes, we will likely not have route times with equal means and covariances, but we should take advantage of this effect of route correlation by assigning arcs to routes in such a manner as to maintain positive correlations among routes as much as possible.

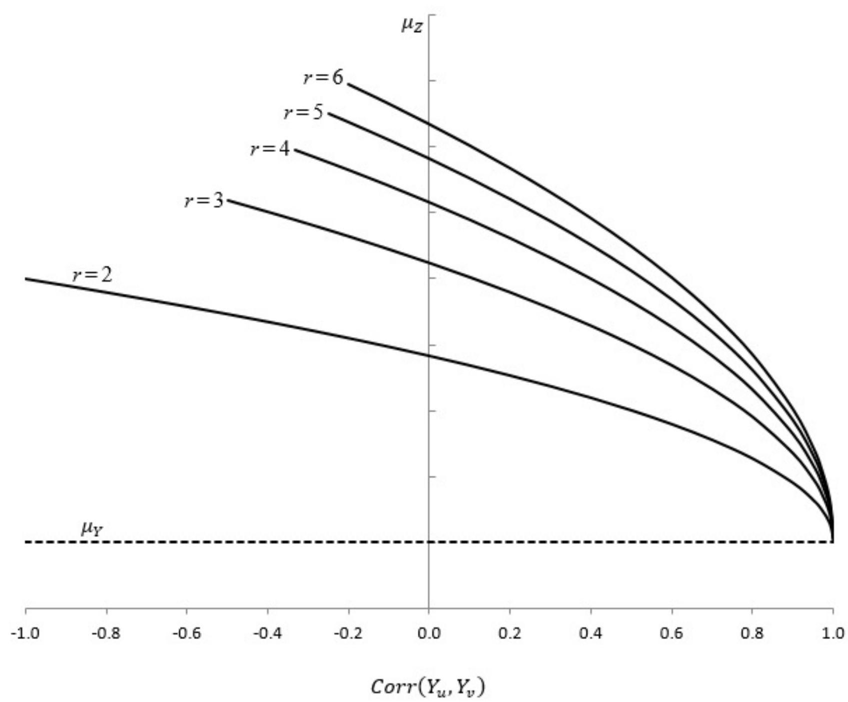


Figure 2: Effect of Route Correlation on the Expected Value of the Makespan

Figure 2 also provides the expected value of the makespan for three through six routes (based on tabulated values provided in Arnold et al. (2008)). However, for more than two routes, *all* of the routes cannot have a strong negative correlation. For example, if A is negatively correlated with B, and B is negatively correlated with C, we would expect A to be positively correlated with C. More formally, a correlation matrix must be positive semidefinite due to the fact that the variance must be positive (Rapisarda et al. (2007)). As indicated by David and Nagaraja (2003), for identically-distributed equicorrelated multinormal variables, the common correlation coefficient must satisfy the following relationship:

$$\frac{-1}{(r-1)} \leq Corr(Y_u, Y_v) \leq 1, \tag{13}$$

where r is the number of routes. Thus, the curves do not extend beyond this range.

As illustrated in Figure 2, the effect of route correlation is even more pronounced as the number of routes increases. The judicious assignment of arcs to routes is critical when the arcs, and therefore the routes, are correlated. Of course, a routing application will not have equal means and covariances, but the assignment of arcs to routes must take into consideration not only the expected times of each route, but the variances and correlations as well. So we now turn our attention to the development of a general heuristic that incorporates the effects of arc correlation on route correlation and of route correlation on the makespan. In the next section, we describe a means of estimating the expected makespan for a given assignment. Then, we describe a heuristic for the min-mean-max vehicle routing problem.

4 Calculating the Expected Makespan and Standard Deviation

Our intent is to develop a general heuristic to determine the expected value, or the expected value plus one standard deviation, of the maximum travel time of several routes with correlated arc travel times. It should be able to be used in a variety of arc- and node-routing algorithms by simply replacing the objective-function computation with the proposed heuristic, and it should easily be incorporated as part of general-purpose software if so desired.

In reviewing the literature on the distributional properties of trips, Noland and Polak (2002) found evidence of normally and lognormally distributed travel times. Taylor (1999) notes that the normal distribution is appropriate for heavily-congested arcs, but the lognormal distribution better describes less congested arcs. Chiang and Roberts (1980) found that a shifted gamma distribution provides reasonable estimates of the travel time for regular-route, less-than-truckload trucking. For normally-distributed arc travel times, the route times will also be normally distributed; for non-normal arc travel times, the route time may tend toward a normal distribution, but the number of arcs in a route may not be sufficient for the Central Limit Theorem to assure convergence to the normal.

However, even if all of the route times are approximately normal, the distribution of the maximum time will not be normal; in fact, it will be skewed. The asymptotic limit distribution of the maximum of IID normal variables is the Gumbel distribution. However, a typical routing problem will not likely have enough routes to converge to the Gumbel distribution, and the route times will neither be identical nor, due to the correlated arc times as illustrated in the previous section, independent.

For two routes with normally-distributed route times, the expected value of the maximum route time can be taken directly from equation (7) above. For more than two routes and/or for non-normal route times, some authors (beginning with Clark (1961)) have suggested a recursive algorithm using extreme-value theory to estimate the expected maximum value. For the routing problem under consideration, this process is incorporated into our proposed approximation algorithm as follows.

4.1 An Approximation Algorithm

Let Π_p be a set of arcs in route p (a given assignment of arcs to routes). Suppose the travel time on each arc follows an arbitrary distribution with a mean of μ_i and a variance of σ_i^2 as well as a correlation of the time to travel arc i and arc k of ρ_{ik} $(i, k \in \Pi_p)$; hence the covariance is $\sigma_{ik} = \rho_{ik}\sigma_i\sigma_k$, where $\sigma_{ii} = \sigma_i^2$ and $\rho_{ii} = 1$.

Step 1: Initial mean and variance of routes. Calculate the mean and variance of the time to complete each route (where R_p is the time to complete route p, and using indices i and k for route p with n_p arcs assigned to it). Then, for each route p:

$$E(R_p) = \sum_{i=1}^{n_p} \mu_i \tag{14}$$

$$Var(R_p) = \sum_{i=1}^{n_p}\sum_{k=1}^{n_p} \sigma_{ik} = \sum_{i=1}^{n_p}\sum_{k=1}^{n_p} \rho_{ik}\sigma_i\sigma_k \tag{15}$$

Step 2: Initial correlation between the routes. Calculate the correlation coefficient of the time to complete each pair of routes (using indices i and k for route p with n_p arcs assigned to it, and using indices j and l for route q with n_q arcs). Then, for each pair of routes p and q:

$$Corr(R_p, R_q) = \frac{\sum_{i=1}^{n_p}\sum_{j=n_p+1}^{n_p+n_q} \rho_{ij}\sigma_i\sigma_j}{\sqrt{\sum_{i=1}^{n_p}\sum_{k=1}^{n_p} \rho_{ik}\sigma_i\sigma_k}\sqrt{\sum_{j=n_p+1}^{n_p+n_q}\sum_{l=n_p+1}^{n_p+n_q} \rho_{jl}\sigma_j\sigma_l}} \tag{16}$$

At this point, we have the distribution of the time to complete each route. To simplify subsequent expressions, let $\mu_{R_p} = E(R_p)$ be the expected (mean) time to complete route p, and $\sigma^2_{R_p} = Var(R_p)$ be the variance of the time to complete route p.

Step 3: Mean and variance for a first combination of routes. Calculate the mean and variance of the maximum time to complete the first two routes. Let $A = max\{R_1, R_2\}$, and ν_{A_1} and ν_{A_2} be the first and the second moment of A, respectively:

$$\begin{aligned} \nu_{A_1} &= \mu_{R_1}\Phi[\alpha] + \mu_{R_2}\Phi[-\alpha] + \sigma_{\Delta R}\phi[\alpha] && (17) \\ \nu_{A_2} &= (\mu^2_{R_1} + \sigma^2_{R_1})\Phi[\alpha] + (\mu^2_{R_2} + \sigma^2_{R_2})\Phi[-\alpha] + (\mu_{R_1} + \mu_{R_2})\sigma_{\Delta R}\phi[\alpha] && (18) \end{aligned}$$

where

$$\begin{aligned} \sigma_{\Delta R} &= \sqrt{\sigma^2_{R_1} + \sigma^2_{R_2} - 2\sigma_{R_1}\sigma_{R_2}Corr(R_1, R_2)} && (19) \\ \alpha &= \frac{(\mu_{R_1} - \mu_{R_1})}{\sigma_{\Delta R}} && (20) \end{aligned}$$

and ϕ and Φ are the density and cumulative distribution functions for the standard normal distribution, respectively.

Then, the mean and variance of the maximum of the two route times will be $\mu_A = \nu_{A_1}$ and $\sigma^2_A = \nu_{A_2} - \nu^2_{A_1}$.

Step 4: Recompute correlation coefficients. Calculate the correlation coefficient of A and each of the remaining routes (for $p = 3, 4, ...$):

$$Corr(A, R_p) = \frac{\sigma_{R_1}Corr(R_1, R_p)\Phi[\alpha] + \sigma_{R_2}Corr(R_2, R_p)\Phi[-\alpha]}{\sqrt{\nu_{A_2} - \nu^2_{A_1}}} \qquad (21)$$

Step 5: Further recombination of routes. Treat A as a normally-distributed route (even though it is not) and calculate the mean and variance of the maximum of A and a third route; that is, $B = max\{R_1, R_2, R_3\} = max\{A, R_3\}$:

ν_{B_1} and ν_{B_2} can be calculated as in Step 3, replacing μ_{R_1} and $\sigma^2_{R_1}$ with μ_A and σ^2_A, respectively, and replacing μ_{R_2} and $\sigma^2_{R_2}$ with μ_{R_3} and $\sigma^2_{R_3}$, respectively.

Step 6: Further recalculation of correlation coefficients. Calculate the correlation coefficient of B and each of remaining routes (for $p = 4, 5, ...$):

$$Corr(B, R_p) = \frac{\sigma_A Corr(R_A, R_p)\Phi[\alpha] + \sigma_{R_3} Corr(R_3, R_p)\Phi[-\alpha]}{\sqrt{\nu_{B_2} - \nu_{B_1}^2}} \tag{22}$$

Note that this will require the use of the correlations in Step 4.

Step 7: Recursion. Repeat Steps 5 and 6 until all routes have been included ($C = max\{B, R_4\}$, $D = max\{C, R_5\}$, etc.). The final values of the mean and variance of the makespan can then be determined from equations (17) and (18).

4.2 Evaluating the Accuracy of the Approximation

For two routes with normally-distributed arc travel times, equations (17) and (18) will provide an exact solution. To evaluate the accuracy of the approximating algorithm for more than two routes and/or for non-normal route times, we compare the results of the algorithm with simulated results. We consider various numbers of routes, routes with varying numbers of arcs as well as various travel-time distributions.

The distributions, including the mean and coefficient of variation, of the arc travel times are randomly generated in the manner of Ehmke et al. (2015). The distance between each pair of customers is the Euclidean distance between randomly-generated coordinates in two-dimensional space with a uniform distribution between 0 and 40. The mean travel time is set equal to this distance, while the coefficient of variation is randomly generated with a uniform distribution between 0.1 and 0.3. Three travel-time distributions are evaluated – a normal distribution, a shifted gamma distribution, and a shifted exponential distribution – with parameters set such that the appropriate mean and coefficient of variation are obtained with a skewness equal to 0 (normal), 1 (shifted gamma), or 2 (shifted exponential). Five instances for each test scenario are generated.

In addition to the situation with uncorrelated travel times (an identity correlation matrix), three types of correlation matrices were constructed for evaluation. For the first scenario, the travel times for all arcs are assumed to be positively equicorrelated; four values representing increasing interrelationships are used ($\rho_{ik} = +0.2, +0.4, +0.6$, and $+0.8$, for $i \neq k$). The other two scenarios incorporate negative correlations; however, due to the need for a positive semidefinite correlation matrix, some of the correlation coefficients are necessarily positive. The second scenario is such that the travel times are positively and negatively correlated within a route and with arcs in all other routes. The third scenario is such that the travel times are positively correlated within a route and positively correlated with all arcs in half of the other routes, negatively correlated with arcs in the other half of the routes. Tables 1–3 of the electronic appendix illustrate the three types of matrices for a 3 route/4 arcs per route instance with a correlation coefficient of 0.2.

To measure its accuracy, the expected makespan of the approximation algorithm is compared to the results of a simulation using $@Risk$ with $10,000$ simulation runs. Tables 1–3 provide the mean absolute percent error (MAPE) of the five instances for each test scenario. Results are provided for 3, 5, and 10 routes; for 2, 4, and 8 arcs per route; as well as for normal, shifted gamma, and shifted exponential travel-time distributions.

Table 1: Mean Absolute Percent Deviation of the Algorithm vs. Simulated Expected Makespan – Normal Distribution

	ρ_{ik}	5 routes/ 2 arcs ea.	3 routes/ 4 arcs ea.	5 routes/ 4 arcs ea.	10 routes/ 4 arcs ea.	5 routes/ 8 arcs ea.
Correlation Matrix 1	+ 0.8	0.0203	0.0141	0.0170	0.0196	0.0265
	+ 0.6	0.0165	0.0191	0.0208	0.0210	0.0193
	+ 0.4	0.0101	0.0147	0.0354	0.0173	0.0264
	+ 0.2	0.0475	0.0209	0.0176	0.0405	0.0232
	0.0	0.0188	0.0410	0.0339	0.0515	0.0528
Correlation Matrix 2	- 0.2	0.0398	0.0419	0.0348	0.0374	0.0233
	- 0.4	0.0405	0.0278	0.0421	0.0241	0.0251
	- 0.6	0.0534	0.0117	0.0213	0.0256	0.0183
	- 0.8	0.0578	0.0096	0.0144	0.0290	0.0244
Correlation Matrix 3	- 0.2	0.0830	0.1175	0.1039	0.1871	0.2557
	- 0.4	0.1518	0.2281	0.2799	0.5126	0.7044
	- 0.6	0.3168	0.3668	0.5043	0.9237	1.1397
	- 0.8	0.5220	0.4859	0.7769	1.3307	1.5874

As illustrated in Tables 1–3, the algorithm is remarkably accurate in estimating the expected makespan. For normally-distributed travel times (Table 1), all of the MAPEs for Correlation Matrices 1 and 2 are less than one-tenth of one percent, with no apparent effect of the number of routes or the number of arcs per route. Not surprisingly, the accuracy is slightly less for the gamma and exponential distributions (Tables 2 and 3), although there is still less than one-half of one percent error for Correlation Matrices 1 and 2. For these distributions, increasing the number of arcs per route (e.g., going from 2 to 4 to 8 arcs per route with 5 routes) improves the accuracy of the algorithm, likely due to the fact that the route times approach normality as the number of arcs grows larger.

The results from Correlation Matrix 3 are somewhat different. Although the algorithm is still reasonably accurate – MAPEs within 1.6, 1.7, and 2.6 percent for the normal, shifted gamma, and shifted exponential travel-time distributions, respectively – the errors are consistently higher. While it is not apparent as to why the algorithm is relatively less

Table 2: Mean Absolute Percent Deviation of the Algorithm vs. Simulated Expected Makespan – Shifted Gamma Distribution

	ρ_{ik}	5 routes/ 2 arcs ea.	3 routes/ 4 arcs ea.	5 routes/ 4 arcs ea.	10 routes/ 4 arcs ea.	5 routes/ 8 arcs ea.
Correlation Matrix 1	+ 0.8	0.1610	0.1766	0.1548	0.1455	0.1064
	+ 0.6	0.1957	0.1841	0.1361	0.0641	0.1287
	+ 0.4	0.2050	0.1544	0.1110	0.0481	0.1158
	+ 0.2	0.2382	0.1204	0.0588	0.1264	0.0960
	0.0	0.2560	0.0833	0.0562	0.2448	0.0999
Correlation Matrix 2	- 0.2	0.2314	0.0797	0.1135	0.1873	0.0682
	- 0.4	0.2147	0.0667	0.1193	0.1656	0.0775
	- 0.6	0.1912	0.0456	0.0750	0.1207	0.0731
	- 0.8	0.1834	0.0418	0.0633	0.1021	0.0290
Correlation Matrix 3	- 0.2	0.1560	0.1540	0.1065	0.1149	0.2239
	- 0.4	0.2016	0.2517	0.2724	0.4790	0.6340
	- 0.6	0.3420	0.3655	0.6102	1.0313	1.1576
	- 0.8	0.7078	0.5305	1.0445	1.5948	1.6693

Table 3: Mean Absolute Percent Deviation of the Algorithm vs. Simulated Expected Makespan – Shifted Exponential Distribution

	ρ_{ik}	5 routes/ 2 arcs ea.	3 routes/ 4 arcs ea.	5 routes/ 4 arcs ea.	10 routes/ 4 arcs ea.	5 routes/ 8 arcs ea.
Correlation Matrix 1	+ 0.8	0.3335	0.3009	0.1845	0.2567	0.1444
	+ 0.6	0.3524	0.3007	0.1988	0.1753	0.1553
	+ 0.4	0.3612	0.2722	0.1856	0.1431	0.1396
	+ 0.2	0.3494	0.2231	0.1677	0.1963	0.1384
	0.0	0.4004	0.1713	0.1994	0.3968	0.1310
Correlation Matrix 2	- 0.2	0.4331	0.1573	0.2478	0.3627	0.1604
	- 0.4	0.4402	0.1278	0.2849	0.3035	0.1407
	- 0.6	0.3925	0.1079	0.2587	0.2485	0.0918
	- 0.8	0.3274	0.0913	0.2081	0.2612	0.0699
Correlation Matrix 3	- 0.2	0.6288	0.6507	0.6340	0.9166	0.8230
	- 0.4	0.4578	0.5062	0.7393	0.7834	0.9837
	- 0.6	0.9347	0.8008	1.3129	1.5489	1.7297
	- 0.8	1.5093	1.1784	1.9794	2.4415	2.5177

effective in this situation, it is interesting to note that Correlation Matrices 1 and 2 provide expected makespans that are smaller than the uncorrelated case ($\rho_{ik} = 0$), while Correlation Matrix 3 results in larger makespans (as shown in Table 4). Correlation Matrix 1 has fairly high, positively-correlated route times, which decreases the expected makespan (see Figure 2). Correlation Matrix 2 has positively- and negatively-correlated arcs within a route, resulting in a lower variance, and also results in route times that have low correlations. Correlation Matrix 3, on the other hand, results in relatively highly-correlated route times, some of which are negative; therefore, increasing the expected makespan. Again, this highlights the need for an appropriate routing algorithm that can take advantage of correlated travel times.

Table 4: Mean Percent Deviation of the Expected Makespan from the Uncorrelated Makespan

	ρ_{ik}	5 routes/ 2 arcs ea.	3 routes/ 4 arcs ea.	5 routes/ 4 arcs ea.	10 routes/ 4 arcs ea.	5 routes/ 8 arcs ea.
Correlation Matrix 1	+ 0.8	- 4.07	- 1.46	- 2.40	- 2.57	- 1.44
	+ 0.6	- 2.90	- 1.11	- 1.77	- 1.85	- 1.05
	+ 0.4	- 1.85	- 0.74	- 1.16	- 1.20	- 0.68
	+ 0.2	- 0.90	- 0.37	- 0.57	- 0.58	- 0.33
	0.0	0.00	0.00	0.00	0.00	0.00
Correlation Matrix 2	- 0.2	- 0.77	- 0.37	- 0.48	- 0.50	- 0.32
	- 0.4	- 1.59	- 0.73	- 0.96	- 1.02	- 0.66
	- 0.6	- 2.45	- 1.07	- 1.43	- 1.57	- 1.00
	- 0.8	- 3.38	- 1.35	- 1.85	- 2.17	- 1.35
Correlation Matrix 3	- 0.2	+ 0.85	+ 1.02	+ 1.82	+ 2.08	+ 2.26
	- 0.4	+ 1.61	+ 2.06	+ 3.42	+ 3.80	+ 4.22
	- 0.6	+ 2.30	+ 3.10	+ 4.88	+ 5.33	+ 6.01
	- 0.8	+ 2.94	+ 4.12	+ 6.25	+ 6.75	+ 7.65

While the expected makespan determined from the approximation algorithm is quite accurate, the same cannot be said of the makespan variance. Thus, to consider an objective that combines the mean as well as the variability, we evaluate the accuracy of the expected value plus one standard deviation of the makespan. As seen in Table 5, the approximation algorithm provides solutions for normally-distributed travel times that are within one percent of the simulated values for all instances. In fact, some instances are better than the deviation of the expected value alone, as an underestimated standard deviation somewhat offsets an overestimated mean. Tables 6 and 7 provide the results for the shifted gamma and shifted exponential distributions. In these situations, the estimate of the makespan variance is considerably worse than that realized with the normal distribution. Thus, the mean absolute percent deviation of the expected value plus one standard deviation of the makespan is as high as 2.5% for the shifted gamma and as high as 4.5% for the shifted exponential.

Table 5: Mean Absolute Percent Deviation of the Approximation Algorithm vs. Simulated Expected Makespan plus Standard Deviation – Normal Distribution

	ρ_{ik}	5 routes/ 2 arcs ea.	3 routes/ 4 arcs ea.	5 routes/ 4 arcs ea.	10 routes/ 4 arcs ea.	5 routes/ 8 arcs ea.
Correlation Matrix 1	0.8	0.0241	0.0224	0.0355	0.0307	0.0670
	0.6	0.0810	0.0419	0.0343	0.0901	0.0526
	0.4	0.1239	0.0482	0.0509	0.1329	0.0621
	0.2	0.2044	0.0552	0.0663	0.1894	0.0757
	0.0	0.2470	0.0972	0.1546	0.2472	0.0911
Correlation Matrix 2	-0.2	0.1956	0.0850	0.1409	0.2387	0.0686
	-0.4	0.1820	0.0672	0.0672	0.1842	0.0929
	-0.6	0.2093	0.0337	0.0629	0.1095	0.0798
	-0.8	0.1929	0.0117	0.0435	0.0477	0.0408
Correlation Matrix 3	-0.2	0.2869	0.1847	0.3799	0.4551	0.3220
	-0.4	0.4031	0.2613	0.4129	0.4780	0.3899
	-0.6	0.4827	0.3575	0.4257	0.4804	0.5633
	-0.8	0.6166	0.5201	0.3731	0.5105	0.8310

5 A Vehicle Routing Heuristic

In Section 4, we presented an approach that approximates the expected makespan and the standard deviation based on extreme-value theory. Next, we want to embed this approximation within a vehicle routing algorithm to find the assignment of customers to routes that optimize the makespan (with and without standard deviation). In this section, we demonstrate how to integrate this approximation in the creation of routes for a fleet of vehicles. To this end, we use a well-known construction heuristic and improve the resulting set of routes by applying two adapted variants of standard local search neighborhoods.

Step 1: Create Input Data. We create the the correlation matrix and the covariance matrix for a given set of depot, customers and their locations to prepare the data input required for the approximation approach presented in Section 4.

Step 2: Initialize Routes. For the creation of routes, we assume that the number of routes is given, and we aim at creating routes for our makespan objective (with or without

Table 6: Mean Absolute Percent Deviation of the Approximation Algorithm vs. Simulated Expected Makespan plus Standard Deviation – Gamma Distribution

	ρ_{ik}	5 routes/ 2 arcs ea.	3 routes/ 4 arcs ea.	5 routes/ 4 arcs ea.	10 routes/ 4 arcs ea.	5 routes/ 8 arcs ea.
Correlation Matrix 1	0.8	0.9252	0.2195	0.3312	0.5669	0.3434
	0.6	1.3587	0.3429	0.5441	0.8493	0.4537
	0.4	1.6199	0.4600	0.7479	1.0353	0.5277
	0.2	1.8680	0.5424	0.8609	1.2410	0.5206
	0.0	2.0779	0.6489	1.0257	1.3847	0.5975
Correlation Matrix 2	-0.2	2.0088	0.6616	1.0119	1.2273	0.6116
	-0.4	1.8753	0.7114	1.0696	1.3000	0.7757
	-0.6	1.8653	0.9726	1.2008	1.4506	1.1342
	-0.8	2.0964	1.5811	1.7455	1.8173	1.7384
Correlation Matrix 3	-0.2	2.3525	0.9742	1.5080	1.7772	0.9149
	-0.4	2.4059	1.2194	1.9053	2.0491	1.1265
	-0.6	2.5010	1.4827	2.2599	2.2236	1.3023
	-0.8	2.4273	1.6897	2.4830	2.3279	1.4936

Table 7: Mean Absolute Percent Deviation of the Approximation Algorithm vs. Simulated Expected Makespan plus Standard Deviation – Exponential Distribution

	ρ_{ik}	5 routes/ 2 arcs ea.	3 routes/ 4 arcs ea.	5 routes/ 4 arcs ea.	10 routes/ 4 arcs ea.	5 routes/ 8 arcs ea.
Correlation Matrix 1	0.8	1.5588	0.3085	0.5311	0.9059	0.5998
	0.6	2.2931	0.4176	0.8448	1.3469	0.7432
	0.4	2.7512	0.5392	1.1494	1.7126	0.8267
	0.2	3.1825	0.8281	1.4687	1.9562	0.8605
	0.0	3.6130	1.1555	1.8766	2.3290	1.0586
Correlation Matrix 2	-0.2	3.5489	1.2247	1.9359	2.3337	1.2421
	-0.4	3.6662	1.6979	2.2456	2.6489	1.8506
	-0.6	3.8829	2.5114	2.9628	3.2389	2.7830
	-0.8	4.4062	3.8882	4.0853	4.0865	4.0980
Correlation Matrix 3	-0.2	3.9053	1.4024	2.6043	2.6593	1.5607
	-0.4	3.8939	1.7617	2.8401	3.0395	1.7323
	-0.6	3.9030	2.1995	3.2908	3.2957	2.1317
	-0.8	3.8615	2.6201	3.6430	3.5390	2.5300

standard deviation) for the given number of routes. We initialize the routes by assigning the farthest, second-farthest, etc. customers to the routes until every route has one initial customer.

Step 3: Insert Customers. We insert the remaining customers following the idea of a greedy randomized adaptive search (GRASP). In each insertion step, we evaluate the consequences of insertion for our complex makespan objective for all remaining customers, routes and insertion positions in these routes. Out of the three "best" customers and insertion points, we randomly choose one for insertion. We continue until all customers have been assigned. We evaluate the current solution with the approximation approach of Section 4 whenever we check if an insertion to a particular insertion point would be beneficial or not. In the end, all customers have been inserted, and a first feasible solution has been created. Due to the random nature of our GRASP approach, we repeat the construction of the initial route plan five times and improve the best of the five created route plans as follows.

Step 4: Improve Routes (1-shift). The created initial solution is subject to improvement with standard local search neighborhood operators. First, we apply the idea of 1-shift, which investigates whether it is beneficial to shift a customer from its current position to all other possible insertion points within the same or the other routes. We identify the most beneficial shift with regard to our objective, keep the improved solution, and continue with 1-shift operations as long as we can find improvements.

Step 5: Improve Routes (Exchange). We use the well-known exchange operator to systematically switch two customers between different routes. We compute the consequences that each switch would have on our makespan objective (with or without standard deviation) and perform the first exchange that leads to an improvement. If we found an improvement, we start the exchange operator again. We continue the exchange operator until no improvement of the objective function is possible any more. Then, we activate 1-shift again, and both operators run alternately until we cannot find improved solutions any more.

Complete evaluation of the current solution is necessary whenever we analyze the consequences of an insertion or a local search move. This leads to a significant increase in run times compared to deterministic routing objectives, where it is often sufficient to perform a delta comparison to evaluate the suitability of a local search move.

6 Experimental Design and Results

In this section, we introduce the instances and present the computational results of our stochastic vehicle routing algorithm.

6.1 Instances

We analyze the impact of correlation in optimizing routes based on well-known benchmark instances with correlation patterns motivated by literature on correlation. In particular, we use the dataset R101 of the Solomon datasets. We use the first customer as depot location. We form four sets of instances based on the next 20 customers, i.e., depot location and customers 2, 3, ..., 21 form the instance R101-1 (short: R1), 22, 23, ..., 41 form R101-2 (short: R2), etc. We focused on fairly small instances so that we could understand and visualize the impact of the correlation on vehicle routing. Solutions for every instance are generated according to the following correlation scenarios:

- Scenario type *incidents (INC)*: Guo (2006) suggests arc travel times may be correlated due to weather effects and secondary incidents. Zockaie et al. (2016) consider different incidents to produce travel time variations in each scenario. In addition, our motivation comes from the real life observation that traffic incidents may often create local traffic jams that are not observed on a regular day-to-day basis. Hence, for the incident scenario, we define three locations of "incidents" randomly in the Euclidean plane. We assume that there is positive correlation between all arcs within 5 unit blocks of the incident locations. If arcs start or finish in the same block, they are positively correlated. This is illustrated in Figure 3.

- Scenario type *inner/outer (I/O)*: Nicholson (2015) suggests that negative correlation can occur when a bottleneck in one location results in increased speeds on the downstream locations. This motivates us to consider an inner/outer scenario: if during a given time period a large amount of traffic is going into the same direction, the reverse direction is more likely to be less loaded. Hence, for our scenario, we designate a block in the middle of the investigated area (coordinates 20-50, 20-50) to be the "downtown". All arcs into the downtown block are positively correlated and all arcs out of the downtown block are positively correlated. The two groups are negatively correlated (i.e. if they are going into different directions). This is illustrated in Figure 4.

We conduct experiments for the above instances and scenarios for four different correlation coefficients, namely zero correlation, 0.5, 0.75 and 0.9 (for negative correlation

Figure 3: Setup for INC Scenario

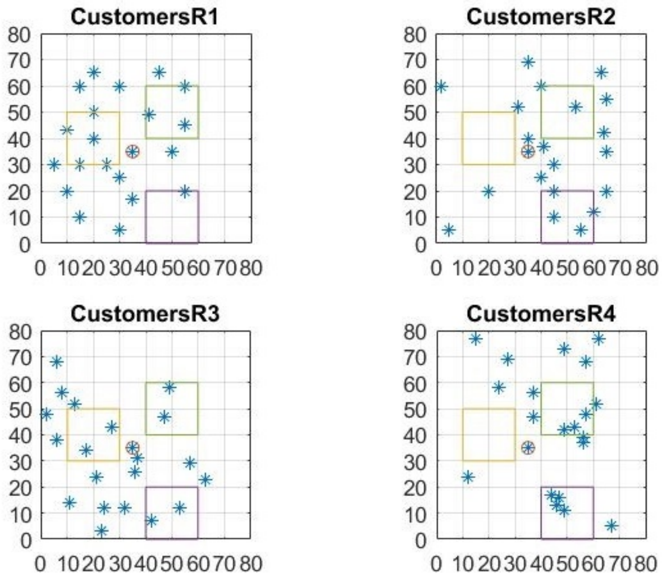
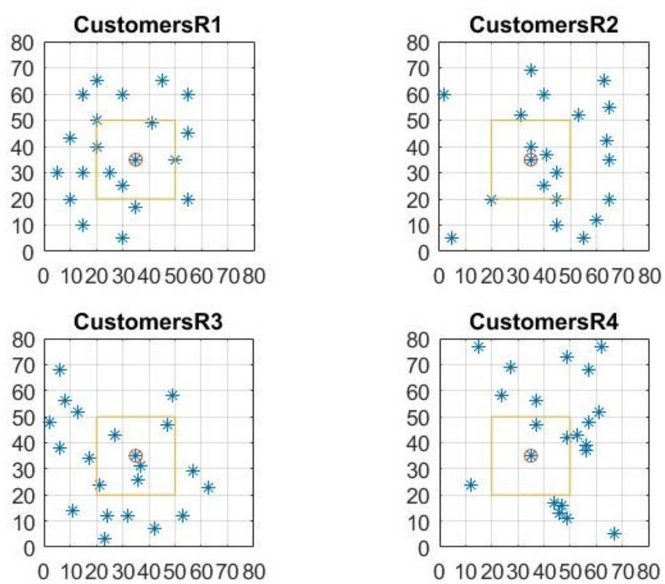


Figure 4: Setup for I/O Scenario



the negative of this). We also consider different coefficient of variation levels of travel times: 25% of mean, 50% of mean, 75% of mean. We assume a normal distribution on travel times. The number of vehicles is fixed and is either 3 or 5. Finally, we investigate two different parameterizations of our objective function (makespan with/without standard deviation). For each result, we report the objective function value and the individual components, i.e. the makespan and standard deviation, and compare it to the zero-correlation solutions.

We implemented our algorithms in Java 8 on a laptop with Win10 OS, Intel i7-7700HQ processor and 16 GB of RAM. We observed that the runtime heavily depends on the number of vehicles. The computation of one solution for a three-vehicle instance takes roughly 30 minutes and is about one hour for a five-vehicle instance.

6.2 Results

We first present summary tables and then discuss illustrative examples of routes created with our stochastic routing algorithm.

6.2.1 Summary Tables

Summary results are given in Tables 8–9. Detailed results are in the electronic appendix in Tables 4–9. For Table 8, we will first define the columns: **#veh** refers to the number of vehicles used, **objective** refers to the objective of the routing problem solved, **base** refers to the no correlation case, **I/O 50, I/O 75, I/O 90** refer to the corresponding inner-outer correlation matrix with 0.5/-0.5, 0.75/-0.75, 0.9/-0.9 correlation values, respectively, and **INC 50, INC 75, INC 90** refer to the corresponding incidence correlation matrix with 0.5/-0.5, 0.75/-0.75, 0.9/-0.9 correlation values, respectively. In terms of the rows, **stdv=0.* mean** is where $*$ is the ratio of the standard deviation of travel time to the mean for each arc (i.e. coefficient of variation), **best sol makespan** is the expected makespan of the best solution obtained for the given correlation matrix, **zero corr sol makespan** is the expected makespan of the zero correlation solution evaluated using the given correlation matrix, and **% different to best solution** is the percentage difference between the best solution for expected makespan and the zero correlation solution evaluated with the given correlation matrix (i.e. the savings in expected makespan from including correlation in our routing algorithm).

From Table 8 that only considers expected makespan, a few patterns are apparent. Increasing the correlation across each correlation scenario leads to increased importance in modeling the correlation in the routing. For example, for 5 vehicles and stdv=0.25 mean, we see savings of 0.34%, 0.65%, and 0.85% for INC 50, INC 75, and INC 90. This is not surprising since more correlation should yield larger impact on solutions. We also see that a larger coefficient of variation yields larger impacts from modeling correlation. For example, for I/O 50 and 3 vehicles, we see the savings go from 0.11% to 0.30% to 0.73% as the standard deviation increases. We will look at an example with a coefficient of variation of 0.75 in the next section to understand the changes in the routes creating the savings. In general, there is no strong relationship between the number of vehicles and the savings from modeling correlation. We see that with 20 customers most experiments yield an average savings of around 1% or less except with the highest coefficient of variation and the INC correlation scenarios. Here we see values ranging as high as 4.20% average improvement from modeling correlation.

Next, we will consider Table 9 that optimizes for expected makespan plus standard deviation. The columns are the same as for Table 8, but there are additional rows. These include **best sol stdv** which is the standard deviation of the best solution obtained the given correlation matrix, **best sol ms+stdv** the sum of the makespan and standard deviation of the best solution obtained for the given correlation matrix, **zero corr sol stdv** the standard deviation of the zero correlation solution evaluated using the given correlation matrix, **zero corr sol ms+stdv** makespan plus standard deviation of the zero correlation solution evaluated using the given correlation matrix, and **% difference to best solution** now represents percentage difference between the zero correlation solution evaluated with the given correlation matrix and the best solution for expected makespan and standard deviation.

Here, we observe generally larger average savings from considering correlation. This is particularly apparent for the INC scenario as we create average savings as high as 2.54% even with a coefficient of variation of only 0.25. We note that the highest average savings across the table is 4.38%, though, which is only slightly higher than 4.20% in Table 8, but many more values in the tables have savings more than 1%. We again see savings increase when considering correlation, but not so clearly with increasing coefficient of variation. This seems to be a change created by the new objective. The dramatic increase in magnitude of savings when incorporating correlation with the INC scenarios is quite interesting and is one of the patterns we try to understand better in the next section.

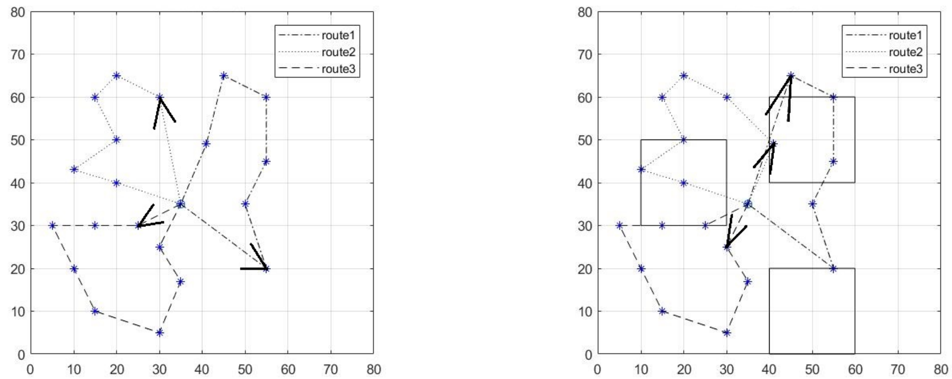
We can also observe differences in the components of the combined objective. In Table 9, we see that the solutions found with the combined objective sometimes have larger expected makespans than the zero correlation solutions (when evaluated with the same correlation matrix), but almost always have an equal or smaller standard deviation (only exceptions are 3 and 5 vehicles, coefficient of variation of 0.25, I/O 50).

Table 8: Summary Results for Minimizing Makespan

#veh	objective	base	I/O 50	I/O 75	I/O 90	INC 50	INC 75	INC 90
				stdv = 0.25 mean				
3	best sol makespan	125.49	124.66	124.19	123.87	126.49	126.84	127.33
	zero corr sol makespan	125.49	124.78	124.42	124.19	126.58	127.06	127.33
	% different to best sol	0.00%	-0.11%	-0.19%	-0.28%	-0.09%	-0.19%	0.00%
5	best sol makespan	100.52	98.81	97.77	97.05	101.67	102.02	102.20
	zero corr sol makespan	100.52	98.98	98.16	97.65	102.01	102.68	103.06
	% different to best sol	0.00%	-0.15%	-0.37%	-0.59%	-0.34%	-0.65%	-0.85%
				stdv = 0.5 mean				
3	best sol makespan	134.35	132.32	131.12	130.34	136.41	137.06	137.44
	zero corr sol makespan	134.35	132.70	131.82	131.26	136.82	137.89	138.49
	% different to best sol	0.00%	-0.30%	-0.55%	-0.73%	-0.31%	-0.62%	-0.78%
5	best sol makespan	110.60	106.99	104.78	102.95	112.82	113.53	113.92
	zero corr sol makespan	110.60	107.23	105.34	104.12	113.65	115.00	115.77
	% different to best sol	0.00%	-0.21%	-0.51%	-1.07%	-0.74%	-1.28%	-1.60%
				stdv = 0.75 mean				
3	best sol makespan	143.62	139.94	138.17	136.30	146.27	147.25	147.68
	zero corr sol makespan	143.62	141.02	139.61	138.72	150.14	152.90	154.45
	% different to best sol	0.00%	-0.73%	-1.01%	-1.69%	-2.44%	-3.53%	-4.20%
5	best sol makespan	121.03	115.39	111.94	109.32	124.73	126.00	126.22
	zero corr sol makespan	121.03	115.64	112.57	110.58	126.54	128.93	130.29
	% different to best sol	0.00%	-0.22%	-0.56%	-1.12%	-1.45%	-2.32%	-3.13%

Table 9: Summary Results for Minimizing Makespan plus Standard Deviation

#veh	objective	base	I/O 50	I/O 75	I/O 90	INC 50	INC 75	INC 90
				stdv = 0.25 mean				
3	best sol makespan	127.67	126.44	125.89	125.49	126.49	126.84	127.04
	best sol stdv	9.08	8.15	7.49	7.06	11.65	12.38	12.79
	best sol ms+stdv	136.75	134.59	133.38	132.55	138.14	139.22	139.83
	zero corr sol makespan	127.67	126.91	126.50	126.24	128.78	129.24	129.49
	zero corr sol stdv	9.08	8.01	7.81	7.52	12.17	13.42	14.12
	zero corr sol ms+stdv	136.75	134.93	134.32	133.77	140.95	142.66	143.60
	% different to best sol	0.00%	-0.24%	-0.71%	-0.94%	-1.98%	-2.35%	-2.54%
5	best sol makespan	102.39	100.81	99.93	99.33	103.62	103.92	104.11
	best sol stdv	8.06	6.88	6.15	5.37	8.91	9.31	9.53
	best sol ms+stdv	110.45	107.68	106.08	104.70	112.53	113.23	113.63
	zero corr sol makespan	102.39	100.87	100.12	99.68	103.93	104.62	105.02
	zero corr sol stdv	8.06	6.87	6.19	5.72	9.32	9.93	10.29
	zero corr sol ms+stdv	110.45	107.74	106.31	105.40	113.25	114.56	115.31
	% different to best sol	0.00%	-0.05%	-0.20%	-0.64%	-0.65%	-1.18%	-1.47%
				stdv = 0.5 mean				
3	best sol makespan	134.55	132.66	131.28	131.63	136.57	137.23	137.62
	best sol stdv	17.57	15.50	14.41	13.54	21.33	22.83	23.70
	best sol ms+stdv	152.12	148.16	145.69	145.18	157.90	160.06	161.32
	zero corr sol makespan	134.55	132.89	132.01	131.46	138.19	139.70	140.55
	zero corr sol stdv	17.57	16.10	15.31	14.80	23.49	25.93	27.28
	zero corr sol ms+stdv	152.12	149.00	147.32	146.27	161.67	165.63	167.83
	% different to best sol	0.00%	-0.58%	-1.12%	-0.68%	-2.20%	-3.17%	-3.66%
5	best sol makespan	110.70	106.92	105.02	103.20	112.84	113.67	114.01
	best sol stdv	14.62	12.17	10.57	9.28	16.54	17.22	17.76
	best sol ms+stdv	125.32	119.10	115.59	112.48	129.38	130.89	131.77
	zero corr sol makespan	110.70	107.29	105.38	104.15	113.97	115.44	116.28
	zero corr sol stdv	14.62	12.42	11.20	10.44	17.45	18.88	19.72
	zero corr sol ms+stdv	125.32	119.71	116.58	114.59	131.42	134.32	136.00
	% different to best sol	0.00%	-0.52%	-0.81%	-1.79%	-1.54%	-2.50%	-3.03%
				stdv = 0.75 mean				
3	best sol makespan	143.29	140.26	138.31	137.51	146.28	147.54	149.02
	best sol stdv	25.59	22.67	20.90	18.98	31.38	33.34	35.36
	best sol ms+stdv	168.88	162.93	159.21	156.49	177.66	180.88	184.38
	zero corr sol makespan	143.29	140.34	138.72	137.69	149.21	151.70	153.09
	zero corr sol stdv	25.59	22.96	21.52	20.59	34.37	37.98	39.99
	zero corr sol ms+stdv	168.88	163.29	160.24	158.29	183.58	189.68	193.08
	% different to best sol	0.00%	-0.20%	-0.59%	-1.03%	-3.04%	-4.38%	-4.12%
5	best sol makespan	121.19	115.47	112.33	110.28	124.68	125.70	126.27
	best sol stdv	21.43	18.07	15.36	12.55	24.44	25.53	26.18
	best sol ms+stdv	142.61	133.54	127.69	122.83	149.11	151.23	152.45
	zero corr sol makespan	121.19	116.03	113.11	111.20	126.31	128.61	129.92
	zero corr sol stdv	21.43	18.12	16.26	15.09	25.92	28.13	29.41
	zero corr sol ms+stdv	142.61	134.15	129.37	126.29	152.23	156.74	159.33
	% different to best sol	0.00%	-0.49%	-1.32%	-2.75%	-2.01%	-3.40%	-4.15%



(a) No Correlation (b) INC Scenario

Figure 5: Routes for R1, Makespan Objective

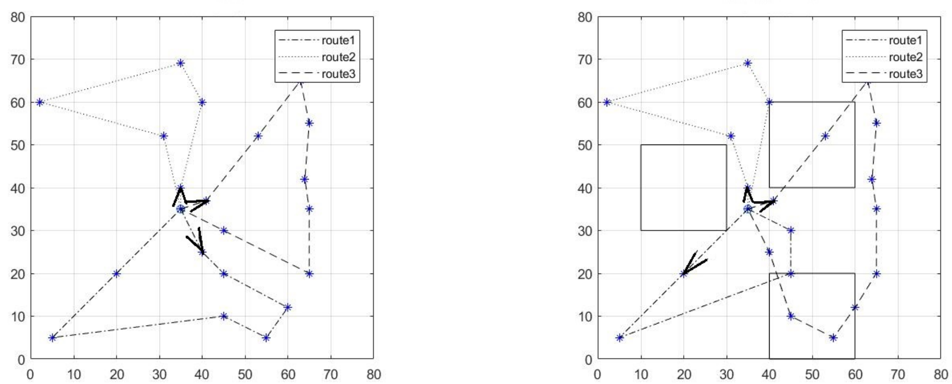
6.2.2 Illustrative Examples of Routes

To understand how correlation is causing these changes in solution values, we next examine a series of illustrative examples. We will look at examples of how the different objectives and correlation patterns change the design of the routes. We will use instances with 3 vehicles and a coefficient of variation of 75%.

We start with the makespan objective and see how an incident correlation pattern changes the resulting routes. For R1 with a maximum positive correlation of 75%, the solution found for the incident correlation instance has a 2.65% improvement over using the routes from the no correlation version. A comparison of the routes found without correlation and with incident correlation is in Figure 5. The two route plans are somewhat different: for instance, it can be observed that one of the customers at (42,48), who was assigned to route 1 with no correlation, has been reassigned to route 2 because of the fact that arcs coming to and leaving from that customer are positively correlated since they are starting and finishing in one of the incident locations.

For R2 with a maximum positive correlation of 90%, the solution found for the incident correlation instance has a 6.51% improvement over using the routes from the no correlation version. The routes found are in Figure 6. It is interesting to see that the solution with correlation here shows a crossing in the solution. The crossing is beneficial because the arcs are in the incident zone and become positively correlated as a result.

We can see the differences that inner/outer correlation can create for instance R3 (see Figure 7). For R3 with a maximum positive correlation of 90%, the solution found for the incident correlation instance has a 2.92% improvement over using the routes from the no



(a) No Correlation

(b) INC Scenario

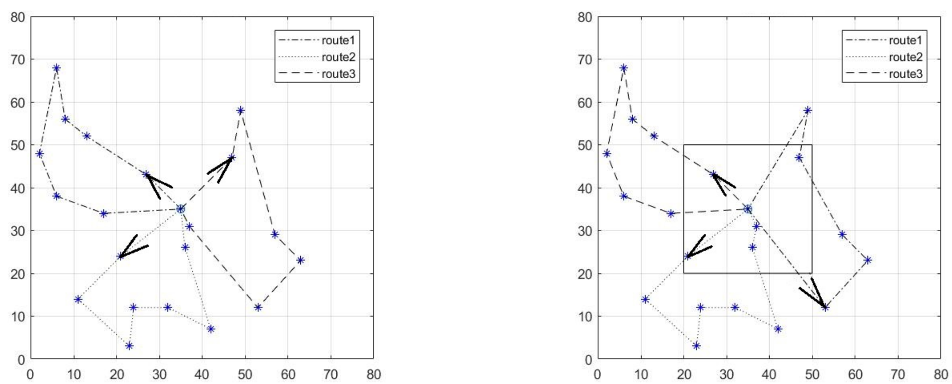
Figure 6: Routes for R2, Makespan Objective

correlation version. For the inner/outer case, the customer that is the closest one to the depot gets reassigned to the different route due to its location inside of the downtown box.

With standard deviation in the objective, we sometimes see similar changes as with only makespan, but at other times the routes change more. For R4, with a maximum positive correlation of 75%, the solution found for the incident correlation instance has a 5.73% improvement over using the routes from the no correlation version with just the makespan and 11.35% with makespan and standard deviation! The routes found with no correlation and the combined objective and the routes found with inner/outer correlation are in Figure 8. For this set of routes, the effect of the incident locations is the most visible. One can notice that all the routes look different in the base case and with the incident correlation. If we look at the top right incident location, we can see that route 1 changed such that the many customers present in route 1 in the base case are reassigned to a different route. We can also notice that those relocated customers were not in the incident location. This fact would suggest that correlation is important in this case and is the most influential. Here the moved customers are "close" to the customers in route 1 in the base case (which we typically want in a routing solution), but the impact of correlation is strong enough to put them on route 3 in the makespan plus standard deviation solution.

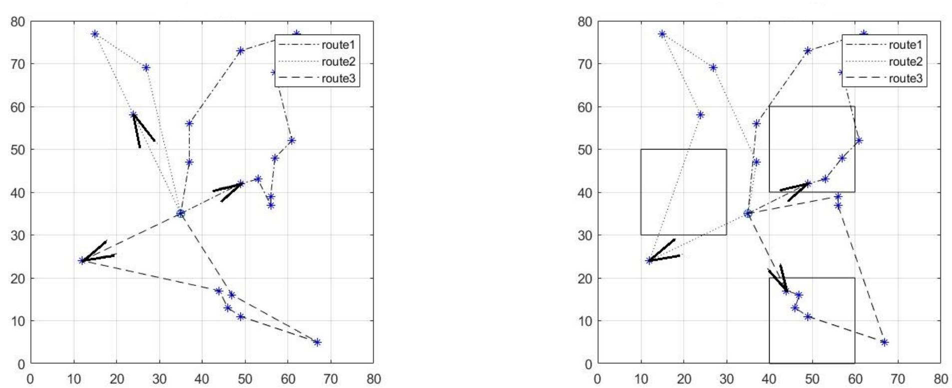
7 Summary and Future Work

Using ideas from extreme-value theory, we developed a method to incorporate arc travel time correlation in a vehicle routing problem with stochastic travel times. We integrated



(a) No Correlation (b) I/O Scenario

Figure 7: Routes for R3, Makespan Objective



(a) No Correlation (b) INC Scenario

Figure 8: Routes for R4, Makespan Plus Standard Deviation Objective

this approximation algorithm into a heuristic for the creation of routes for a fleet of vehicles. Using real world motivated scenarios, we explored the effect of the different correlation types/levels on the value of the objective function along with the set of resulting routes. We observed that correlation actually plays a significant role even for small instances with 20 customers and 3/5 vehicles. Taking correlation into account can result in an 11.35% decrease in the objective function value for these relatively small cases. These savings result from changes in the routes to take advantage of the impact of correlation that are counter-intuitive to typical route structures.

Future research directions include algorithmic and implementation improvements to decrease the computation time so that bigger datasets could be tested. Currently, we evaluate the full objective for every change in the routes, but we will may be able to use clever storage techniques to make this faster. We also want to explore routing with different travel time distributions since our approximation algorithm showed the makespan to be accurate for distributions other than normal.

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Modeling Correlation in Vehicle Routing Problems with Makespan Objectives and Stochastic Travel Times

May 2, 2018

1 Electronic Appendix

Table 1: Correlation Matrix 1

	1	2	3	4	5	6	7	8	9	10	11	12
1	1.0	+0.2	+0.2	+0.2	+0.2	+0.2	+0.2	+0.2	+0.2	+0.2	+0.2	+0.2
2	+0.2	1.0	+0.2	+0.2	+0.2	+0.2	+0.2	+0.2	+0.2	+0.2	+0.2	+0.2
3	+0.2	+0.2	1.0	+0.2	+0.2	+0.2	+0.2	+0.2	+0.2	+0.2	+0.2	+0.2
4	+0.2	+0.2	+0.2	1.0	+0.2	+0.2	+0.2	+0.2	+0.2	+0.2	+0.2	+0.2
5	+0.2	+0.2	+0.2	+0.2	1.0	+0.2	+0.2	+0.2	+0.2	+0.2	+0.2	+0.2
6	+0.2	+0.2	+0.2	+0.2	+0.2	1.0	+0.2	+0.2	+0.2	+0.2	+0.2	+0.2
7	+0.2	+0.2	+0.2	+0.2	+0.2	+0.2	1.0	+0.2	+0.2	+0.2	+0.2	+0.2
8	+0.2	+0.2	+0.2	+0.2	+0.2	+0.2	+0.2	1.0	+0.2	+0.2	+0.2	+0.2
9	+0.2	+0.2	+0.2	+0.2	+0.2	+0.2	+0.2	+0.2	1.0	+0.2	+0.2	+0.2
10	+0.2	+0.2	+0.2	+0.2	+0.2	+0.2	+0.2	+0.2	+0.2	1.0	+0.2	+0.2
11	+0.2	+0.2	+0.2	+0.2	+0.2	+0.2	+0.2	+0.2	+0.2	+0.2	1.0	+0.2
12	+0.2	+0.2	+0.2	+0.2	+0.2	+0.2	+0.2	+0.2	+0.2	+0.2	+0.2	1.0

Table 2: Correlation Matrix 2

	1	2	3	4	5	6	7	8	9	10	11	12
1	1.0	+0.2	-0.2	-0.2	-0.2	-0.2	+0.2	+0.2	+0.2	+0.2	-0.2	-0.2
2	+0.2	1.0	-0.2	-0.2	-0.2	-0.2	+0.2	+0.2	+0.2	+0.2	-0.2	-0.2
3	-0.2	-0.2	1.0	+0.2	+0.2	+0.2	-0.2	-0.2	-0.2	-0.2	+0.2	+0.2
4	-0.2	-0.2	+0.2	1.0	+0.2	+0.2	-0.2	-0.2	-0.2	-0.2	+0.2	+0.2
5	-0.2	-0.2	+0.2	+0.2	1.0	+0.2	-0.2	-0.2	-0.2	-0.2	+0.2	+0.2
6	-0.2	-0.2	+0.2	+0.2	+0.2	1.0	-0.2	-0.2	-0.2	-0.2	+0.2	+0.2
7	+0.2	+0.2	-0.2	-0.2	-0.2	-0.2	1.0	+0.2	+0.2	+0.2	-0.2	-0.2
8	+0.2	+0.2	-0.2	-0.2	-0.2	-0.2	+0.2	1.0	+0.2	+0.2	-0.2	-0.2
9	+0.2	+0.2	-0.2	-0.2	-0.2	-0.2	+0.2	+0.2	1.0	+0.2	-0.2	-0.2
10	+0.2	+0.2	-0.2	-0.2	-0.2	-0.2	+0.2	+0.2	+0.2	1.0	-0.2	-0.2
11	-0.2	-0.2	+0.2	+0.2	+0.2	+0.2	-0.2	-0.2	-0.2	-0.2	1.0	+0.2
12	-0.2	-0.2	+0.2	+0.2	+0.2	+0.2	-0.2	-0.2	-0.2	-0.2	+0.2	1.0

Table 3: Correlation Matrix 3

	1	2	3	4	5	6	7	8	9	10	11	12
1	1.0	+ 0.2	+ 0.2	+ 0.2	- 0.2	- 0.2	- 0.2	- 0.2	+ 0.2	+ 0.2	+ 0.2	+ 0.2
2	+ 0.2	1.0	+ 0.2	+ 0.2	- 0.2	- 0.2	- 0.2	- 0.2	+ 0.2	+ 0.2	+ 0.2	+ 0.2
3	+ 0.2	+ 0.2	1.0	+ 0.2	- 0.2	- 0.2	- 0.2	- 0.2	+ 0.2	+ 0.2	+ 0.2	+ 0.2
4	+ 0.2	+ 0.2	+ 0.2	1.0	- 0.2	- 0.2	- 0.2	- 0.2	+ 0.2	+ 0.2	+ 0.2	+ 0.2
5	- 0.2	- 0.2	- 0.2	- 0.2	1.0	+ 0.2	+ 0.2	+ 0.2	- 0.2	- 0.2	- 0.2	- 0.2
6	- 0.2	- 0.2	- 0.2	- 0.2	+ 0.2	1.0	+ 0.2	+ 0.2	- 0.2	- 0.2	- 0.2	- 0.2
7	- 0.2	- 0.2	- 0.2	- 0.2	+ 0.2	+ 0.2	1.0	+ 0.2	- 0.2	- 0.2	- 0.2	- 0.2
8	- 0.2	- 0.2	- 0.2	- 0.2	+ 0.2	+ 0.2	+ 0.2	1.0	- 0.2	- 0.2	- 0.2	- 0.2
9	+ 0.2	+ 0.2	+ 0.2	+ 0.2	- 0.2	- 0.2	- 0.2	- 0.2	1.0	+ 0.2	+ 0.2	+ 0.2
10	+ 0.2	+ 0.2	+ 0.2	+ 0.2	- 0.2	- 0.2	- 0.2	- 0.2	+ 0.2	1.0	+ 0.2	+ 0.2
11	+ 0.2	+ 0.2	+ 0.2	+ 0.2	- 0.2	- 0.2	- 0.2	- 0.2	+ 0.2	+ 0.2	1.0	+ 0.2
12	+ 0.2	+ 0.2	+ 0.2	+ 0.2	- 0.2	- 0.2	- 0.2	- 0.2	+ 0.2	+ 0.2	+ 0.2	1.0

Table 4: Minimizing Makespan with stdv = 0.25 * mean

3 veh	objective	base	I/O 50	I/O 75	I/O 90	INC 50	INC 75	INC 90
1	best solution	113.03	112.23	111.81	111.57	114.76	115.18	116.56
	zero correlation	113.03	112.23	111.82	111.57	115.16	116.05	116.56
	% different to best sol	0.00%	0.00%	-0.01%	0.00%	-0.34%	-0.76%	0.00%
2	best solution	138.78	137.93	137.49	137.21	138.96	139.04	139.09
	zero correlation	138.78	137.93	137.49	137.21	138.96	139.04	139.09
	% different to best sol	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
3	best solution	117.24	116.07	115.32	114.72	118.97	119.72	120.14
	zero correlation	117.24	116.57	116.22	115.99	118.97	119.73	120.16
	% different to best sol	0.00%	-0.43%	-0.77%	-1.10%	0.00%	-0.01%	-0.01%
4	best solution	132.90	132.40	132.15	132.00	133.25	133.43	133.53
	zero correlation	132.90	132.40	132.15	132.00	133.25	133.43	133.53
	% different to best sol	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
5 veh	**objective**	**base**	**I/O 50**	**I/O 75**	**I/O 90**	**INC 50**	**INC 75**	**INC 90**
1	best solution	89.28	87.92	87.22	86.88	91.52	91.94	92.13
	zero correlation	89.28	87.94	87.27	86.88	92.11	93.29	93.95
	% different to best sol	0.00%	-0.03%	-0.06%	0.00%	-0.64%	-1.44%	-1.93%
2	best solution	105.22	103.48	102.67	102.07	105.99	106.36	106.57
	zero correlation	105.22	103.70	103.01	102.66	106.01	106.40	106.62
	% different to best sol	0.00%	-0.21%	-0.34%	-0.58%	-0.03%	-0.04%	-0.05%
3	best solution	104.51	103.59	94.63	93.60	98.58	98.96	99.19
	zero correlation	104.51	103.70	103.31	103.09	104.80	104.95	105.04
	% different to best sol	0.00%	-0.11%	-8.40%	-9.21%	-5.93%	-5.71%	-5.58%
4	best solution	109.82	107.85	106.58	105.66	110.60	110.82	110.92
	zero correlation	109.82	108.26	107.37	106.79	111.36	112.08	112.50
	% different to best sol	0.00%	-0.38%	-0.74%	-1.05%	-0.68%	-1.13%	-1.41%

Table 5: Minimizing Makespan with stdv = 0.5 * mean

3 veh	**objective**	**base**	**I/O 50**	**I/O 75**	**I/O 90**	**INC 50**	**INC 75**	**INC 90**
1	best solution	120.09	118.03	116.90	116.17	123.57	124.40	124.87
	zero correlation	120.09	118.03	116.90	116.17	124.68	126.57	127.63
	% different to best sol	0.00%	0.00%	0.00%	0.00%	-0.89%	-1.72%	-2.16%
2	best solution	149.09	147.17	146.13	145.49	149.30	149.05	148.85
	zero correlation	149.09	147.33	146.39	145.81	149.80	150.11	150.29
	% different to best sol	0.00%	-0.11%	-0.18%	-0.22%	-0.34%	-0.71%	-0.96%
3	best solution	126.04	123.43	121.80	120.63	129.50	130.97	131.90
	zero correlation	126.04	124.65	123.90	123.44	129.50	131.03	131.90
	% different to best sol	0.00%	-0.98%	-1.69%	-2.27%	0.00%	-0.04%	0.00%
4	best solution	142.19	140.64	139.64	139.05	143.29	143.83	144.15
	zero correlation	142.19	140.80	140.08	139.64	143.29	143.83	144.15
	% different to best sol	0.00%	-0.11%	-0.31%	-0.42%	0.00%	0.00%	0.00%
5 veh	**objective**	**base**	**I/O 50**	**I/O 75**	**I/O 90**	**INC 50**	**INC 75**	**INC 90**
1	best solution	99.33	96.83	95.12	94.35	102.60	103.38	103.70
	zero correlation	99.33	96.93	95.61	94.79	104.09	106.05	107.13
	% different to best sol	0.00%	-0.10%	-0.52%	-0.46%	-1.44%	-2.51%	-3.20%
2	best solution	115.91	111.24	108.66	106.73	117.59	118.37	118.84
	zero correlation	115.91	111.29	108.73	107.15	117.59	118.38	118.83
	% different to best sol	0.00%	-0.04%	-0.07%	-0.40%	0.00%	0.00%	0.01%
3	best solution	107.76	103.97	101.74	99.64	109.47	110.25	110.71
	zero correlation	107.76	103.98	101.74	100.22	109.61	110.49	111.02
	% different to best sol	0.00%	-0.01%	0.00%	-0.58%	-0.12%	-0.22%	-0.28%
4	best solution	119.40	115.92	113.58	111.09	121.61	122.12	122.42
	zero correlation	119.40	116.73	115.27	114.33	123.33	125.09	126.10
	% different to best sol	0.00%	-0.70%	-1.46%	-2.83%	-1.39%	-2.38%	-2.92%

Table 6: Minimizing Makespan with stdv = 0.75 * mean

3 veh	objective	base	I/O 50	I/O 75	I/O 90	INC 50	INC 75	INC 90
1	best sol makespan	127.86	124.62	122.79	121.61	132.72	133.94	134.64
	zero corr sol makespan	127.86	124.62	122.79	121.61	134.74	137.58	139.17
	% difference to best sol	0.00%	0.00%	0.00%	0.00%	-1.50%	-2.65%	-3.25%
2	best sol makespan	159.71	154.74	153.70	151.30	158.02	157.70	157.49
	zero corr sol makespan	159.71	157.39	156.17	155.42	164.88	167.16	168.45
	% difference to best sol	0.00%	-1.69%	-1.59%	-2.65%	-4.16%	-5.66%	-6.51%
3	best sol makespan	134.94	130.98	128.58	126.68	140.09	142.31	143.10
	zero corr sol makespan	134.94	132.57	131.29	130.49	140.12	142.41	143.71
	% difference to best sol	0.00%	-1.20%	-2.06%	-2.92%	-0.02%	-0.07%	-0.43%
4	best sol makespan	151.98	149.42	147.62	145.61	154.26	155.04	155.48
	zero corr sol makespan	151.98	149.48	148.17	147.37	160.83	164.46	166.48
	% difference to best sol	0.00%	-0.04%	-0.38%	-1.19%	-4.09%	-5.73%	-6.61%
5 veh	**objective**	**base**	**I/O 50**	**I/O 75**	**I/O 90**	**INC 50**	**INC 75**	**INC 90**
1	best solution	108.12	104.55	102.60	100.63	114.03	114.42	114.90
	zero correlation	108.12	104.64	102.78	101.61	118.24	122.30	124.54
	% different to best sol	0.00%	-0.09%	-0.17%	-0.97%	-3.55%	-6.44%	-7.74%
2	best solution	128.20	120.55	115.93	113.07	130.63	131.78	132.46
	zero correlation	128.20	120.59	116.18	113.28	130.63	131.78	132.46
	% different to best sol	0.00%	-0.04%	-0.21%	-0.18%	0.00%	0.00%	0.00%
3	best solution	118.15	112.22	108.44	106.28	120.93	122.11	122.82
	zero correlation	118.15	112.80	109.74	107.72	121.15	122.61	123.47
	% different to best sol	0.00%	-0.52%	-1.18%	-1.34%	-0.19%	-0.41%	-0.53%
4	best solution	129.67	124.23	120.78	117.31	133.32	135.67	134.70
	zero correlation	129.67	124.51	121.61	119.70	136.14	139.04	140.69
	% different to best sol	0.00%	-0.22%	-0.68%	-2.00%	-2.08%	-2.42%	-4.26%

Table 7: Minimizing Makespan plus Standard Deviation with stdv = 0.25 * mean

3 veh	objective	base	I/O 50	I/O 75	I/O 90	INC 50	INC 75	INC 90
1	best sol makespan	119.55	118.37	117.70	117.27	114.76	115.18	115.41
	best sol stdv	7.82	6.64	5.95	5.49	10.46	11.27	11.73
	best sol ms+stdv	127.37	125.01	123.65	122.76	125.23	126.44	127.14
	zero corr sol makespan	119.55	118.37	117.70	117.27	119.30	119.17	119.08
	zero corr sol stdv	7.82	6.64	5.95	5.49	10.82	12.02	12.67
	zero corr sol ms+stdv	127.37	125.01	123.65	122.76	130.12	131.18	131.75
	% different to best sol	0.00%	0.00%	0.00%	0.00%	-3.76%	-3.61%	-3.50%
2	best sol makespan	138.88	137.93	137.57	137.34	138.96	139.04	139.09
	best sol stdv	9.66	8.98	8.45	8.11	12.42	13.54	14.17
	best sol ms+stdv	148.54	146.91	146.02	145.45	151.39	152.59	153.26
	zero corr sol makespan	138.88	138.11	137.71	137.46	139.06	139.14	139.17
	zero corr sol stdv	9.66	8.85	8.42	8.14	12.33	13.45	14.09
	zero corr sol ms+stdv	148.54	146.97	146.13	145.60	151.39	152.59	153.26
	% different to best sol	0.00%	-0.04%	-0.07%	-0.11%	0.00%	0.00%	0.00%
3	best sol makespan	117.28	116.07	115.38	114.74	118.98	119.73	120.14
	best sol stdv	8.17	6.86	5.86	5.18	10.47	11.46	12.01
	best sol ms+stdv	125.45	122.93	121.24	119.92	129.46	131.18	132.16
	zero corr sol makespan	117.28	116.60	116.24	116.01	118.98	119.73	120.14
	zero corr sol stdv	8.17	6.55	7.23	7.03	10.47	11.46	12.01
	zero corr sol ms+stdv	125.45	123.15	123.47	123.04	129.46	131.18	132.16
	% different to best sol	0.00%	-0.18%	-1.80%	-2.53%	0.00%	0.00%	0.00%
4	best sol makespan	134.96	133.37	132.89	132.59	133.25	133.43	133.53
	best sol stdv	10.67	10.12	9.71	9.46	13.24	13.24	13.24
	best sol ms+stdv	145.64	143.49	142.60	142.06	146.49	146.67	146.78
	zero corr sol makespan	134.96	134.56	134.36	134.24	137.77	138.92	139.56
	zero corr sol stdv	10.67	10.01	9.65	9.43	15.05	16.75	17.69
	zero corr sol ms+stdv	145.64	144.57	144.01	143.67	152.82	155.67	157.24
	% different to best sol	0.00%	-0.75%	-0.98%	-1.12%	-4.14%	-5.79%	-6.66%
5 veh	**objective**	**base**	**I/O 50**	**I/O 75**	**I/O 90**	**INC 50**	**INC 75**	**INC 90**
1	best sol makespan	89.28	87.93	87.28	86.89	91.62	91.95	92.13
	best sol stdv	5.99	5.44	5.21	5.13	8.15	8.79	9.15
	best sol ms+stdv	95.27	93.37	92.49	92.02	99.77	100.74	101.29
	zero corr sol makespan	89.28	87.95	87.28	86.89	92.11	93.29	93.95
	zero corr sol stdv	5.99	5.42	5.21	5.13	8.90	9.99	10.58
	zero corr sol ms+stdv	95.27	93.37	92.49	92.02	101.01	103.28	104.53
	% different to best sol	0.00%	0.00%	0.00%	0.00%	-1.23%	-2.46%	-3.10%
2	best sol makespan	105.23	103.72	102.78	102.26	106.03	106.41	106.63
	best sol stdv	8.34	7.17	6.57	5.69	9.05	9.43	9.65
	best sol ms+stdv	113.57	110.89	109.36	107.95	115.08	115.84	116.29
	zero corr sol makespan	105.23	103.73	103.05	102.70	106.03	106.41	106.63
	zero corr sol stdv	8.34	7.16	6.51	6.06	9.05	9.43	9.65
	zero corr sol ms+stdv	113.57	110.89	109.56	108.76	115.08	115.84	116.29
	% different to best sol	0.00%	-0.01%	-0.18%	-0.75%	0.00%	0.00%	0.00%
3	best sol makespan	105.23	103.72	102.78	102.26	106.03	106.41	106.63
	best sol stdv	8.34	7.17	6.57	5.69	9.05	9.43	9.65
	best sol ms+stdv	113.57	110.89	109.36	107.95	115.08	115.84	116.29
	zero corr sol makespan	105.23	103.73	103.05	102.70	106.03	106.41	106.63
	zero corr sol stdv	8.34	7.16	6.51	6.06	9.05	9.43	9.65
	zero corr sol ms+stdv	113.57	110.89	109.56	108.76	115.08	115.84	116.29
	% different to best sol	0.00%	-0.01%	-0.18%	-0.75%	0.00%	0.00%	0.00%
4	best sol makespan	109.83	107.86	106.88	105.89	110.80	110.92	111.03
	best sol stdv	9.58	7.74	6.24	4.97	9.40	9.58	9.64
	best sol ms+stdv	119.41	115.60	113.13	110.87	120.19	120.50	120.67
	zero corr sol makespan	109.83	108.08	107.09	106.45	111.55	112.39	112.87
	zero corr sol stdv	9.58	7.74	6.53	5.62	10.29	10.88	11.27
	zero corr sol ms+stdv	119.41	115.82	113.62	112.07	121.84	123.27	124.14
	% different to best sol	0.00%	-0.19%	-0.44%	-1.08%	-1.35%	-2.24%	-2.80%

Table 8: Minimizing Makespan plus Standard Deviation with stdv = 0.5 * mean

3 veh	objective	base	I/O 50	I/O 75	I/O 90	INC 50	INC 75	INC 90
1	best sol makespan	120.09	118.05	116.90	116.99	123.57	124.40	124.87
	best sol stdv	15.19	13.17	12.03	10.42	20.04	21.83	22.85
	best sol ms+stdv	135.28	131.22	128.93	127.41	143.61	146.23	147.72
	zero corr sol makespan	120.09	118.05	116.92	116.20	124.68	126.57	127.63
	zero corr sol stdv	15.19	13.17	12.01	11.24	21.19	23.63	24.98
	zero corr sol ms+stdv	135.28	131.22	128.93	127.44	145.87	150.20	152.61
	% different to best sol	0.00%	0.00%	-0.01%	-0.02%	-1.55%	-2.64%	-3.20%
2	best sol makespan	149.13	147.44	146.21	147.33	149.30	149.11	148.94
	best sol stdv	19.11	17.57	16.75	13.50	23.23	25.28	26.46
	best sol ms+stdv	168.24	165.01	162.96	160.82	172.53	174.39	175.40
	zero corr sol makespan	149.13	147.48	146.60	146.05	149.80	150.08	150.23
	zero corr sol stdv	19.11	17.57	16.74	16.23	24.15	26.31	27.53
	zero corr sol ms+stdv	168.24	165.05	163.34	162.28	173.94	176.39	177.77
	% different to best sol	0.00%	-0.02%	-0.23%	-0.90%	-0.81%	-1.13%	-1.33%
3	best sol makespan	126.09	123.60	121.84	123.17	129.50	130.97	131.90
	best sol stdv	15.94	13.31	11.84	13.75	20.45	22.38	23.50
	best sol ms+stdv	142.03	136.91	133.67	136.92	149.94	153.35	155.40
	zero corr sol makespan	126.09	124.53	123.69	123.17	129.50	130.97	131.81
	zero corr sol stdv	15.94	14.76	14.14	13.75	20.45	22.38	23.46
	zero corr sol ms+stdv	142.03	139.29	137.83	136.92	149.94	153.35	155.27
	% different to best sol	0.00%	-1.71%	-3.02%	0.00%	0.00%	0.00%	0.08%
4	best sol makespan	142.89	141.57	140.18	139.05	143.92	144.45	144.77
	best sol stdv	20.04	17.93	17.03	16.51	21.60	21.83	21.98
	best sol ms+stdv	162.92	159.50	157.21	155.56	165.52	166.28	166.75
	zero corr sol makespan	142.89	141.51	140.84	140.45	148.78	151.19	152.52
	zero corr sol stdv	20.04	18.91	18.34	17.99	28.16	31.38	33.15
	zero corr sol ms+stdv	162.92	160.43	159.18	158.44	176.94	182.57	185.67
	% different to best sol	0.00%	-0.58%	-1.24%	-1.82%	-6.46%	-8.92%	-10.19%
5 veh	**objective**	**base**	**I/O 50**	**I/O 75**	**I/O 90**	**INC 50**	**INC 75**	**INC 90**
1	best sol makespan	99.33	96.36	95.43	93.44	102.60	103.71	103.98
	best sol stdv	12.32	10.35	10.02	9.44	15.88	16.98	17.77
	best sol ms+stdv	111.65	106.71	105.45	102.88	118.48	120.69	121.75
	zero corr sol makespan	99.33	96.75	95.35	94.47	104.09	106.05	107.13
	zero corr sol stdv	12.32	10.98	10.24	9.76	18.10	20.57	21.95
	zero corr sol ms+stdv	111.65	107.73	105.59	104.23	122.19	126.62	129.08
	% different to best sol	0.00%	-0.94%	-0.13%	-1.29%	-3.04%	-4.69%	-5.67%
2	best sol makespan	115.91	111.29	108.99	107.04	117.59	118.47	118.89
	best sol stdv	15.42	12.66	10.93	9.99	16.74	17.35	17.74
	best sol ms+stdv	131.34	123.94	119.91	117.03	134.33	135.81	136.63
	zero corr sol makespan	115.91	111.43	108.94	107.38	117.59	118.38	118.84
	zero corr sol stdv	15.42	12.54	11.02	10.15	16.74	17.46	17.89
	zero corr sol ms+stdv	131.34	123.97	119.96	117.52	134.33	135.83	136.73
	% different to best sol	0.00%	-0.02%	-0.04%	-0.42%	0.00%	-0.01%	-0.07%
3	best sol makespan	107.72	103.86	101.64	100.39	109.44	110.26	110.71
	best sol stdv	13.66	11.49	9.95	8.52	15.13	15.86	16.30
	best sol ms+stdv	121.39	115.35	111.59	108.92	124.57	126.12	127.01
	zero corr sol makespan	107.72	104.45	102.58	101.36	109.53	110.42	110.94
	zero corr sol stdv	13.66	11.51	10.28	9.48	15.36	16.19	16.68
	zero corr sol ms+stdv	121.39	115.96	112.87	110.84	124.89	126.60	127.61
	% different to best sol	0.00%	-0.53%	-1.13%	-1.74%	-0.26%	-0.38%	-0.48%
4	best sol makespan	119.83	116.18	114.03	111.91	121.73	122.25	122.48
	best sol stdv	17.07	14.20	11.39	9.17	18.40	18.70	19.21
	best sol ms+stdv	136.90	130.39	125.42	121.08	140.13	140.96	141.69
	zero corr sol makespan	119.83	116.51	114.64	113.41	124.68	126.93	128.23
	zero corr sol stdv	17.07	14.66	13.27	12.35	19.62	21.31	22.36
	zero corr sol ms+stdv	136.90	131.18	127.90	125.77	144.29	148.24	150.59
	% different to best sol	0.00%	-0.60%	-1.94%	-3.73%	-2.88%	-4.91%	-5.91%

Table 9: Minimizing Makespan plus Standard Deviation with stdv = 0.75 * mean

3 veh	objective	base	I/O 50	I/O 75	I/O 90	INC 50	INC 75	INC 90
1	best sol makespan	127.86	124.62	122.79	122.45	132.72	133.94	139.17
	best sol stdv	22.24	19.32	17.69	15.47	29.75	32.53	36.91
	best sol ms+stdv	150.11	143.94	140.48	137.92	162.47	166.47	176.08
	zero corr sol makespan	127.86	124.62	122.79	121.61	134.74	137.58	139.17
	zero corr sol stdv	22.24	19.32	17.69	16.63	31.20	34.87	36.91
	zero corr sol ms+stdv	150.11	143.94	140.48	138.24	165.94	172.45	176.08
	% different to best sol	0.00%	0.00%	0.00%	-0.23%	-2.09%	-3.47%	0.00%
2	best sol makespan	157.97	155.24	153.84	153.29	158.02	157.78	157.63
	best sol stdv	27.63	25.36	23.78	21.25	34.19	37.09	38.72
	best sol ms+stdv	185.60	180.60	177.61	174.53	192.21	194.87	196.35
	zero corr sol makespan	157.97	155.92	154.88	154.25	160.36	161.48	162.13
	zero corr sol stdv	27.63	25.88	25.03	24.54	34.93	37.96	39.65
	zero corr sol ms+stdv	185.60	181.80	179.91	178.79	195.28	199.44	201.77
	% different to best sol	0.00%	-0.66%	-1.28%	-2.38%	-1.58%	-2.29%	-2.69%
3	best sol makespan	135.16	131.05	128.54	127.40	140.09	143.25	143.54
	best sol stdv	23.53	19.90	17.60	16.09	30.44	32.04	33.75
	best sol ms+stdv	158.69	150.95	146.14	143.49	170.53	175.29	177.29
	zero corr sol makespan	135.16	131.19	128.91	127.40	140.42	142.69	143.98
	zero corr sol stdv	23.53	19.78	17.58	16.09	30.38	33.31	34.95
	zero corr sol ms+stdv	158.69	150.98	146.49	143.49	170.80	176.00	178.93
	% different to best sol	0.00%	-0.02%	-0.24%	0.00%	-0.16%	-0.40%	-0.92%
4	best sol makespan	152.16	150.12	148.08	146.89	154.28	155.20	155.74
	best sol stdv	28.95	26.09	24.53	23.10	31.14	31.71	32.06
	best sol ms+stdv	181.12	176.22	172.60	169.99	185.42	186.91	187.80
	zero corr sol makespan	152.16	149.60	148.30	147.52	161.33	165.05	167.11
	zero corr sol stdv	28.95	26.85	25.77	25.11	40.98	45.79	48.44
	zero corr sol ms+stdv	181.12	176.45	174.07	172.63	202.31	210.84	215.54
	% different to best sol	0.00%	-0.13%	-0.84%	-1.53%	-8.35%	-11.35%	-12.87%
5 veh	**objective**	**base**	**I/O 50**	**I/O 75**	**I/O 90**	**INC 50**	**INC 75**	**INC 90**
1	best sol makespan	108.60	104.67	102.28	102.01	113.86	114.55	114.97
	best sol stdv	18.10	15.70	14.29	10.86	23.62	25.55	26.62
	best sol ms+stdv	126.70	120.37	116.57	112.88	137.48	140.11	141.59
	zero corr sol makespan	108.60	105.66	104.10	103.13	115.72	118.65	120.27
	zero corr sol stdv	18.10	16.28	15.32	14.74	27.34	31.19	33.31
	zero corr sol ms+stdv	126.70	121.94	119.42	117.87	143.07	149.84	153.58
	% different to best sol	0.00%	-1.28%	-2.39%	-4.24%	-3.90%	-6.49%	-7.81%
2	best sol makespan	128.20	120.55	116.09	113.40	130.63	131.85	132.47
	best sol stdv	22.82	18.42	16.01	14.30	24.65	25.50	26.07
	best sol ms+stdv	151.02	138.96	132.10	127.70	155.28	157.35	158.55
	zero corr sol makespan	128.20	120.85	116.59	113.78	130.63	131.78	132.46
	zero corr sol stdv	22.82	18.17	15.64	14.14	24.65	25.66	26.29
	zero corr sol ms+stdv	151.02	139.02	132.23	127.92	155.28	157.45	158.74
	% different to best sol	0.00%	-0.04%	-0.09%	-0.17%	0.00%	-0.06%	-0.12%
3	best sol makespan	118.09	112.35	108.94	107.63	120.90	122.23	122.91
	best sol stdv	19.91	16.85	14.64	11.06	22.28	23.31	23.94
	best sol ms+stdv	138.00	129.20	123.59	118.70	143.19	145.54	146.85
	zero corr sol makespan	118.09	112.96	110.02	108.08	120.90	122.28	123.10
	zero corr sol stdv	19.91	16.81	15.03	13.88	22.28	23.46	24.16
	zero corr sol ms+stdv	138.00	129.76	125.05	121.95	143.19	145.74	147.25
	% different to best sol	0.00%	-0.43%	-1.17%	-2.67%	0.00%	-0.14%	-0.27%
4	best sol makespan	129.85	124.29	121.99	118.07	133.32	134.18	134.70
	best sol stdv	24.88	21.32	16.48	13.96	27.19	27.75	28.09
	best sol ms+stdv	154.73	145.61	138.48	132.03	160.51	161.93	162.79
	zero corr sol makespan	129.85	124.66	121.73	119.80	137.98	141.72	143.86
	zero corr sol stdv	24.88	21.23	19.06	17.61	29.42	32.20	33.88
	zero corr sol ms+stdv	154.73	145.89	140.79	137.42	167.40	173.92	177.74
	% different to best sol	0.00%	-0.19%	-1.64%	-3.92%	-4.12%	-6.89%	-8.41%

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